Physics fundamentals

Jan Helm
Technical University Berlin
Email: jan.helm@alumni.tu-berlin.de

Abstract
We present here in a concise schematic form the basic theories and models in theoretical physics from a mathematical point of view, with two extensions
-Ashtekar-Kodama (AK) gravity, an extension of Quantum Loop Gravity, in its classical regime it contains General Relativity in the limit $\Lambda \rightarrow 0$ for the cosmological constant, in its quantum regime it is a complete renormalizable quantum field theory with the Lie(ε) Lie-group with 4 generators as symmetry group
-an extension of the Standard Model of particle physics: SU(4)-preon model (SU4PM), with hyper-color (hc) interaction based on SU(4), where the conventional weak force is its Yukawa-interaction mediated by massive Z- and W-bosons, and all massive particles are built from preons *q* and *r* .
The presentation consists of six chapters:
- basic principles: minimum principles, classical and quantum mechanics, basic algebraic structure, complex functions and differential equations, geometry: metric and gravity, geometric symmetry group
- interactions: long-range (classical) interactions (AK-gravity, electrodynamics), quantum interactions (AK-QFT, QED, QCD, hc-SU4)
-statistical mechanics (thermodynamics)
- models: model of macrocosm (LambdaCDM), model of microcosm(extended SM)
-interactions and symmetry groups
-problems solved by the model extensions
In chapter 0 we present a compressed version in form of flowcharts

Contents

0 Flowcharts

Minimum principles classical: path $q^\mu(\tau)$

action relativistic $S[q^\mu]=\int L\left(q^\mu,\frac{dq^\mu}{d\tau},\tau\right)d^3x\,dt=\min$, i.e. variation $\delta S=0$, $\tau=t\sqrt{1-v^2/c^2}$ proper time

relativistic Euler-Lagrange equations: $0=\frac{\delta S}{\delta q}=-\frac{d}{d\tau}\left(\frac{\partial L}{\partial\dot{q}^\mu}\right)+\frac{\partial L}{\partial q^\mu}$ $\dot{q}^\mu=\frac{dq^\mu}{d\tau}$

Hamiltonian density $H\left(q^\mu,\frac{dq^\mu}{d\tau},\tau\right)=\frac{dq^\mu}{d\tau}\pi_\mu-L\left(q^\mu,\frac{dq^\mu}{d\tau},\tau\right)$ where $\pi_\mu=\frac{\partial L}{\partial\left(\frac{\partial q^\mu}{\partial\tau}\right)}$ conj. momentum

Hamiltonian field equations $\frac{dq^\mu}{d\tau}=\frac{\partial H}{\partial\pi_\mu}$ $\frac{d\pi_\mu}{d\tau}=-\frac{\partial H}{\partial q^\mu}$, non-relativistic $\tau=t$

Hamiltonian is $H_0=\int H\,d^3x$ and Lagrangian $L_0=\int L\,d^3x$

Classical relativistic and Newtonian mechanics

relativistic energy: $E_L=\frac{mc^2}{\sqrt{1-v^2}}$, in general $E=\frac{mc^2}{dt/d\tau}$, $\tau=t\sqrt{1-v^2/c^2}$ proper time

Newtonian approximation $v<<1$ $E_{kin}=\frac{mv^2}{2}$

Lagrangian relativistic $L_L=-mc^2\frac{d\tau}{dt}-V(x(t))=-mc^2\sqrt{1-\left(\frac{dx(t)}{cdt}\right)^2}-V(x(t))$

Lagrangian Newtonian $L_N=\frac{m\left(\frac{dx(t)}{dt}\right)^2}{2}-V(x(t))$, eom Newton equation $m\ddot{\vec{x}}(t)=\vec{F}=-\nabla V(\vec{x})$

fully relativistic gravitational force equation $\frac{d}{dt}\left(\frac{m\dot{\vec{r}}}{\sqrt{1-\frac{v^2}{c^2}}^{2}}\right)=-GMm\frac{\vec{r}}{r^3}\frac{1}{\left(1-\frac{v^2}{c^2}\right)}$

gravitational potential correction $\delta V=V\frac{v^2}{c^2}=-\frac{GMm}{r}\frac{(L/m)^2}{c^2r^2}$ the same as GR correction

relativistic Navier –Stokes equations , Euler-equations $\mu=0$

$\left(\gamma\rho c^2+p\right)u^\mu\partial_\mu u^\nu=-\partial^\mu p+\mu\partial_\nu\partial^\nu u^\mu+f^\mu$

ordinary Navier –Stokes $\rho\left(\partial_t+\vec{v}\cdot\nabla\right)\vec{v}=-\nabla p+\mu\nabla^2\vec{v}$

relativistic Hooke's equation in displacement u^μ , stress potential σ , stiffness tensor C

$\frac{d}{d\tau}mv_\mu-\partial_\mu\sigma=K_\mu$, where $\vec{K}=\frac{\vec{F}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$,

$\frac{d}{dt}\frac{m\vec{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}-\nabla\sigma=\vec{F}$ $\sigma=C_{\mu\nu}\varepsilon^{\mu\nu}=C_{\mu\nu}\frac{1}{2}\left(\partial_\mu u^\nu+\partial_\nu u^\mu\right)$

ordinary general Hooke's law $\rho\ddot{\vec{u}}-\nabla\sigma=\vec{F}$ $\sigma=C:\varepsilon$ $\varepsilon=\frac{1}{2}\left(\nabla\circ\vec{u}+\left(\nabla\circ\vec{u}\right)^T\right)$

General relativity (GR) is based on two principles

-General covariance: GR equations are **invariant under arbitrary (analytic) coordinate transformations**

Therefore GR is the *differential geometry of the metric* in the sense of Gauss and Riemann, the *GR-equations-of-motion* are the Gaussian *geodesic equations* of the shortest connection $\frac{d^2x^\kappa}{d\tau^2} = -\Gamma^\kappa{}_{\mu\nu} \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$.

-GR metric equations are derived from the action minimum principle with the GR invariant curvature *R*, and this GR **invariant curvature and energy density are equivalent**

The simplest possible Lagrangian density is Einstein-Hilbert-Lagrangian density $\frac{R-2\Lambda}{2\kappa}$

The complete GR Lagrangian density has the form $\frac{R-2\Lambda}{2\kappa} + L_M$, L_M = mass-energy density

with the GR-action $S = \int \left(\frac{R-2\Lambda}{2\kappa} + L_M \right) \sqrt{-g}\, d^4x$, and the resulting variational equations are

$$\delta S = 0 \ , \ \frac{1}{2\kappa}\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{2\kappa}\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} - \frac{\Lambda}{\kappa}\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} + \frac{\delta L_M}{\delta g^{\mu\nu}} + \frac{L_M}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = 0$$

which give the Einstein equations

$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$, with $\kappa = \frac{8\pi G}{c^4}$

with Minkowski metric $g_{\mu\nu} = diag(+1,-1,-1,-1)$

$T_{\mu\nu} = L_M g_{\mu\nu} - 2\frac{\delta L_M}{\delta g^{\mu\nu}}$ is energy-momentum tensor

Quantum mechanics

The energy-momentum $p^\mu = \left(\frac{E}{c}, \vec{p}\right)$ 4-vector with energy relation $p^\mu p_\mu = \left(\frac{E}{c}\right)^2 - \vec{p}^2 = m^2c^2$

$E = \frac{mc^2}{\sqrt{1-(v/c)^2}}$, Newtonian limit $E \approx mc^2 + E_{kin}$ $E_{kin} = \frac{mv^2}{2}$

classical 4-momentum $p_\mu = (\frac{E}{c}, \vec{p}) \rightarrow$ quantum momentum operator $p_\mu = i\hbar(\frac{1}{c}\partial_t, -\vec{\nabla}) = i\hbar\partial_\mu$

Heisenberg commutator $[x,p] = i\hbar$ $\rightarrow$ uncertainty relation $\Delta x \Delta p \geq \frac{\hbar}{2}$

quantum bound systems: discrete energy E_i = eigenvalues of hamiltonian operator $H\psi = E_i\psi$

Newtonian energy → Schrödinger equation $i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r},t)\right)\Psi(\vec{r},t)$

relativistic energy → Dirac equation $\left(i\hbar\gamma^\mu\partial_\mu - mc\right)\Psi = 0$ with Dirac γ $\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}$

quantum orbit =probability density $\left|\Psi(x,t)\right|^2$, wave function $\Psi(x,t)$,

mean orbit location $\overline{x}(t) = \int \Psi^*(x',t)\, x'\, \Psi(x',t)\, dx'$,

orbit width $\Delta x(t) = \sqrt{\int \Psi^*(x',t)\left(\overline{x}(t) - x'\right)^2 \Psi(x',t)\, dx'}$

classical orbit = function $x(t)$, orbit width $\Delta x(t) = 0$

orbit transition = WKB-approximation, $\frac{d\lambda}{dx} << 1$, $\Psi(x) = C_\pm \frac{\exp\left(\pm\frac{i}{\hbar}\int\sqrt{2m\left(E-V(x)\right)}dx\right)}{\left(2m\left(E-V(x)\right)\right)^{1/4}}$, $\frac{\Delta x(t)}{\left|\overline{x}(t)\right|} << 1$

Fundamental equations from relativistic invariants

wave equation $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta\right)\psi=0$ for $m=0$ (massless field carriers)

Klein-Gordon equation for massive scalars $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta+\frac{m^2c^2}{\hbar^2}\right)\psi=0$

mass current density $j^\mu=\left(\rho,\vec{j}\right)$ yields via $\partial_\mu j^\mu=0$

continuity equation $\frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j}=0$

heat equation $\frac{\partial u}{\partial t}+\nabla\cdot\vec{q}=0$ or $\frac{\partial u}{\partial t}-K\,\Delta u=0$ for temperature u and heat flow $\vec{q}$

Fick's **diffusion equation** $\frac{\partial\varphi}{\partial t}-D\Delta\varphi=0$ for the concentration φ

Quantum-classical transition

Quantum	Classical
energy-momentum $p^{\mu}=\left(\frac{E}{c},\vec{p}\right)$	energy-momentum $p^{\mu}=\left(\frac{E}{c},\vec{p}\right)$
energy relation $p^{\mu}p_{\mu}=\left(\frac{E}{c}\right)^2-\vec{p}^2=m^2c^2$	energy relation $p^{\mu}p_{\mu}=\left(\frac{E}{c}\right)^2-\vec{p}^2=m^2c^2$
	nonrel. $E=\frac{p^2}{2m}+V$
quantum momentum operator $p_{\mu}=i\hbar(\frac{1}{c}\partial_t,-\vec{\nabla})=i\hbar\partial_{\mu}$, $\vec{p}=-i\hbar\nabla$	classical 4-momentum $p_{\mu}=(\frac{E}{c},\vec{p})$
variables are operators commutator $[x,p]=i\hbar$	variables commute , are complex numbers , commutator $[x,p]=0$, $\hbar\to 0$
uncertainty relation $\Delta x\Delta p\geq\frac{\hbar}{2}$	no uncertainty relation
discrete energy E_i E_i = eigenvalue(H) $H\psi=E_i\psi$	continuous energy E
Schrödinger equation =energy relation $i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t}=\left(-\frac{\hbar^2}{2m}\nabla^2+V(\vec{r},t)\right)\Psi(\vec{r},t)$	
eom Dirac equation $\left(i\hbar\gamma^{\mu}\partial_{\mu}-mc\right)\Psi=0$	eom Newtonian $\frac{d\vec{p}}{dt}=-\nabla V$
orbit =probability density $\left\|\Psi(x,t)\right\|^2$	sharp orbit $x(t)$
wave function $\Psi(x,t)$	
mean orbit location $\bar{x}(t)=\int\Psi^*(x',t)\,x'\,\Psi(x',t)\,dx'$	
orbit width $\Delta x(t)=\sqrt{\int\Psi^*(x',t)\left(\bar{x}(t)-x'\right)^2\Psi(x',t)\,dx'}$	orbit width $\Delta x(t)=0$
orbit transition = WKB-approximation $\frac{d\lambda}{dx}<<1$, $\Psi(x)=C_{\pm}\frac{\exp\left(\pm\frac{i}{\hbar}\int\sqrt{2m(E-V(x))}dx\right)}{\left(2m(E-V(x))\right)^{1/4}}$, $\frac{\Delta x(t)}{\left\|\bar{x}(t)\right\|}<<1$	
gravitational decoherence $\tau=\frac{1}{N\lambda_{\Lambda}e^{r/r_{gr}}}$, $\lambda_{\Lambda}=\frac{E_{\Lambda}}{\hbar}=\frac{c}{R_{\Lambda}}=0.33\times10^{-13}s^{-1}$, $r_{gr}=\sqrt{l_P\sqrt{\frac{1}{\Lambda}}}=3.9\times10^{-5}m$, $\tau(S-cat)=1.1\times10^{-12}s$	no entanglement, permanent decoherence
signal time $t_s=\frac{d}{c}$, $t_s(S-cat)=0.17\times10^{-8}s$	classical $\tau<t_s$
thermal decoherence $\tau\approx\frac{\hbar}{E_{th}\rho_n r^2 r_B}$	

Schematics of SO(1,3) (Lorentzian) world

-- classical regime --

SO(1,3) **Lorentzian rotation group**

flat metric $\eta_{\mu\nu} = diag(+1,-1,-1,-1)$

$$\Lambda = \exp\left(-\frac{i}{2}\omega_{\mu\nu} M^{\mu\nu}\right) = \exp(-i(\vec{\phi}\,\vec{J} + \vec{v}\,\vec{K}))$$

$\vec{\varphi}$ =3 spatial rotation angles $\vec{v}$ =3 boost angles

$M^{\mu\nu} = (\mu,\nu)$ coordinate swaps

$$(M^{\mu\nu})^{\rho}_{\sigma} = -i(g^{\mu\rho}\delta^{\nu}_{\sigma} - g^{\nu\rho}\delta^{\mu}_{\sigma})$$

$J^k = \frac{1}{2}\varepsilon^{lmk} M^{lm}$ rotations $K^k = M^{0k}$ boosts

GR=(geometry(metrics),energy-tensor)

Einstein equations

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

spherical Schwarzschild solution

$$g_{\mu\nu} = diag((1-r_s/r), -1/(1-r_s/r), -r^2, -r^2\sin^2\theta)$$

boundary cond. r→∞

$$E\eta E^t = g^{-1}/(-\det g)^{3/4}$$

AK-gravity

$$F_{\mu\nu}{}^{\kappa} = \partial_\mu A_\nu{}^{\kappa} - \partial_\nu A_\mu{}^{\kappa} + \varepsilon^{\kappa}{}_{\kappa_1\kappa_2} A_\mu{}^{\kappa_1} A_\nu{}^{\kappa_2}$$

32 AK-equations $AK(A_\mu{}^\nu, E^{\mu\nu}, \Lambda)$

$$H_{(\mu,\nu)}{}^{\kappa} \equiv F_{\mu\nu}{}^{\kappa} + \frac{\Lambda}{3}\varepsilon_{\mu\nu\rho} E^{\rho\kappa} = 0$$

$$G^{\mu} \equiv \partial_\nu E^{\nu\mu} + \varepsilon^{\mu}{}_{\kappa\lambda} A_\nu{}^{\kappa} E^{\nu\lambda} = 0$$

$$I_\mu \equiv E^{\kappa}{}_{\nu} F_{\mu\kappa}{}^{\nu} = 0$$

$$\frac{du^\mu}{d\tau} = -\Gamma^{\mu}_{\nu\lambda} u^\nu u^\lambda, \quad E^{\mu\kappa} E^{\nu}{}_{\kappa} / (\det(E))^{3/4} = g^{\mu\nu}$$

$\Lambda \ll 1/r_s^2$

Einstein equations $r - r_s \gg l_0 = r_s^3 \Lambda/3$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R_0 + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} ,$$

v<<c Newtonian $E_N = -mc^2\, r_s/r$

GR-cosmology FLRW equation *R(t)*

$$\dot{R}^2 + \left(-\frac{K}{R^2} - \frac{D}{R} + \frac{\Lambda}{3}R^2\right)c^2 = -kc^2$$

$\Lambda > \Lambda_c$ expanding

galaxies

gas-clouds, stars, planets

gases, fluids, solids

classical mechanics

Lagrangian relativistic

$$L = mc^2 g_{\mu\nu} \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} - -V(x)$$

Lagrangian Newtonian $L_N = \dfrac{m\left(\dfrac{dx(t)}{dt}\right)^2}{2} - V(x(t))$

Euler-Lagrange, $\dfrac{d}{d\tau}\dfrac{\partial L}{\partial \dfrac{dx^\mu}{d\tau}} - \dfrac{\partial L}{\partial x^\mu} = 0$

$S = \int L(x, \partial_\mu x)\,dx$ variation of action

$$\frac{\delta S}{\delta x} = -\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu x)}\right) + \frac{\partial L}{\partial x}$$

classical electrodynamics

Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$

$E_i = -cF^{0i}$, $B_k = -\varepsilon_{kij}F^{ij}$ duality

$$m\frac{du^\mu}{d\tau} = F^{\mu\nu}u_\nu$$

thermodynamics

canonical partition function $Z = \sum_k e^{-E_k/k_BT}$

entropy $S = k_B \ln W$, $\dfrac{\partial S}{\partial \tau} > 0$

statistics fermions, bosons, Maxwell

$$N(\varepsilon) = \frac{g(\varepsilon)}{\exp\left(\frac{\varepsilon-\mu}{k_BT}\right)+1} \qquad N(\varepsilon) = \frac{g(\varepsilon)}{\exp\left(\frac{\varepsilon-\mu}{k_BT}\right)-1}$$

$$N(\varepsilon) = \frac{g(\varepsilon)}{\exp\left(\frac{\varepsilon}{k_BT}\right)}$$

temp. T

-- quantum regime --

basic entities
bispinors s=1/2: quarks, leptons
L-,R-Weyl-spinors s=1/2: preons
scalars: cosm. Λ-generator s=0 φ_Λ
massless fields: graviton s=2 $A_\mu{}^\nu$, s=1 photon $A_e{}^\mu$,
8 gluons $A_g{}^\mu$, 15 hc-bosons $A_h{}^\mu$

temp. T

Dirac equation
$\left(i\hbar\gamma^\mu\partial_\mu - mc\right)\Psi = 0$, bispinor Ψ, $\{\gamma^\mu,\gamma^\nu\} = 2diag(1,-1,-1,-1)$
$v<<c$ Schrödinger equation: $i\hbar\frac{\partial\Psi(\vec r,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec r,t)\right)\Psi(\vec r,t)$

quantum electrodynamics
scale $r_B = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = 5.29*10^{-11}m$,
E- scale $E_{em} = \frac{\hbar c}{r_B} = 3.7keV$
$D_\mu = \partial_\mu + ieA_\mu$ SU(1)
Dirac eq. $\left(i\gamma^\mu D_\mu - m\right)\Psi = 0$
Maxwell equations $\partial_\mu F^{\mu\nu} = 0$
field eq. $\partial_\mu F^{\mu\nu} = j^\nu = -e\,\bar\psi\gamma^\nu\psi$
Lagrangian $L = \bar\psi\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}F^a{}_{\mu\nu}F_a{}^{\mu\nu}$
approx $E_{el}(r) = \frac{e_0{}^2}{r}$

atoms. molecules

photon $A_e{}^\mu$

electrons, nuclei

SM composite particles
leptons
hadrons: baryons s=1/2, mesons s=0,1

color (strong) interaction SU(3)
field carrier gluon $A^a{}_\mu$ a=1..8 S=1
charge color=(r,g,b)
energy scale $E_{col} = 220MeV$, conf-scale
$r_{col} = \frac{\hbar c}{E_{col}} = 0.89*10^{-15}m \approx r_{pr}$
cov. derivative $D_\mu = \partial_\mu - igA^a{}_\mu T^a$
Dirac eq. $\left(i\ D_\mu\gamma^\mu - m\right)\psi = 0$
$\psi = \left(u_1,u_2,u_3\right)$, e.g. proton $p = \left(u,u,d\right)$
baryon $b:8A_g$ meson $m:2A_g$ v-meson $m:4A_g$
field tensor $F^a{}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,f^{abc}A^b_\mu A^c_\nu$
Lagrangian $L = \bar\psi\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}F^a{}_{\mu\nu}F_a{}^{\mu\nu}$
Yukawa-boson π^0,π^+,π^-

8 gluons $A_g{}^\mu$

SM basic particles
leptons=rr, quarks=qr, s=1/2, color=rgb

weak hc-Yukawa force W/Z-bosons
self-interaction Higgs H

--- sub-quark regime ---

hyper-color interaction SU(4)
4 charges (L-, L+, R-, R+)
15 hc-bosons $A^a{}_\mu$ a=1...15, S=1
cov. derivative $D_\mu = \partial_\mu - ig A_\mu^a T_a$, $T_a = \lambda_a / 2$
Dirac eq. $\left(i\gamma^\mu D_\mu - m\right)\Psi = 0$
field eq.
$$\frac{1}{2}\partial_\nu\left(F^{a\mu\nu} - F^{a\nu\mu}\right) = j^a{}_\nu = g\,\bar{\psi}\gamma^\mu T^a\psi - \frac{g}{2}\left(f^{kac}A_\nu^c + f^{kba}A_\nu^b\right)F^{k\mu\nu}$$
$\Psi = \left(u_{L-}, u_{L+}, u_{R-}, u_{R+}\right)$, e.g. electron $e^- = \left(r_{L-}, 0, r_{R-}, 0\right)$
generations $f_1 : 1A_h$ $f_2 : 4A_h$ $f_3 : 15A_h$
field tensor $F^a{}_{\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\, f^{abc} A_\mu^b A_\nu^c$
Lagrangian $L = \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}F^a{}_{\mu\nu}F_a{}^{\mu\nu}$
coupling Callan-Symanzik $g_{hc}(\mu) = 4\pi\sqrt{3 / \left(76\sqrt{\left(\log(\frac{\mu}{\Lambda_{hc}})\right)^2 + c_{GE1}{}^2}\right)}$
E-scale $\Lambda_{hc} = 2m(Z_0) = 180 GeV$
$r_{hc} = \frac{\hbar c}{\Lambda_{hc}} = 1.08*10^{-18} m$
symmetry breaking
$SU(4) = SU(2)_L \otimes SU(1)_R \otimes SU(1)_{em}$

15 hcb's $A_h{}^\mu$

preons= r, q, s=1/2, hcolor=(L-,L+,R-,R+)

AK-quantum-gravity
$r_{gr} = \sqrt{l_P\sqrt{\frac{1}{\Lambda}}} = 39\mu m$, E-scale $E_{gr} = \frac{\hbar c}{r_{gr}} = 5.04*10^{-3} eV$, $\sqrt{\alpha_{gr}} = 0.29\times 10^{-30}$ interaction constant
covariant derivative $D_\mu = \partial_\mu - i\sqrt{\alpha_{gr}}\left(\tau_a A^a\right)_\mu$, $\tau^a = i\varepsilon_\nu{}^a{}_\lambda$, $\left(D_\mu\right)^\lambda{}_\kappa = \partial_\mu + \varepsilon^\lambda{}_{a\kappa}\sqrt{\alpha_{gr}}A_\mu{}^a$
gauge Lie group= Lie(ε) , $[\tau^a, \tau^b] = i\varepsilon_c{}^{ab}\tau^c$
equivalent to Lie3(ε) Lie-algebra with 3 generators τ^a and the commutator $\left[\tau_a, \tau_b\right] = i\sum_k\left(1 + \varepsilon_{abk}\right)\tau_k$,
a=1,2,3 which is isomorphic to SU(2)
Dirac eq. $\left(i\hbar\gamma^\mu D_\mu - mc\right)\Psi = 0$
field eq.= $AK(A_\mu{}^\nu, E^{\mu\nu}, \Lambda)$, graviton tensor $A_\mu{}^\nu$, tetrad $E^{\mu\nu}$
boundary condition at $r = r_{gr}$ $E^{\mu\kappa}E^\nu{}_\kappa = g^{\mu\nu} / (-\det(g))^{3/4}$, with Minkowski metric $g = \eta$
approx. quantum grav. potential $E_{gr}(r) = \frac{r_\Lambda}{r}mc^2$, $r_\Lambda = 2.03*10^{-57} m$
grav. quantum noise $P_p = \frac{r_\Lambda}{r_p}m_p c^2 \frac{c}{r_p} \sim 10^{-22} eV / s$, (r_p , m_p particle distance, mass)

Statistical mechanics

entropic principle of thermodynamics

$\frac{dS}{dt} \geq 0$ entropy is non-decreasing in time *t*,

or (Lorentz-invariant) $\frac{dS}{d\tau} \geq 0$ entropy is non-decreasing in proper time τ

Fundamental variable: partition function $Z = \sum_i \exp\left(-\frac{E_i}{k_B T}\right)$

important thermodynamic variables

$\langle E \rangle = -k_B T \frac{\partial \ln Z}{\partial \beta}$ mean energy

$F = \langle E \rangle - TS = -k_B T \ln Z$ Helmholtz free energy

$S = \frac{\partial}{\partial T}\left(k_B T \ln Z\right)$ entropy

$C_v = \frac{\partial \langle E \rangle}{\partial T} = \frac{1}{k_B T^2}\left\langle (\Delta E)^2 \right\rangle$ heat capacity

Thermodynamic **classical and quantum ensembles** with their partition function and the macroscopic function

	Microcanonical	Canonical	Grand canonical
Fixed variables	*N, V, E*	*N, V, T*	μ, *V, T*
Microscopic features	Number of microstates W, with width ω $W = \sum_k f\left(\frac{H_k - E}{\omega}\right)$	Canonical partition function $Z = \sum_k exp\left(-\frac{E_k}{k_B T}\right)$ $Tr\left(exp\left(-\beta H\right)\right) = \sum_k exp\left(-\beta E_k\right) = Z(\beta)$	Grand partition function $Z = \sum_k exp\left(-\frac{E_k - \mu N_k}{k_B T}\right)$ $Z(\beta, \mu_1, \mu_2, ...) = Tr\left(exp\left(\beta\left(\sum_k \mu_k N_k - H\right)\right)\right)$
Probability and density matrix	$\hat{\rho} = \frac{1}{W}\sum_k f\left(\frac{H_k - E}{\omega}\right)\lvert\psi_k\rangle\langle\psi_k\rvert$	$p(E_m) = \frac{exp\left(-\beta E_m\right)}{\sum_k exp\left(-\beta E_k\right)}$ $\rho = \frac{e^{-\beta H}}{Tr\, e^{-\beta H}}$	$P(E_m) = \frac{e^{-\beta\left(E_m - \sum_i \mu_i N_i\right)}}{\sum_n e^{-\beta\left(E_n - \sum_i \mu_i N_i\right)}}$ $\rho = \frac{e^{-\beta\left(H - \sum_i \mu_i N_i\right)}}{Tr\, e^{-\beta\left(H - \sum_i \mu_i N_i\right)}}$
Macroscopic function =min	Boltzmann entropy $S = k_B \ln W$ v.Neumann entropy $S = -k_B Tr(\rho \ln \rho)$	Helmholtz free energy $F = -k_B T \ln Z$	Grand potential $\Omega = -k_B T \ln Z$

The three **probability distributions** for the *average number in a state* ε: $N(\varepsilon)$ with degeneracy *g(ε)*

fermions $S = n + 1/2$ (Pauli principle) Fermi-Dirac statistics $W(\nu_i) = \prod_r \frac{g_r!}{\nu_r!(g_r - \nu_r)!}$ $\nu_r = 0{,}1$

$$N(\varepsilon) = \frac{g(\varepsilon)}{\exp\left(\frac{\varepsilon - \mu}{k_B T}\right) + 1}$$

bosons $S=n$ Bose-Einstein statistics $W(\nu_i)=\prod_r \frac{(g_r+\nu_r-1)!}{\nu_r!(g_r-1)!}$ $\nu_r=0,1,2,...$

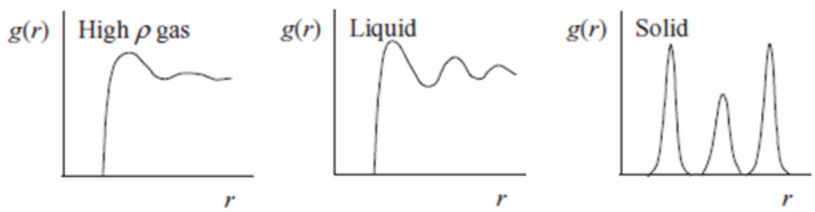
$$N(\varepsilon)=\frac{g(\varepsilon)}{\exp\left(\frac{\varepsilon-\mu}{k_B T}\right)-1}$$

classical Boltzmann-Maxwell statistics $N(\varepsilon)=\exp\left(-\frac{\varepsilon}{k_B T}\right)$

Phases of matter in first-order solid-fluid-gas transitions

Phases of matter differ in their number $N(r)$ of particles at radius *r*, described by pair correlation function *g(r)*

$$N(r)=n_0\int_0^r g(r)4\pi r^2 dr$$



and in their temperature range: solid $T<T_{freeze}$, fluid $T_{freeze}<T<T_{boil}$, gas $T>T_{boil}$

-Fluid-gas eos

Peng-Robinson eos describes the fluid-gas phase

$$p=p(\beta,v,p_c,\beta_c)=\frac{1}{\beta(v-b_1)}-\frac{\left(1+\left(0.480+1.574\omega(p_c,\beta_c)-0.176\,\omega(p_c,\beta_c)^2\right)\left(1-\sqrt{\beta/\beta_c}\right)\right)^2 a_1}{v(v+b_1)+b_1(v-b_1)},$$

with vdWaals parameters *a*, *b*, in terms of critical parameters p_c, inverse thermal energy $\beta_c=\frac{1}{k_B T}$,

-Solid phase eos

Mie-Grueneisen equation describes the solid phase

$$p(\eta,\beta)=p_0+\frac{(y_1\eta/v_0-y_2)(\eta-1)\left(\eta-\frac{\Gamma_0}{2}(\eta-1)\right)}{\eta\left(\eta-s(\eta-1)\right)^2}+3\frac{\Gamma_0}{v_0}\left(\frac{1}{\beta}-\frac{1}{\beta_0}\right),$$

where $\eta=\frac{v_0}{v}$, $\Gamma_0\approx 2$, $s\approx 3/2$, $\beta_0=\beta_t$, $p_0=p_t$, $v_0=v_t$ are the triple-point values

- Fluid-gas curve

Maxwell condition: equal pressure and equal Gibbs energy

$p=p_{PR}(v_f,\beta)$, $p=p_{PR}(v_g,\beta)\rightarrow v_f=v_{PR,1}(p,\beta)$, $v_g=v_{PR,2}(p,\beta)$ cubic roots of *p*-equation

Maxwell-Gibbs equation $G_{PR}(v_f(p,\beta),\beta)-G_{PR}(v_g(p,\beta),\beta)=0\rightarrow p_{fg}=p(\beta)$ fluid-gas curve

where $G_{PR}(v,\beta)$ and $G_{MG}(v,\beta)$ is the Gibbs energy of Peng-Robinson resp. Mie-Grueneisen eos.

- Solid-fluid curve

Maxwell condition: equal pressure and equal Gibbs energy

$p=p_{PR}(v_f,\beta)$, $p=p_{MG}(v_s,\beta)\rightarrow v_f=v_{PR,1}(p,\beta)$, $v_s=v_{MG,1}(p,\beta)$ cubic roots of *p*-equation

Maxwell-Gibbs equation $G_{PR}(v_f(p,\beta),\beta)-G_{MG}(v_s(p,\beta),\beta)=0\rightarrow p_{sf}=p(\beta)$ solid-fluid curve

where $G_{PR}(v,\beta)$ and $G_{MG}(v,\beta)$ is the Gibbs energy of Peng-Robinson resp. Mie-Grueneisen eos.

- Triple point

At the triple point, the saturation curve ends, fluid-->gas , fluid-->solid , gas-->solid curves meet, i.e. $p_{PR}(v_g,T)=p_{PR}(v_f,T)=p_{MG}(v_s,T)$

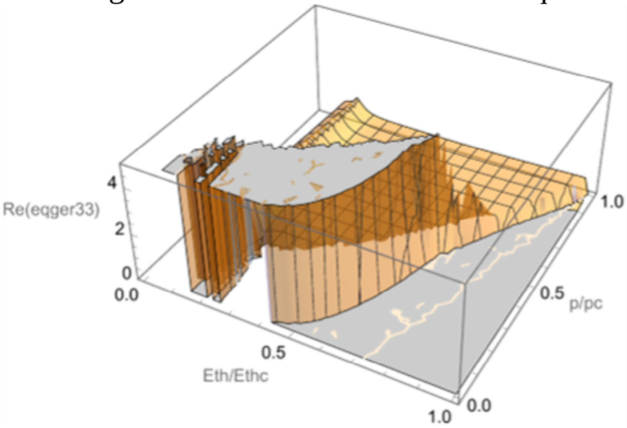
The triple point can be found as the branching point of saturation curve in the $p(\lambda, \beta)$ diagram.

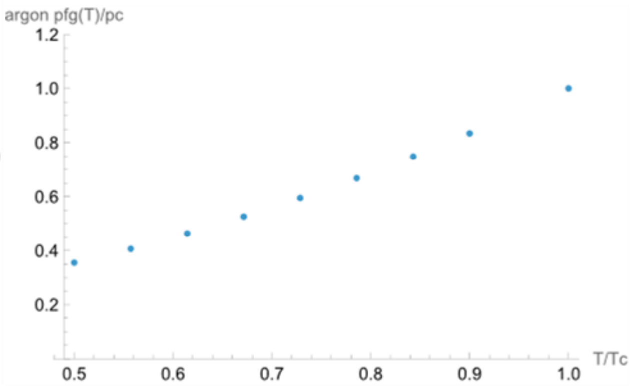
- Critical point

At the critical point, only one phase exists, the isotherm has a turning point there.

$$\frac{\partial p}{\partial v} = 0 \ , \ \frac{\partial^2 p}{\partial v^2} = 0$$

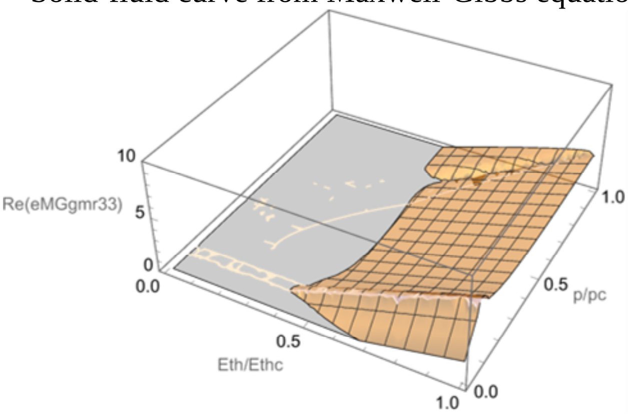
- Fluid-gas curve from Maxwell-Gibbs equation

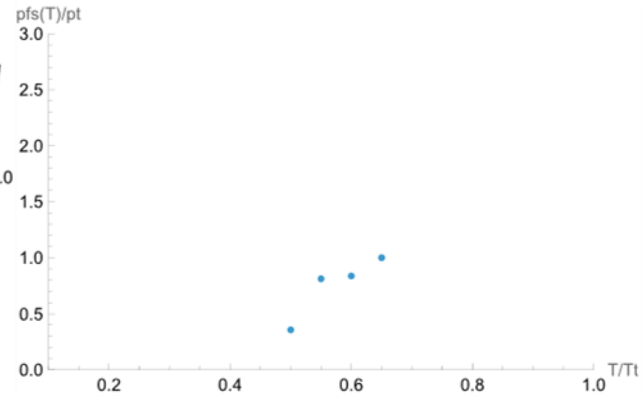




fg-curve: $G_{PR}\left(v_f\left(p, \beta\right), \beta\right) - G_{PR}\left(v_g\left(p, \beta\right), \beta\right) = 0 \qquad p_{fg} = p\left(\beta\right)$

- Solid-fluid curve from Maxwell-Gibbs equation





sf-curve: $G_{PR}\left(v_f\left(p, \beta\right), \beta\right) - G_{MG}\left(v_s\left(p, \beta\right), \beta\right) = 0 \qquad p_{sf} = p\left(\beta\right)$

Macrocosmic **phenomenological ΛCDM model**

The fundamental Friedmann equations for world radius *R(t)* , density *ρ(t)* and pressure *p(t)*

$$\dot{\rho} = -3\frac{\dot{R}}{R}(\rho + \frac{P}{c^2})$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\Lambda c^2}{3}$$

eos $p(\rho) = const * \rho^w$ matter , eos $P = \rho_r c^2 / 3$ radiation

relation for the Hubble constant

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}$$

ΛCDM model is based on six *dependent fundamental parameters* $H, k, \Lambda, \Omega, t_0, \rho_r, \rho_m$

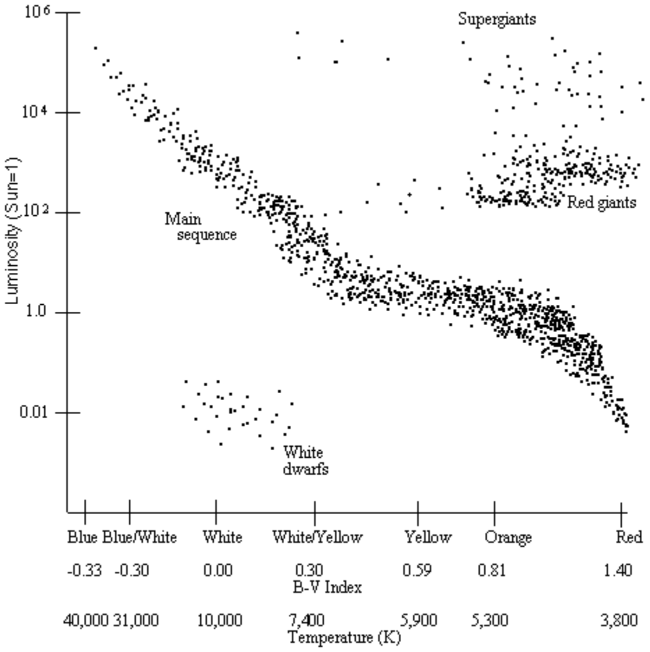
resp. four *independent fundamental parameters* D, K, k, Λ

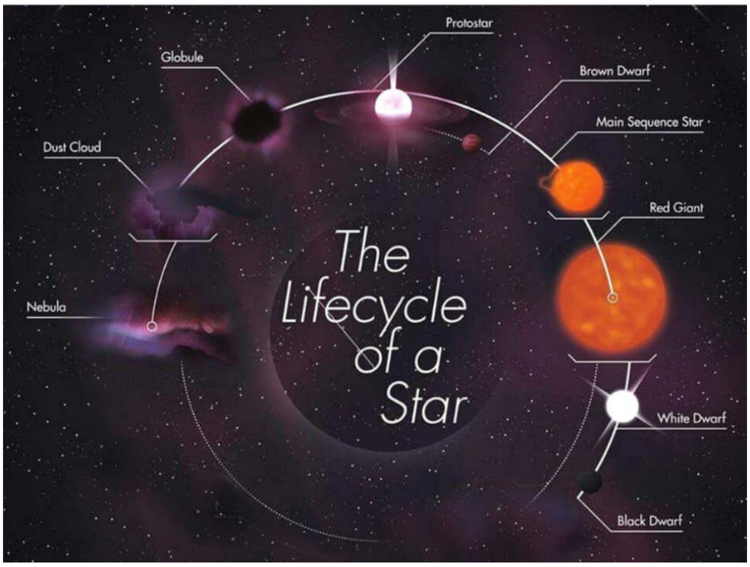
D=fundamental length, K fundamental area, k=sign(curvature), Λ cosmological constant

Parameters of the ΛCDM model

Description	Symbol	Value
Physical baryon density parameter	$\Omega_b h^2$	0.02230±0.00014
Physical dark matter density parameter	$\Omega_c h^2$	0.1188±0.0010
Scalar spectral index	n_s	0.9667±0.0040
Curvature fluctuation amplitude, $k_0 = 0.002$ Mpc^{-1}	Δ_R^2	$2.441 \pm 0.092 \times 10^{-9}$
Reionization optical depth	τ	0.066±0.012
Age of the universe	t_0	$13.799 \pm 0.021 \times 10^9$ years
Reduced Hubble constant	h	0.704±0.025
Type of curvature	k	uncertain
Cosmological constant	Λ	1.1 10^{-52} m^{-2}
Density of matter	ρ_m	0.3089 ρ_c
Density of radiation	ρ_r	3.79 10^{-5} ρ_c
Relative density	Ω	1.01±0.05 ρ_c
Critical density	ρ_c	3.79 10^{-26} kg/m^3

Star models





Hertzsprung-Russell diagram

Sun-like stars

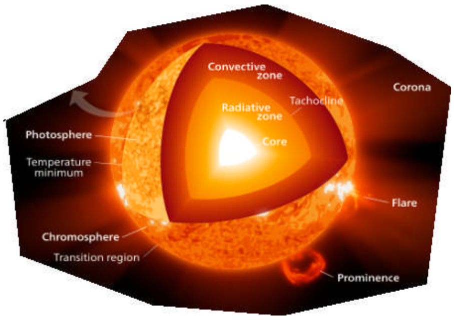
mass $M \sim M_{sun} = 1.99 \times 10^{30}\, kg$, radius $R \sim R_{sun} = 0.7 \times 10^{6}\, km$

density $\rho \sim \rho_{sun} = 1.4\, g\, cm^{-3}$, age $t_0 \sim t_{sun} = 4.6 \times 10^{9}\, y \quad t_0 \le 10 \times 10^{9}\, y$

eos **ideal gas** $PV = n_m RT \quad P = nk_B T = \rho \dfrac{k_B T}{m_0}$

temperature $T = \begin{cases} 15.7 \times 10^{7}\, K\ core \\ 5.8 \times 10^{3}\, K\ surface \end{cases}$, gravitational acceleration $g \sim 275\, m\, s^{-2}$,

energy production p-p-chain $4H_1^{+} + 2e^{-} \rightarrow He_4^{2+} + 2\bar{\nu}_e + 26.7\, MeV$



White-dwarf stars

mass $0.17M_{sun} \le M \le 1.33M_{sun}$, Chandrasekhar mass limit $M < M_{max} = 1.4M_{sun}$

radius $R \propto M^{1/3}$ $6000km \le R \le 12000km$ calc. $R\left(M = 0.6M_{sun}\right) = 9000km$

density calc. $\rho\left(0.6M_{sun}\right) = 1.6\times10^{6}\, g\, cm^{-3}$, typical age $t_0 \sim 6\times10^{9}\, y$

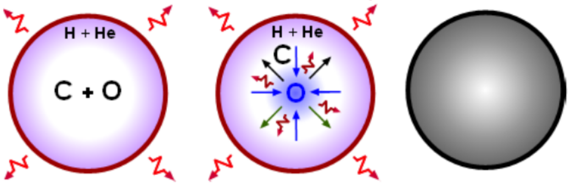
magnetic field 0.2 T to 100 kT $0.2T \le B \le 100kT$

eos pressure Fermi **electron gas**

low-density approximation $P = 8\pi P_0 \dfrac{x_F^{\ 5}}{15} K_1 \rho^{5/3}$, $x_F = \dfrac{p_F}{m_e c} = \dfrac{h}{2m_e c}(3\pi)^{1/3} n^{1/3}$

p_F Fermi-angular-momentum, *n* particle density

temperature core $T \sim 10^{7} K$



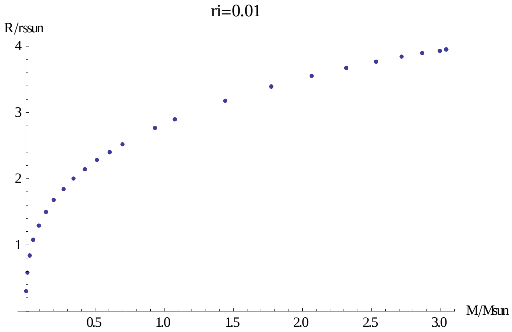
Neutron stars

mass calc. $0.14 M_{sun} \le M \le 3.04 M_{sun}$

radius $R \propto M^{1/3}$ calc. $4.5km \le R \le 12km$ $R\left(M = 0.5M_{sun}\right) = 6.9km$

sun-units Schwarzschild-radius $r_{ssun} = 3km$, and Schwarzschild-density $\rho_{ssun} = 0.74 \times 10^{17}\, g\, cm^{-3}$

R-M relation



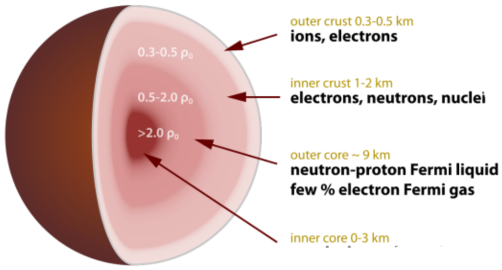
density calc. $3.5 \times 10^{14} \le \rho \le 4 \times 10^{14}\, g\, cm^{-3}$, typical age $t_0 \sim 6 \times 10^{9}\, y$

magnetic field 0.2 T to 100 kT $0.2T \le B \le 100kT$

eos pressure Fermi **neutron fluid**

$P_1(r_1) = c_1 \rho_1(r_1) V_{nn}((E_n / \rho_1(r_1))^{1/3})$

temperature $T \sim 10^{6} K$, gravitational acceleration $g \sim 2 \times 10^{12}\, ms^{-2}$



Tolman–Oppenheimer–Volkoff (TOV) equation

for mass *M(r)*, pressure *P(r)* and density *ρ(r)*

$$P'(r) = -\left(\frac{GM(r)\rho(r)}{r^2}\right)\left(1 + \frac{P(r)}{\rho(r)c^2}\right)\left(1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right)\left(1 - \frac{2GM(r)}{rc^2}\right)^{-1}$$

with

$4\pi r^2 \rho(r) = M'(r)$ and eos: usually $P(r) = K\, \rho(r)^{\gamma}$

Stellar black holes

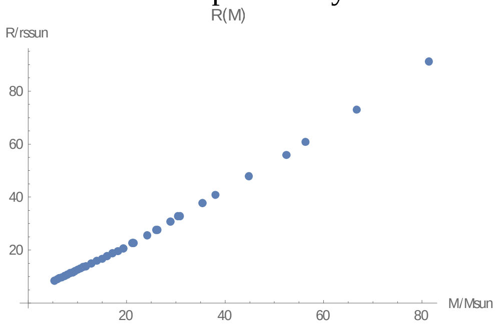
shell star with mass M, outer radius R, inner radius $r_i > r_s(M)$

sun-units Schwarzschild-radius $r_{ssun} = 3km$, and Schwarzschild-density $\rho_{ssun} = 0.74 \times 10^{17}\, g\, cm^{-3}$

mass calc. $5.35 M_{sun} \le M \le 100 M_{sun}$

radius $R \propto M$ calc. $25.5km \le R \le 336km$, inner radius calc. $21km \le r_i \le 330km$

R-M-relation is practically linear



relative flight velocity deviation from 1 is $\frac{\Delta v}{c} \approx d_{Rsrel} = \frac{R-M}{M} \sim 0.07$, redshift is $z \approx \frac{1}{d_{Rsrel}} \sim 14$

eos pressure Fermi **neutron gas**

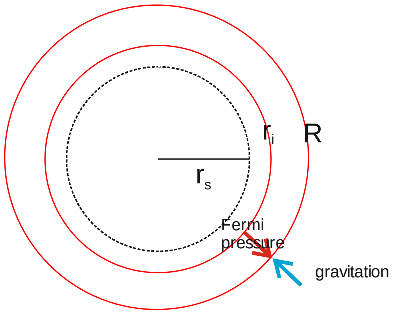
low-density approximation $P = 8\pi P_0 \frac{x_F^{\;5}}{15} K_1 \rho^{5/3}$, $x_F = \frac{p_F}{m_n c} = \frac{h}{2 m_n c}(3\pi)^{1/3} n^{1/3}$

p_F is the Fermi-angular-momentum, n the particle density

magnetic field probably like neutron stars $B = 1kT \ldots 100kT$ (form relativistic jets at poles)

temperature probably like neutron stars $T \sim 10^6 K$

entropy of a shell star is $S = \frac{k_B A}{\pi L_P^{\;2}} \log 2$ $\approx$ Hawking- Bekenstein entropy of black-hole $S = \frac{k_B A}{4 L_P^{\;2}}$



Supermassive (galactic) black holes

supermassive shell star with mass M, outer radius R, inner radius $r_i > r_s(M)$

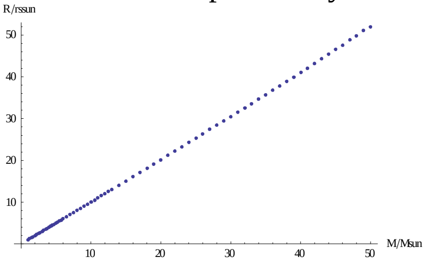
mega sun-mass $M_{Msun} = 10^6 M_{sun}$ mega-Schwarzschild-radius $r_{Mssun} = 3\times 10^6 km$,

mega-Schwarzschild-density $\rho_{Mssun} = 10^{-12}\rho_{ssun} = 0.74\times 10^5 g\, cm^{-3}$

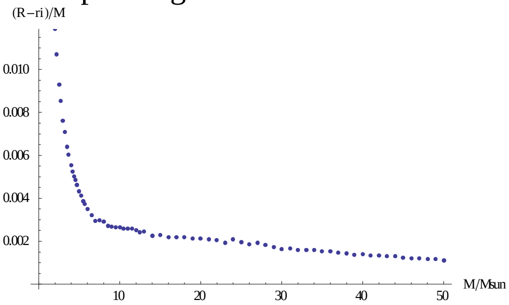
mass calc. $2\times 10^6 M_{sun} \le M \le 50\times 10^6 M_{sun}$

radius $R \approx r_s(M)$ calc. $6\times 10^6 km \le R \le 150\times 10^6 km$, inner radius $r_i \approx r_s(M)$

R-M-relation is practically linear



corresponding relative shell thickness $dR_{rel}=dR/M$ is



relative flight velocity deviation from 1 is $\frac{\Delta v}{c} \approx d_{Rsrel} = \frac{R-M}{M} \sim 0.002$, redshift is $z \approx \frac{1}{d_{Rsrel}} \sim 500$

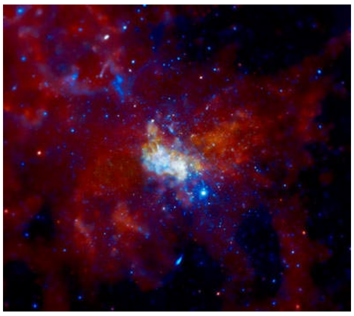
eos pressure Fermi **electron gas** like for a white-dwarf

low-density approximation $P = 8\pi P_0 \frac{x_F^{\ 5}}{15} K_1 \rho^{5/3}$,

$x_F = \frac{p_F}{m_e c} = \frac{h}{2m_e c}(3\pi)^{1/3} n^{1/3}$, p_F Fermi-angular-momentum, n particle density

temperature probably in x-ray region, i.e. $T \sim 100keV = 10^8 K$

magnetic field probably scaling with redshift $B_{sm}(M) \approx \frac{z_{sm}}{z_{st}} B_{st} \approx 35 B_{st}(M)$, $B = 30kT...3MT$



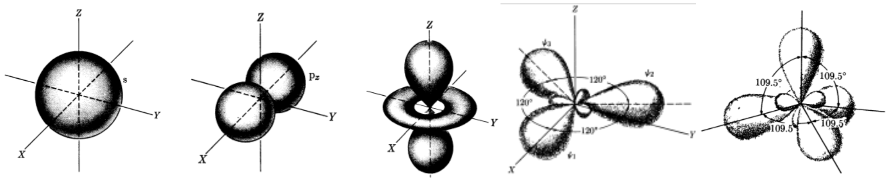
Models of conglomerates

Structural base

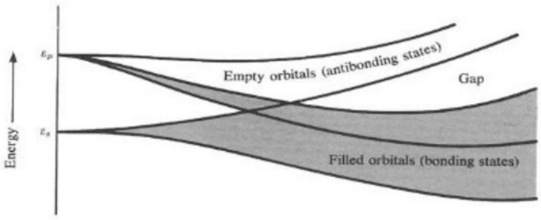
80 atoms with characteristic electronic orbital configuration

orbitals $\Psi_{n_A,n_I,l_1,l_2}(\vec{r})$, n_A= atomic number, n_I= ionization ,

l_1 orbit am , l_2 second orbit am



Interaction energy of orbitals $E(O_1,O_2,r)$



Transport phenomena

-Diffusion

Fick's law $\frac{\partial c(x,t)}{\partial t} = D\frac{\partial^2 c(x,t)}{\partial x^2}$

-Thermal conduction

$\vec{j}(\vec{x}) = -\kappa\nabla T$ Fourier's law

-Electrical conduction

$\vec{j}(\vec{x}) = \sigma\ \nabla U$

Chemical reactions

reaction $\nu_A A + \nu_B B \leftrightarrow \nu_X X + \nu_Y Y$

rate $r = k_{for}[A]^{\nu_A}[B]^{\nu_B}$ Arrhenius

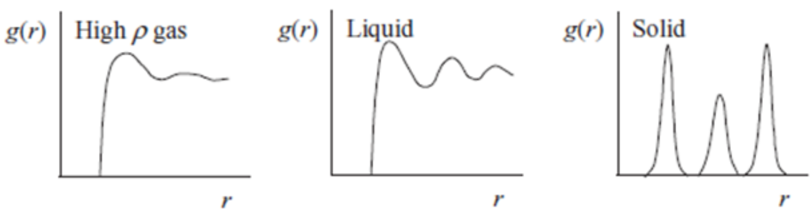
$k_{for} = A'T\exp(-E_{act}/RT)$

Phases

Phases differ by number $N(r)$ of particles at radius r ,

described by pair correlation function $g(r)$

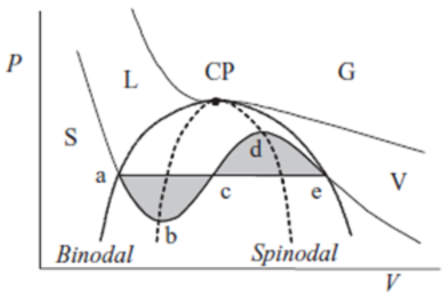
$$N(r) = n_0\int_0^r g(r)4\pi r^2 dr$$



temperature range:

solid $T < T_{freeze}$, fluid $T_{freeze} < T < T_{boil}$, gas $T > T_{boil}$

-Phase diagram



Mass and energy

specific weight $\rho = \frac{M}{V}$

specific heat $c_Q = \frac{Q}{m\Delta T}$

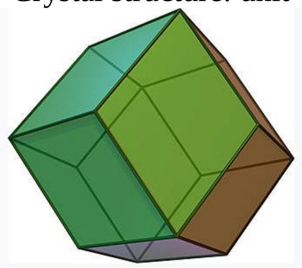
Phase solid

-Fundamental material parameters

geometric parameters of the Wigner-Seitz cell: medium vertical vectors $\vec{r}_i$, $i = 1,\ldots,n_{WS}$,of the n_{WS} surfaces of the Wigner-Seitz cell

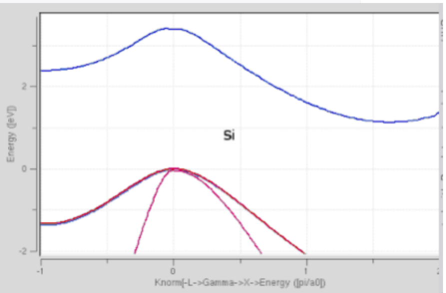
interaction energy of orbitals within the Wigner-Seitz cell $E\left(O_{k_1}, O_{k_2}, r\right)$ $k_1, k_2 = 1,\ldots,n_{WS}$

-Crystal structure: unit Wigner-Seitz cell



-Electronic band structure

energy in momentum space $E(k)$



-Elastic

stress $\sigma = E\varepsilon$, with strain ε,

shear stress $\tau = G\,\gamma$, shear modulus $G = \dfrac{E}{2(1+\nu)}$

friction $F = \mu F_n$

-Electromagnetic

magnetization $\vec{M} = \chi_m \vec{H}$

electric polarization $\vec{P} = \varepsilon_0 \chi_e \vec{E}$

-Optical

refraction $n_i \sin\theta_i = const$ Snell's law

absorption: intensity $I = I_0 \exp(-\mu x)$

Phase gaseous and fluid

-Fundamental material parameters

geometric parameters of the participating atoms/molecules

interaction energy of orbitals $E\left(O_1, O_2, r\right)$

-Eos real gas (van der Waals),

$\left(P + a\rho^2\right)(1 - b\rho) = \rho RT$ material

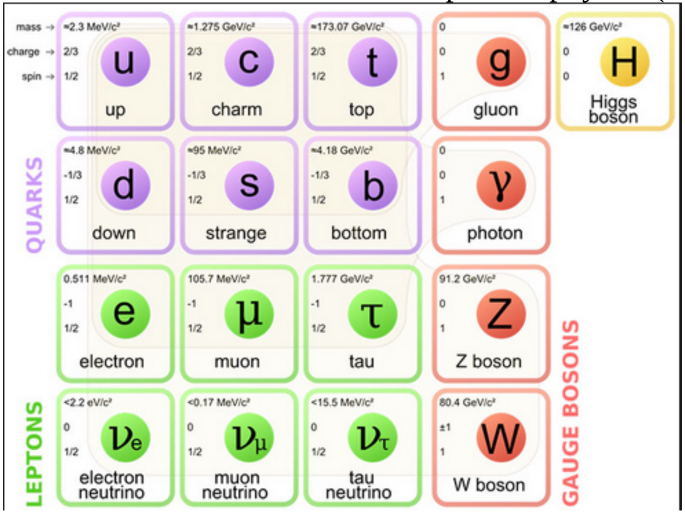
-Eos real fluid $\dfrac{1}{V_0}\dfrac{dV}{dP} = -\dfrac{A}{B+P}$ (Tait)

-Dynamics equation gas and fluid

velocity distribution $\vec{u}(\vec{x},t)$ Navier-Stokes

$$\rho\frac{D\vec{u}}{Dt} = \rho g - \nabla P + \eta\nabla^2\vec{u}\,,\quad \frac{D\vec{u}}{Dt} = \frac{\partial\vec{u}}{\partial t} + \vec{u}\nabla\bullet\vec{u}$$

Microcosmic **Standard Model** of particle physics (SM)



Parameters of the Standard Model 28 parameters

Symbol	Description	Renormalization scheme (point)	Value
m_e	Electron mass		511 keV
m_μ	Muon mass		105.7 MeV
m_τ	Tau mass		1.78 GeV
m_u	Up quark mass	$\mu_{\underline{MS}}$ = 2 GeV	1.9 MeV
m_d	Down quark mass	μ_{MS} = 2 GeV	4.4 MeV
m_s	Strange quark mass	μ_{MS} = 2 GeV	87 MeV
m_c	Charm quark mass	$\mu_{MS} = m_c$	1.32 GeV
m_b	Bottom quark mass	$\mu_{MS} = m_b$	4.24 GeV
m_t	Top quark mass	On-shell scheme	172.7 GeV
θ_{12}	CKM 12-mixing angle	q flavor mixing	13.1°
θ_{23}	CKM 23-mixing angle		2.4°
θ_{13}	CKM 13-mixing angle		0.2°
δ_{13}	CKM CP-violating Phase		0.995
θ_{12}	PMNS 12-mixing angle	ν flavor mixing	33.6±0.8°
θ_{23}	PMNS 23-mixing angle		47.2±4°
θ_{13}	PMNS 13-mixing angle		8.5±0.15°
δ_{13}	PMNS CP-violating Phase		4.1±0.75
g_1 or g'	U(1) gauge coupling	$\mu_{MS} = m_Z$	0.357
g_2 or g	SU(2) gauge coupling	$\mu_{MS} = m_Z$	0.652
g_3 or g_s	SU(3) gauge coupling	$\mu_{MS} = m_Z$	1.221
Λ	crit. energy in SU(3)		220MeV
c_{gE0}	additional log in col-coupling		0.69
θ_{QCD}	QCD vacuum angle		~0
v	Higgs vacuum expectation value		246 GeV
m_H	Higgs mass		125.36±0.41 GeV
$m_{\nu e}$	electron neutrino mass		<= 0.12 eV
$m_{\nu\mu}$	mu neutrino mass		<= 0.12 eV
$m_{\nu\tau}$	tau neutrino mass		<= 0.12 eV

Extended SM = SU(4) preon model (SU4PM)

The basic particle families are
-leptons L=r⊗r , hc-tetra-spinor = doublet of two r-preons, spin S=1/2
-quarks Q=r⊗q , hc-tetra-spinor = doublet of an r- and a q-preon, color Q_c=1 , spin S=1/2

Parameters of the SU4 preon model (6 parameters)

Description	Value
r mass	0.033meV
q mass	0.77 MeV
char. SU(4) en. Λ	180GeV
zero correction c_{GE}	0.095
char . SU(3) en. Λ_g	220MeV
zero corr. c_{gE0}	0.69

Configuration of the SM in the SU4PM

charged leptons {e, mu, tau}

$$x=\left(\begin{pmatrix} rL- \\ 0 \end{pmatrix},0,\begin{pmatrix} rR- \\ 0 \end{pmatrix},0\right)$$

$e=x+A13$ flavor F=1 one boson m=0.511MeV

$mu=x+A13+\overline{A}13+A24+\overline{A}24$ F=2: four bosons m=105.7MeV

$tau=x+A$ F=3: all bosons m=1.78GeV

lepton neutrinos {nue, num, nut}

$$x=\left(\begin{pmatrix} rL- \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ rL+ \end{pmatrix},0,0\right)$$

$nue=x+A12$ m=0.30meV

$num=x+A12+\overline{A}12+A34+\overline{A}34$ m=11meV

$nut=x+A$ m=98meV

u-quarks {u,c,t}

$$x=\left(0,\begin{pmatrix} (rL++qL+)/\sqrt{2} \\ (rL++qL+)/\sqrt{2} \end{pmatrix},0,\begin{pmatrix} (rL++qR+)/\sqrt{2} \\ (rL++qR+)/\sqrt{2} \end{pmatrix}\right)$$

$u=x+A24$ m=2.3MeV

$c=x+A24+\overline{A}24+A13+\overline{A}13$ m=1.34GeV

$t=x+A$ m=171GeV

d-quarks {d, s, b}

$$x=\left(\begin{pmatrix} (rR-+qL+)/\sqrt{2} \\ (rR-+qL+)/\sqrt{2} \end{pmatrix},0,\begin{pmatrix} (rR-+qR+)/\sqrt{2} \\ (rR-+qR+)/\sqrt{2} \end{pmatrix},0\right)$$

$d=x+A13$ m=4.8MeV

$s=x+A13+\overline{A}13+A24+\overline{A}24$ m=100 MeV

$b=x+A$ m=4.2 GeV

weak massive bosons {W, Z, ZL, H}
F=3, all A

$$W^-=\left(0,0,\begin{pmatrix} u1 \\ -u1 \end{pmatrix},0\right)\sqrt{2}\quad u1=((rR-)-(rR-))/\sqrt{2},\ (W^-)^\dagger=W^+ \text{ m=80.4GeV}$$

$$Z0=\left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},0,0,\begin{pmatrix} u1 \\ u1 \end{pmatrix}\right)/\sqrt{2}\quad u1=((rL-)+(rR+))/\sqrt{2},\ (Z0)^\dagger=Z0 \text{ m=91.2GeV}$$

participates in neutron decay and similar weak decays

$$ZL=\left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},0,0\right)/\sqrt{2}\quad u1=((rL-)+(rL+))/\sqrt{2} \text{ m=91GeV}$$

SU(4) mass generator

$$H=\left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix}\right)/2\quad u1=((rL-)+(rL+)+(rR-)+(rR+))/2 \text{ m=125GeV}$$

strong neutrinos {qnue, qnum, qnut} self-interaction

$$x=\left(\begin{pmatrix} qL- \\ 0 \end{pmatrix},0,0,\begin{pmatrix} 0 \\ qR+ \end{pmatrix}\right)$$

$qnue=x+A14$ m=23.2MeV

$qnum=x+A14+\overline{A}14+A23+\overline{A}23$ m=205MeV

$qnut=x+A$ m=2.4GeV

strong massive bosons {Zq, Hq}
F=3, all A
neutral strong exchange boson

$$Zq=\left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} 0 \\ 0 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right)/\sqrt{2}$$

$u1=((qL-)+(qR-))/\sqrt{2}$, $(Zq)^\dagger=Zq$ m=644GeV

SU(3) mass generator

$$Hq=\left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix}\right)/2$$

1 Basic principles

Minimum principles
classical: path $q^{\mu}(\tau)$

action relativistic $S[q^{\mu}]=\int L\left(q^{\mu},\frac{dq^{\mu}}{d\tau},\tau\right)d^{3}x\,dt=\min$, i.e. variation $\delta S=0$, $\tau=t\sqrt{1-v^{2}/c^{2}}$ proper time

relativistic Euler-Lagrange equations: $0=\frac{\delta S}{\delta q}=-\frac{d}{d\tau}\left(\frac{\partial L}{\partial\dot{q}^{\mu}}\right)+\frac{\partial L}{\partial q^{\mu}}$ $\dot{q}^{\mu}=\frac{dq^{\mu}}{d\tau}$

Hamiltonian density $H\left(q^{\mu},\frac{dq^{\mu}}{d\tau},\tau\right)=\frac{dq^{\mu}}{d\tau}\pi_{\mu}-L\left(q^{\mu},\frac{dq^{\mu}}{d\tau},\tau\right)$ where $\pi_{\mu}=\frac{\partial L}{\partial\left(\frac{\partial q^{\mu}}{\partial\tau}\right)}$ conj. momentum

Hamiltonian field equations $\frac{dq^{\mu}}{d\tau}=\frac{\partial H}{\partial\pi_{\mu}}$ $\frac{d\pi_{\mu}}{d\tau}=-\frac{\partial H}{\partial q^{\mu}}$, non-relativistic $\tau=t$

Hamiltonian is $H_{0}=\int H\,d^{3}x$ and Lagrangian $L_{0}=\int L\,d^{3}x$

quantum: wave-function $\varphi(x^{\mu})$ scalar, vector, spinor

action $S[\varphi]=\int L\left(\varphi,\frac{\partial\varphi}{\partial x^{\mu}},x^{\mu}\right)d^{3}x\,dt=\min$ i,e, variation $\delta S=0$,

relativistic Euler-Lagrange equations: $0=\frac{\delta S}{\delta\varphi}=-\partial_{\mu}\left(\frac{\partial L}{\partial(\partial_{\mu}\varphi)}\right)+\frac{\partial L}{\partial\varphi}$

Hamiltonian density $H\left(\varphi,\pi,x^{\mu}\right)=\frac{\partial\varphi}{\partial\tau}\pi-L\left(\varphi,\frac{\partial\varphi}{\partial x^{\mu}},x^{\mu}\right)$ where $\pi=\frac{\partial L}{\partial\left(\frac{\partial\varphi}{\partial\tau}\right)}$ conj. momentum

Hamiltonian field equations $\frac{\partial\varphi}{\partial\tau}=\frac{\partial H}{\partial\pi}$ $\frac{\partial\pi}{\partial\tau}=-\frac{\partial H}{\partial\varphi}$, non-relativistic $\tau=t$

Classical and quantum mechanics and the energy relation
Classical relativistic and Newtonian mechanics has the following energy formula and Lagrangian [8]:

Relativistic energy: $E=\frac{mc^{2}}{\sqrt{1-\left(v/c\right)^{2}}}$, in general $E=\frac{mc^{2}}{dt/d\tau}$, $\tau=t\sqrt{1-v^{2}/c^{2}}$ proper time

results from the relativistic invariant $mc^{2}=E^{2}-p^{2}c^{2}$,
with the Einstein formula as approximation $E=mc^{2}$ for massive objects
and $E=pc$ for massless fields like the photon

Newtonian approximation *v<<1* $E_{kin}=\frac{mv^{2}}{2}$

Lagrangian Newtonian $L_{N}=\frac{mv^{2}}{2}-V(x(t))=\frac{m\left(\frac{dx(t)}{dt}\right)^{2}}{2}-V(x(t))$,

gives the Newtonian eom (Euler-Lagrange equations) $\frac{d}{d\tau}\frac{\partial L}{\partial u^{\nu}}-\frac{\partial L}{\partial x^{\nu}}=0$,

Lagrangian relativistic $L=-mc^{2}\frac{d\tau}{dt}-V(x(t))=-mc^{2}\sqrt{1-\left(\frac{dx(t)}{cdt}\right)^{2}}-V(x(t))$

gives the relativistic eom (Euler-Lagrange equations) $\frac{d}{d\tau}\frac{\partial L}{\partial u^{\nu}}-\frac{\partial L}{\partial x^{\nu}}=0$,

i.e. $\frac{d}{d\tau} mu_\nu = K_\nu = -\partial_\nu V$ with the the Minkowski force K_ν (space part $K_i\sqrt{1-(\vec{v}/c)^2} = F_i$),
non-relativistic force $\vec{F} = (F_i)$, and 4-velocity u_ν , from which follows :

space eom $\frac{d}{dt}\frac{m\vec{v}}{\sqrt{1-(\vec{v}/c)^2}} = -\nabla V(\vec{x})$ with $\vec{v} = \frac{d\vec{x}}{dt}$, time eom $\frac{d}{dt}\frac{mc^2}{\sqrt{1-(\vec{v}/c)^2}} = \frac{dT}{dt} = \vec{F}\cdot\vec{v}$

with the kinetic energy $T = \frac{mc^2}{\sqrt{1-(\vec{v}/c)^2}}$

Fundamental classical and quantum 4-vectors

energy-momentum classical $p^\mu = \left(\frac{E}{c}, \vec{p}\right)$ quantum $p_\mu = i\hbar(\frac{1}{c}\partial_t, -\vec{\nabla}) = i\hbar\partial_\mu$

angular momentum $L^\mu = p_\nu x_\kappa \varepsilon^{\mu\nu\kappa}$ non-relativistic pseudo-3-vector $\vec{L} = \vec{p}\times\vec{x}$ quantum $L^\mu = i\hbar\partial_\nu x_\kappa \varepsilon^{\mu\nu\kappa}$

Fundamental symmetry operators

charge conjugation $C = q \to -q$ quantum $C\psi = i\gamma^2\psi^*$

parity=spatial reversal $P = \vec{x} \to -\vec{x}$ quantum $P = i\gamma^0$

time reversal $T = t \to -t$ quantum $T = i\gamma^1\gamma^3$

the product $CPT = -i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma^5$ is a Lorentz-invariant and CPT is conserved under all interactions
CP weakly violated under weak interactions (kaon decay rel. deviation ~10^{-6})

Classical relativistic and Newtonian hydrodynamics obey the (viscous) Navier-Stokes and (non-viscous) Euler equations [9]

relativistic Navier –Stokes $\left(\gamma\rho c^2 + p\right)u^\mu\partial_\mu u^\nu = -\partial^\mu p + \mu\partial_\nu\partial^\nu u^\mu + f^\mu$

ordinary Navier –Stokes $\rho\left(\partial_t + \vec{v}\cdot\nabla\right)\vec{v} = -\nabla p + \mu\nabla^2\vec{v}$

relativistic Euler-equations $\left(\gamma\rho c^2 + p\right)u^\mu\partial_\mu u^\nu = -\partial^\nu p - u^\nu u^\mu\partial_\mu p$

ordinary Euler-equations $\rho\left(\partial_t + \vec{v}\cdot\nabla\right)\vec{v} = -\nabla p$

Classical relativistic and Newtonian linear elastomechanics obey the generalized Hooke's law [10]
general Newtonian Hooke's law with displacement $\vec{u}$, stress potential σ

$$\rho\ddot{\vec{u}} - \nabla\sigma = \vec{F} \quad \sigma = C_{ij}\varepsilon_{ij} \quad \varepsilon = \frac{1}{2}\left(\nabla\circ\vec{u} + \left(\nabla\circ\vec{u}\right)^T\right)$$

relativistic Hooke's equation in displacement u^μ , stress potential σ , stiffness tensor C

$\frac{d}{d\tau} mv_\mu - \partial_\mu\sigma = K_\mu$, relativistic force K_μ , where $\vec{K} = \frac{\vec{F}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$$\frac{d}{dt}\frac{m\vec{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - \nabla\sigma = \vec{F} \quad \sigma = C_{\mu\nu}\varepsilon^{\mu\nu} = C_{\mu\nu}\frac{1}{2}\left(\partial_\mu u^\nu + \partial_\nu u^\mu\right)$$

General relativity (GR) is based on two principles

-General covariance: GR equations are **invariant under arbitrary (analytic) coordinate transformations**
Therefore GR is the *differential geometry of the metric* in the sense of Gauss and Riemann, the *GR-equations-of-motion* are the Gaussian *geodesic equations* of the shortest connection $\frac{d^2x^\kappa}{d\tau^2} = -\Gamma^\kappa{}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$.

-GR metric equations are derived from the action minimum principle with the GR invariant curvature *R*, and this GR **invariant curvature and energy density are equivalent**

This simplest possible Lagrangian density is the celebrated Einstein-Hilbert-Lagrangian density $\frac{R-2\Lambda}{2\kappa}$,
where R is the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$ derived from the Ricci tensor $R_{\mu\nu} = R^{\kappa}{}_{\mu\kappa\nu}$ and the Riemann tensor $R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}{}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}$, where $\Gamma^{\lambda}{}_{\mu\nu}$ are the Christofffel symbbols, and Λ is the cosmological constant.
This means that the GR Lagrangian density has the form $\frac{R-2\Lambda}{2\kappa} + L_M$,
where $\frac{R-2\Lambda}{2\kappa}$ is the metric part, and the dimensional factor $\frac{1}{2\kappa}$ (κ is the Einstein constant) converts the curvature R into energy density, L_M is the relativistic mass-energy density,
with the GR-action $S = \int\left(\frac{R-2\Lambda}{2\kappa} + L_M\right)\sqrt{-g}\,d^4x$, and the resulting variational equations are

$$\delta S = 0$$

$$\frac{1}{2\kappa}\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{2\kappa}\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} - \frac{\Lambda}{\kappa}\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} + \frac{\delta L_M}{\delta g^{\mu\nu}} + \frac{L_M}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = 0$$

which give the Einstein equations
$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$, with $\kappa = \frac{8\pi G}{c^4}$ and $T_{\mu\nu} = L_M g_{\mu\nu} - 2\frac{\delta L_M}{\delta g^{\mu\nu}}$ being the energy-momentum tensor .

A **gauge field theory** e.g. electrodynamics is formulated with covariant derivative $D_{\mu} = \partial_{\mu} + i\frac{e}{\hbar}A_{\mu}$ we get the classical momentum $p_{\mu} = m\frac{dx_{\mu}}{d\tau} + eA_{\mu}$, quantum momentum $p_{\mu} = i\hbar\partial_{\mu} + eA_{\mu}$
the relativistic Lagrangian becomes $L_c = \frac{1}{2}m\frac{dx^{\mu}}{d\tau}\frac{dx_{\mu}}{d\tau} - V\left(x^{\mu}\right)$, $L_c = \frac{1}{2}m\frac{dx^{\mu}}{d\tau}\frac{dx_{\mu}}{d\tau} + e\frac{dx^{\mu}}{d\tau}A_{\mu}$
For **relativistic Newtonian gravity** , the ansatz for the exact relativistic Newtonian gravitational force equation is with the Schwarzschild radius $r_s = \frac{2GM}{c^2}$

$$\frac{d}{d\tau}mu_{\nu} = \partial_{\nu}\frac{GMm}{\sqrt{x^{\mu}x_{\mu}}} = \partial_{\nu}\frac{mr_s c^2}{2\sqrt{x^{\mu}x_{\mu}}} \quad \text{where } x^{\mu}x_{\mu} = r^2 - c^2t^2$$

after integration over t and using the infinitesimal velocity addition theorem $dv \to dv\left(1 - \frac{v^2}{c^2}\right)$

we get the space part $\frac{d}{dt}\left(\frac{m\dot{\vec{r}}}{\sqrt{1-\frac{v^{2}}{c^2}}^{\,2}}\right) = -GMm\frac{\vec{r}}{r^3}\frac{1}{\left(1-\frac{v^2}{c^2}\right)}$, the fully **relativistic gravitational force equation** [5] and the corresponding **1-order correction** to effective potential becomes $\delta V = V\frac{v^2}{c^2} = -\frac{GMm}{r}\frac{\left(L/m\right)^2}{c^2r^2}$,
which is **the same as the GR-correction** [4:32].

Lagrangian Newtonian is $L_N = \frac{m\left(\frac{dx(t)}{dt}\right)^2}{2} - V(x(t))$, eom Newton equation $\ddot{\vec{x}}(t) = F = -\nabla V(\vec{x})$

Newtonian-relativistic transition

Mathematically speaking, the special theory of relativity (STR) is an embedding of the Galilean RxR3 space, with the ordinary rotations of R3 as symmetry, into the R4 space, with the Lorentz group as the rotational symmetry group.

This embedding is nothing other than a mapping (scalar, 3-vector) -> 4-vector, which, however, must be *structurally conformal*, i.e., mathematically expressed, a homomorphism. This embedding is nothing other than a mapping

(scalar, 3-vector) → 4-vector

which, however, must be structurally conformal, i.e., mathematically expressed, a homomorphism.

The mapping in this case is:

$\left(t,\left(\vec{x}\right)\right)\rightarrow x^{\mu}=\left(ct,\vec{x}\right)$ for spacetime

$\left(E,\left(\vec{p}\right)\right)\rightarrow p^{\mu}=\left(E/c,\vec{p}\right)$ for momentum

Conformal mapping means that the *minimal principle of least action* and the resulting *Euler-Lagrange equations* are preserved. This is shown in the two tables (see below). The action integral must be a relativistic invariant, so the Lagrangian must be a function of proper time τ:

$S=\int_{\tau_1}^{\tau_2} f\left(\tau\right)d\tau=\int_{t_1}^{t_2} f\left(\tau\left(t\right)\right)dt\sqrt{1-\frac{v^2}{c^2}}$, the latter because of $d\tau=dt\sqrt{1-\frac{v^2}{c^2}}$ (Lorentz transformation) .

The simplest choice is $f=const=\varepsilon$, and Lagrangian becomes $L=\varepsilon\sqrt{1-\frac{v^2}{c^2}}$.

According to the correspondence principle $\lim\left(L_{rel}\left(v\right)|\frac{v}{c}\rightarrow 0\right)=L_N\left(v\right)$, the relativistic Lagrangian becomes is the limit of low velocity the Newtonian Lagrangian. From this follows expanding the root

$\lim\left(\varepsilon\left(-\frac{v^2}{2c^2}\right)|\frac{v}{c}\rightarrow 0\right)=m\frac{v^2}{2}$, so necessarily $L=-mc^2\sqrt{1-\frac{v^2}{c^2}}$.

The general expression is $L=mc^2 g_{\mu\nu}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}$ .where $d\tau=dt\sqrt{1-v^2/c^2}$ is the proper time

Due to the validity of the Euler-Lagrange equations, we obtain the relativistic energy relation (see Relativistic Lagrange equations)

$$E=cp^0=c\frac{\partial L}{\partial\frac{dx^0}{d\tau}}=mc^2\frac{dt}{d\tau}=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Table: Newtonian Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial\frac{dx_j}{dt}}-\frac{\partial L}{\partial x_j}=0$$

$\vec{p}=m\frac{d\vec{x}}{dt}$, $p_j=\frac{\partial L}{\partial\frac{dx_j}{dt}}$

Lagrangian $L=\frac{mv^2}{2}$

Table: Relativistic Euler-Lagrange equations

$$\frac{d}{d\tau}\frac{\partial L}{\partial\frac{dx^{\mu}}{d\tau}}-\frac{\partial L}{\partial x^{\mu}}=0$$

$p^\mu = m\frac{dx^\mu}{d\tau}$, $p^\mu = \frac{\partial L}{\partial \frac{dx^\mu}{d\tau}}$

Lagrangian $L = -mc^2\sqrt{1-\frac{v^2}{c^2}}$

Quantum-classical transition

Quantum mechanics transforms into classical mechanics under the condition $\frac{\hbar}{E\,\Delta t} << 1$

The energy-momentum $p^\mu = \left(\frac{E}{c}, \vec{p}\right)$ is a 4-vector and its invariant length yields the relativistic energy relation

$p^\mu p_\mu = \left(\frac{E}{c}\right)^2 - \vec{p}^2 = m^2c^2$

E transforms like t (0-component of a 4-vector) , so we have the relativistic Einstein energy formula

$E = \frac{mc^2}{\sqrt{1-(v/c)^2}}$, which becomes $E \approx mc^2 + \frac{mv^2}{2}$, or $E_{kin} = \frac{mv^2}{2}$ in the Newtonian approximation.

Classical 4-momentum $p_\mu = (\frac{E}{c}, \vec{p})$ **transforms into quantum momentum** operator

$p_\mu = i\hbar(\frac{1}{c}\partial_t, -\vec{\nabla}) = i\hbar\partial_\mu$

Classical variables commute, quantum operators obey commutator relations for general coordinates and conjugated momenta

$[x,p] = i\hbar$, which lead to uncertainty relation $\Delta x \Delta p \geq \frac{\hbar}{2}$

Classical bound systems have continuous energy E, quantum bound systems (total energy E<0) have discrete energy E_i = eigenvalues of hamiltonian operator $H\psi = E_i\psi$

From the quantum transition we get with the Newtonian energy formula the **Schrödinger equation**

$i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r},t)\right)\Psi(\vec{r},t)$

and from the relativistic energy relation the **Dirac equation**

$\left(i\hbar\gamma^\mu\partial_\mu - mc\right)\Psi = 0$ with the 4-component bi-spinor wave function Ψ for signature $\eta_{\mu\nu} = diag(+1,-1,-1,-1)$

The anti-commutation relations for the Dirac 4x4-matrices

$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\eta\nu}$

arise from the requirement for the squared equation to become the relativistic energy relation

$-\hbar^2\frac{\partial^2\Psi}{c^2\partial t^2} - \left(i\hbar\gamma^0\gamma^k\partial_k + \gamma^0 mc\right)^2\Psi = 0$

The Dirac Lagrangian is $L_D\left(x^\mu(\tau)\right) = \bar{\Psi}\left(x^\mu(\tau)\right)\left(i\hbar c\gamma^\mu\partial_\mu - mc^2\right)\Psi\left(x^\mu(\tau)\right)$ or

$L_D\left(x^\mu(\tau)\right) = \bar{\Psi}\left(x^\mu(\tau)\right)\left(i\gamma^\mu\partial_\mu - m\right)\Psi\left(x^\mu(\tau)\right)$ ($\hbar = c = 1$ convention)

with the action $S(L) = \int_{\tau_1}^{\tau_2} L\left(x^\mu(\tau), \tau\right)d\tau$

The eom for a massive vector boson is the **Proca equation** as generalized Maxwell equation

$\Box A^\nu - \partial^\nu\left(\partial_\mu A^\mu\right) + \frac{m^2c^4}{\hbar^2}A^\nu = j^\nu$

with the Lagrangian $L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2c^4}{2\hbar^2}A_\mu A^\mu - j^\mu A_\mu$

The eom for a massive 3/2-spinor is the **Rarita-Schwinger equation**

$\left(-i\hbar\varepsilon^{\mu\kappa\rho\nu}\gamma_5\gamma_\kappa\partial_\rho - mc\sigma^{\mu\nu}\right)\psi_\nu = 0$ for the 4-vector-spinor ψ_ν with the Dirac-sigma matrices $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu,\gamma^\nu\right]$

with the Lagrangian $L = -\frac{1}{2}c\bar{\psi}_\mu\left(-i\hbar\varepsilon^{\mu\kappa\rho\nu}\gamma_5\gamma_\kappa\partial_\rho - mc\sigma^{\mu\nu}\right)\psi_\nu$

A **quantum orbit** is a probability density $\left|\Psi(x,t)\right|^2$ of the wave function $\Psi(x,t)$, with mean orbit location $\bar{x}(t) = \int \Psi^*(x',t)x'\Psi(x',t)dx'$, and orbit width $\Delta x(t) = \sqrt{\int \Psi^*(x',t)\left(\bar{x}(t)-x'\right)^2\Psi(x',t)dx'}$.

A **classical orbit** is a function $x(t)$ with orbit width $\Delta x(t) = 0$.

The **transition between the two** is described by the semi-classical WKB-approximation

For $\frac{d\lambda}{dx} << 1$ (de Broglie wavelength slowly varying)

$$\Psi(x) = C_\pm \frac{\exp\left(\pm\frac{i}{\hbar}\int\sqrt{2m\left(E-V(x)\right)}dx\right)}{\left(2m\left(E-V(x)\right)\right)^{1/4}},$$

where $\Psi(x)$ is fast-oscillating in the classical limit $\hbar << \int\sqrt{2m\left(E-V(x)\right)}dx$,

and therefore the orbit width $\frac{\Delta x(t)}{\left|\bar{x}(t)\right|} << 1$

Collapse of wavefunction and quantum-gravitational decoherence

The collapse of wavefunction in the quantum regime is described by the *objective collapse theory* put forward Ghirardi-Rimini-Weber (GRW) [13] , the wave function collapse is characterized by the delocalization width r_c and by the decay rate λ .

The decay rate is $\Lambda_d = N\lambda$, modified by delocalization width decay rate is $\Lambda_d = N\lambda e^{r/r_c}$, where r is the average distance in the ensemble.

The decay time is $\tau = 1/\Lambda_d = \frac{1}{N\lambda e^{r/r_c}}$.

The GRW theory is supported by AK quantum gravity with GRW parameters

$r_C = r_{gr} = \sqrt{l_P\sqrt{\frac{1}{\Lambda}}} = 3.9*10^{-5}m = 39\mu m$ and $\lambda = \lambda_\Lambda = \frac{E_\Lambda}{\hbar} = 0.33*10^{-13}s^{-1}$ with the Λ energy scale $E_\Lambda = \frac{\hbar c}{R_\Lambda} = 2.07*10^{-33}eV$, and r_{gr} quantum gravitational limit scale. In this interpretation, quantum gravitational decoherence is the effect of a *graviton bath with mean energy* E_Λ , and maximum mean particle distance r_{gr} .

The spontaneous collapse of the wave function is the basis for fast quantum-gravitational decoherence of quantum entanglement in the classical regime.

The *transition from the quantum to the classical regime happens by decoherence*, which becomes effectively momentary, when the decoherence time is *below the signal time of the object* $\tau = \frac{1}{\Lambda_d} < t_{obj} = \frac{r_{obj}}{c}$ (r_{obj} is the object diameter).

Thermal decoherence

Approximate formula for thermal decoherence is [14]

$\tau \approx \frac{\hbar}{E_{th}\rho_n r^2 r_B}$, with Bohr radius $r_B = 5.3\times10^{-9}cm$, thermal energy $E_{th} = k_B T$, particle density ρ_n , mean particle distance r .

Example

- For $N = N_A = 10^{23}$, $r = 0.3nm$ (e,g, in water),

$$\Lambda_d = N\,\lambda\, e^{r/r_c} = 10^{23}\, 0.33\times10^{-13} s^{-1}\, exp\left(0.3\times10^{-9} / 3.9\times10^{-5}\right) = 0.33\times10^{10} s^{-1}$$

- For large average distance $r = 1cm$ (diluted gas in quasi-vacuum),

$$\Lambda_d = N\,\lambda\, e^{-r/r_c} = 10^{23}\, 0.33\times10^{-13} s^{-1}\, exp\left(10^{-2} / 3.9\times10^{-5}\right) = 0.33\times10^{10} s^{-1}\, 10^{0.434\times256.4} = 0.62\times10^{121} s^{-1} ,$$

decay time $\tau = 1/\Lambda_d = 1.6\times10^{-121} s$ is smaller than Planck time $\tau \ll t_{Pl} = 0.54\times10^{-43} s$, so it is effectively zero: *there is no entanglement for average distance well above the quantum gravitational limit scale.*

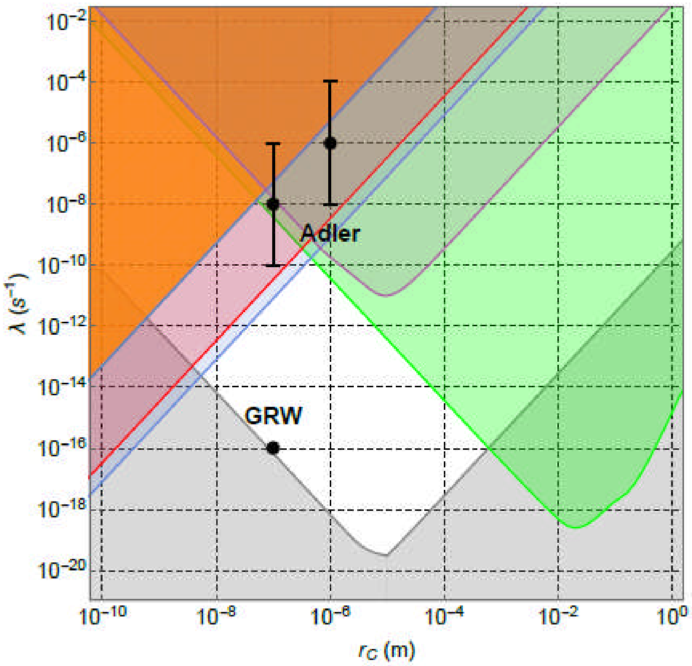
- Schrödinger's cat , quantum-gravitational decoherence , *m*=5kg (mainly water, molecular weight 18), $N=N_A$*5000g/18g=2.8*10^{25}$, $r = 0.3nm$, $r_{cat} = 50cm$, $t_{cat} = r_{cat} / c = 0.17\times10^{-8} s$

$$\Lambda_d = N\,\lambda\, e^{r/r_c} = 2.8\times10^{25}\, 0.33\times10^{-13} s^{-1}\, exp\left(0.3\times10^{-9} / 3.9\times10^{-5}\right) = 0.92\times10^{12} s^{-1} ,\ \tau = 1.1\times10^{-12} s$$

decoherence is momentary, as $\tau < t_{cat}$

- Schrödinger's cat at lab conditions, thermal decoherence [14], *m*=5kg, $r_{cat} = 50cm$, signal time $t_{cat} = r_{cat} / c = 0.17\times10^{-8} s$, decoherence time $\tau_{cat} = 10^{-28} s$

Recently, possible GRW parameter values have been constrained experimentally [12], as shown in the plot below.



The values $r_C = r_{gr} = 3.9*10^{-5} m$ and $\lambda = \lambda_\Lambda = \frac{E_\Lambda}{\hbar} = 0.33*10^{-13} s^{-1}$ lie in the experimentally allowed white area, whereas the original GRW values lie at its boundary, and the modified Adler's values are excluded.

Fundamental equations based on relativistic scalars

The relativistic energy relation $p^\mu p_\mu = \left(\frac{E}{c}\right)^2 - \vec{p}^2 = m^2c^2$ yields by the transformation

$p_\mu = i\hbar(\frac{1}{c}\partial_t, -\vec{\nabla}) = i\hbar\partial_\mu$ the **wave equation** $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right)\psi = 0$ for *m=0* (massless field carriers)

and the **Klein-Gordon equation** for massive scalars $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta + \frac{m^2c^2}{\hbar^2}\right)\psi = 0$

With the mass current density $j^\mu = \left(\rho, \vec{j}\right)$ we get the relativistic invariant $\partial_\mu j^\mu = \frac{\partial\rho}{\partial t} + \nabla\cdot\vec{j} = \sigma$ and

the continuity equation $\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$ for $\sigma = 0$

From this follow transport equations

heat equation $\frac{\partial u}{\partial t} + \nabla \cdot \vec{q} = 0$ or $\frac{\partial u}{\partial t} - K\,\Delta u = 0$ for temperature u and heat flow $\vec{q}$

and Fick's **diffusion equation** $\frac{\partial \varphi}{\partial t} - D\Delta\varphi = 0$ for the concentration φ

Basic algebraic structure

Classical physics is based on the (algebraic) field of real numbers R with ist algebraic and topological structure.

Quantum physics operates on complex numbers C for vectors, and on quaternions $Q = C(\sigma_\mu)$ (= complex numbers with Pauli-matrices added) for spinors.

Complex functions and differential equations

Physical variables in quantum mechanics are *complex* analytic functions $f(x_\mu, \{\kappa_i\})$ of (real) position x_μ and parameters κ_i , most important: wave function $\psi(x_\mu)$.

Physical variables in classical mechanics are *real* analytic functions $f(x_\mu, \{\kappa_i\})$ of (real) position x_μ and parameters κ_i , most important: orbit $x_\mu(\tau)$.

The fundamental Euler-Lagrange equations, which are orbit and wave function equations, are normally *linear partial differential equations of second order* in position x_μ .

There are 3 fundamental pdeq's in physics

elliptic: Poisson equation $\nabla^2\varphi = f$, e.g. gravity $\nabla^2\varphi = 4\pi G\rho$, electrostatics $\nabla^2\varphi = -\frac{\rho}{\varepsilon}$

hyperbolic: wave equation $\Box A^\mu = 0$ where $\Box = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is the d'Alembert operator

parabolic: heat equation $\frac{\partial u}{\partial t} - \alpha\nabla^2 u = 0$

Dimension and fundamental parameters

There are **6 fundamental independent dimensional constants**

reduced Planck constant $\hbar = h/2\pi$

light velocity c

dielectrical constant ε_0

Boltzmann constant k_B

Planck length $l_p = \sqrt{\frac{\hbar G}{c^3}}$ derived from Newton's gravitational constant G

cosmological length $R_\Lambda = 1/\sqrt{\Lambda}$ derived from the cosmological constant Λ

The first four are purely dimensional constants

energy= $\hbar$ /time

time=length/c

charge= $\sqrt{4\pi\varepsilon_0}\,\sqrt{\text{energy length}}$

temperature=energy/ k_B

and the Planck length $l_p = \sqrt{\frac{\hbar G}{c^3}}$ makes everything dimensionless, and defines the minimal length (and maximal energy)

$R_\Lambda = 1/\sqrt{\Lambda}$ defines the maximal length (and minimal energy)

The truly universal parameters are **8 energy-mass constants** plus **2 cosmological length** and **1 cosmological curvature sign**:

-masses of preons: $m_r = 0.033 meV$ $m_q = 0.77 MeV$
-characteristic energies of interaction
hypercolor SU(4) interaction $E = 180 GeV$ and zero-shift constant $c_{GE} = 0.095$
color (strong) SU(3) interaction $E = 220 MeV$ and zero-shift constant $c_{GE} = 0.69$
electromagnetic interaction $E = \frac{\hbar c}{r_B} = \frac{m_e c e^2}{4\pi\varepsilon_0 \hbar} = 3.7 keV$ (derived from elementary charge *e₀*)
Ashtekar-Kodama gravitation $E = \hbar c\sqrt{\Lambda} = \frac{m_e c e^2}{4\pi\varepsilon_0 \hbar} = 2.06*10^{-33} eV$ derived from the cosmological constant Λ
-cosmological characteristic lengths
constant $D = \frac{8\pi G}{3c^2}\rho_m R^3 = 1.61*10^{26} m$
constant $\sqrt{K} = \sqrt{\frac{8\pi G}{3c^2}\rho_r R^2} = 1.6*10^{24} m$
-sign of curvature $k = 0, -1, +1$ (best present value=0)

The fourth independent cosmological parameter: the age of universe t_0 is not a true fundamental parameter of our world, since it shows only our temporal position in it.

Geometry: metric and gravity

Geometry of classical space-time is determined by the metric, the empty space-time has the flat Minkowski metric
in Cartesian coordinates
$\eta_{\mu\nu} = diag(+1, -1, -1, -1)$
in spherical coordinates
$\eta_{\mu\nu} = diag(+1, -1, -r^2, -r^2 \sin^2\theta)$
spherically symmetrical space-time with mass and scale r_s has the Schwarzschild metric

$$g_{\mu\nu} = \begin{pmatrix} \left(1-\frac{r_s}{r}\right) & & & \\ & -\frac{1}{\left(1-\frac{r_s}{r}\right)} & & \\ & & -r^2 & \\ & & & -r^2\sin^2\theta \end{pmatrix}$$

and axially symmetric space-time with rotating mass and scale r_s has the Kerr metric

$$g_{\mu\nu} = \begin{pmatrix} \left(1-\frac{r r_s}{\rho_{12}}\right) & & & \frac{r r_s a \sin^2\theta}{\rho_{12}} \\ & -\rho_{12}/\Lambda_{12} & & \\ & & -\rho_{12} & \\ \frac{r r_s a \sin^2\theta}{\rho_{12}} & & & -\sin^2\theta(r^2 + a^2 + \frac{r r_s a^2 \sin^2\theta}{\rho_{12}}) \end{pmatrix}$$

where $\Lambda_{12} = r^2 - r r_s + a^2$, $\rho_{12} = (r^2 + a^2\cos^2\theta)$

In quantum regime $r << r_{gr}$, $r_{gr} = \sqrt{l_P\sqrt{\frac{1}{\Lambda}}} = 3.1*10^{-5} m = 31 \mu m$, the metric is the Minkowski metric .

The valid geometry is the *inner geometry of the metric* (in the sense of Gauss, i.e. those properties of the geometry, which are coordinate-independent). This geometry is *generally invariant* (under arbitrary coordinate transformations), and also physical interactions at large scales.

The only long-range interaction coupled to the tetrad (metric) is gravity, therefore in order for gravity to come out in the limit, the **Einstein equations** arise as a combination of (generally covariant) tensors $G_{\mu\nu}$, $T_{\mu\nu}$, $g_{\mu\nu}$:

$G_{\mu\nu}+\Lambda g_{\mu\nu}=\kappa T_{\mu\nu}$, with $G_{\mu\nu}=R_{\mu\nu}-\frac{R}{2}g_{\mu\nu}$ and $\kappa=\frac{8\pi G}{c^4}=\frac{8\pi l_p^{\ 2}}{\hbar c}=\frac{8\pi l_p}{m_p c^2}$, with Planck length l_p and Planck mass m_p, where κ and $G_{\mu\nu}$ are determined in such a way that in the limit the Poisson equation of gravity results: $g_{00}=\left(1+\frac{\varphi}{c^2}\right)^2\approx 1+\frac{2\varphi}{c^2}$, $\nabla^2\varphi(r)=4\pi G\rho(r)$,

it follows $\varphi(r)=-\frac{GM}{r}=-\frac{r_s}{2r}c^2$, so the Schwarzschild space-time has $g_{00}=1-\frac{r_s}{r}$ and it follows from the above ansatz $|g_{11}|=\frac{1}{g_{00}}=\frac{1}{1-\frac{r_s}{r}}$.

The Einstein equation are derived via minimum principle from the **Einstein-Hilbert action**

$$S=\int\left(\frac{(R-2\Lambda)}{2\kappa}+L_M\right)\sqrt{-g}\,d^4x$$

where L_M is the interaction Lagrangian for the stress-energy tensor (energy density)

$$T_{\mu\nu}\equiv-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_M\right)}{\delta g^{\mu\nu}}=-2\frac{\delta L_M}{\delta g^{\mu\nu}}+g_{\mu\nu}L_M$$

We get for the Einstein-Hilbert Lagrangian in Planck units with the Planck energy density $\frac{m_p c^2}{l_p^{\ 3}}$ and the 3-dimensional space angle $\Omega_3=4\pi$ and the 1-dimensional time angle $\Omega_1=\int_{-1}^{1}dt=2$

the physical-geometrical density relation

$\frac{L_{EH}}{m_p c^2/l_p^{\ 3}}=\frac{1}{2\Omega_1\Omega_3}\frac{(R-2\Lambda)}{1/l_p^{\ 2}}$ i.e. the physical density equals half the Gaussian geometrical metric curvature normalized over the 4-space angle.

Geometric symmetry group

The fundamental symmetry group is the Lorentz group (rotation *SO(1,3)*) + displacements=Poincare group. All physical equations are Lorentz-covariant or Lorentz-invariant.

SO(1,3) is the group of orthogonal rotations of the real 4-dimensional vector space R^4 with the Minkowski metric $g_{\mu\nu}=diag(+1,-1,-1,-1)$, which satisfies the orthogonality relation $\Lambda^t g\Lambda=g$.

Lorentz-group of rotations Λ 4x4 (1-vector) representation:

$$\Lambda=\exp\left(-\frac{i}{2}\omega_{\mu\nu}M^{\mu\nu}\right)=\exp(-i(\vec{\phi}\,\vec{J}+\vec{\nu}\,\vec{K}))$$

where $\vec{\phi}$ is the rotation angle, $\vec{\nu}$ is the boost angle, where v is the velocity, and elementary space- and time-rotations are

elementary plane rotation is $U_{rot}(\varphi)=\begin{pmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{pmatrix}$

elementary boost (x,t)-rotation is $U_{boost}(\nu)=\begin{pmatrix}\cosh\nu & -\sinh\nu\\ -\sinh\nu & \cosh\nu\end{pmatrix}$ with $\nu=arctanh(\frac{v}{c})$

or explicitly $t'=\frac{t-v\,x/c^2}{\sqrt{1-v^2/c^2}}$ $x'=\frac{x-v\,t}{\sqrt{1-v^2/c^2}}$

pure rotations $\vec{J}=(J_1,J_2,J_3)$ boosts $\vec{K}=(K_1,K_2,K_3)$

$$J^i = M^{\pi 3(i)} \quad \text{e.g.} \quad J^1 = M^{23} = i\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

and $K^i = M^{0i}$ e.g. $K^1 = M^{01} = i\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

π3(i) is the i-rest of the 3-permutation , e.g. π3(1)=(2,3)
J^i and K^i satisfy the commutator algebra
$[J^i, J^j] = i\,\varepsilon^{ijk} J^k \quad [J^i, K^j] = i\,\varepsilon^{ijk} K^k \quad [K^i, K^j] = -i\,\varepsilon^{ijk} K^k$
$(M^{\mu\nu})^\rho_\sigma = -i\,(g^{\mu\rho}\delta^\nu_\sigma - g^{\nu\rho}\delta^\mu_\sigma)$ are the 6 generators in the tensor representation.

Physical particles differ in their *transformation property*: they are scalars (Lorentz-invariant) or vectors-tensors (integer spin *S= n*, n*1-vector-representation of Lorentz group) or spinors (half-integer spin *S=n+1/2*, (2n+1)*1/2-Weyl-representation of Lorentz group, *S=1/2*: Weyl-L or Weyl-R spinor).
Physical particles are distinguished as *field carriers* (massless bosons, *S=1* or *2* , moving with *v=c* : graviton *S=2*, photon, gluon, hc-boson *S=1*, cosm. Λ-generator φ_Λ *S=0*) or *basic proper particles* (massive fermions, *S=1/2, v<c* : preons, leptons, quarks) or *composite proper particles* (massive fermions or bosons, *S=n+1/2* or *S=n* , *v<c* : all remaining particles including hc-Yukawa W-boson, Z-boson (*S=1*), and higgs H (*S=0*) .

Lorentz-transformation behavior of **vectors** (spin s=1) and **spinors** (spin s=1/2) :
rotation 4-vector *X* with *Λ* : $X' = \Lambda X$

$$\Lambda = \exp\left(-\frac{i}{2}\omega_{\mu\nu} M^{\mu\nu}\right) = \exp(-i(\vec{\phi}\,\vec{J} + \vec{v}\,\vec{K}))$$, where $(M^{\mu\nu})^\rho_\sigma = -i\,(g^{\mu\rho}\delta^\nu_\sigma - g^{\nu\rho}\delta^\mu_\sigma)$

rotation left-handed, right-handed spinor *Ψ* with A_L , A_R : $\Psi' = A_L \Psi \quad \Psi' = A_R \Psi$

$$A_L = \exp\left(-\frac{i}{2}\omega_{\mu\nu}\,\sigma^{\mu\nu}\right) = \exp(-\frac{i}{2}(\vec{\phi} - i\vec{v})\vec{\sigma})$$, left-handed Weyl-spinor

$$A_R = \exp\left(-\frac{i}{2}\omega_{\mu\nu}\,\tilde{\sigma}^{\mu\nu}\right) = \exp(-\frac{i}{2}(\vec{\phi} + i\vec{v})\vec{\sigma})$$, right-handed Weyl-spinor

rotation Dirac bi-spinor *Ψ* with *S* : $\Psi' = S\,\Psi$

$$S = \exp\left(-\frac{i}{2}\omega_{\mu\nu}\,\Sigma^{\mu\nu}\right) = \begin{pmatrix} A_L & \\ & A_R \end{pmatrix}$$, where

Pauli 3-vector $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$

$\Sigma^{\mu\nu} = \begin{pmatrix} \sigma^{\mu\nu} & \\ & \tilde{\sigma}^{\mu\nu} \end{pmatrix} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$, generators Clifford-algebra of Dirac-matrices $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\,1(4,4)$

Pauli spin-tensor $\sigma^{\mu\nu} = \frac{i}{4}[\sigma^\mu, \tilde{\sigma}^\nu]$, $\tilde{\sigma}^{\mu\nu} = \frac{i}{4}[\tilde{\sigma}^\mu, \sigma^\nu]$, $\sigma^\mu = \frac{1}{2}(1, \vec{\sigma})$, $\tilde{\sigma}^\mu = \frac{1}{2}(1, -\vec{\sigma})$

Spin and representations
Spin *S* is the (fractional to standard dimension 4) *dimension of the corresponding transformation matrix representation* of the Lorentz group
S=1/2 for left- or right Weyl-spinors with 2x2 matrix representation A_L for the left-handed Weyl-spinor Lorentz rotation $\Psi_L(x) \rightarrow \Psi'_L(x') = A_L \Psi_L(x)$
and A_R for the right-handed Weyl-spinor Lorentz rotation $\Psi_R(x) \rightarrow \Psi'_R(x') = A_R \Psi_R(x)$
S=1 for the 4x4 vector representation for contravariant vector Lorentz rotation $\Psi(x) \rightarrow \Psi'(x') = \Lambda\,\Psi(x)$

and for the 4x4 vector representation for covariant vector Lorentz rotation $\Psi(x) \to \Psi'(x') = \left(\Lambda^t\right)^{-1}\Psi(x)$,
where $\left(\Lambda^t\right)^{-1}$ is the transposed inverse of the Lorentz rotation Λ .
S=2 for the 8x8 vector-tensor representation for contravariant 2-tensor Lorentz rotation
$\Psi(x) \to \Psi'(x') = \Lambda\Psi(x)\Lambda$

Gauge fields
All four fundamental interactions: gravity, electromagnetism, color interaction, weak (hyper-color) interaction are described by gauge field theory.
-Electromagnetism
Gauge transformation is basically a phase-shift of the wave function induced by a phase $\Lambda(x)$:
$\Psi(x) \to \Psi'(x) = \exp\left(ie\Lambda(x)\right)\Psi(x) = \Omega(x)\Psi(x)$, where $\Omega(x) = \exp\left(ie\Lambda(x)\right)$ is the gauge transformation.
The momentum operator should be covariant under the gauge transformation, so we have to introduce the covariant derivative
$D_\mu = \partial_\mu + ieA_\mu$ instead of the original 4-derivative vector ∂_μ .
The covariant derivative transforms correctly under the gauge transformation
$\left(D_\mu\Psi(x)\right)' \to \Omega(x)D_\mu\Psi(x)$
The Maxwell field tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
In QFT, we use cgs units and “natural units” defined by the convention $\hbar = 1 \quad c = 1$, and $e = \sqrt{4\pi\alpha}$ is the dimensionless interaction constant of the gauge theory (the interaction constant must be dimensionless, otherwise the theory is non-renormalizable).
In cgs units without the QFT-convention, the covariant derivative reads $D_\mu = \partial_\mu + i\frac{e}{\hbar}A_\mu$

-General non-commutative gauge theory
The general gauge transformation, valid for gravity, color interaction, weak (hyper-color) interaction, is based on operators τ^a with the Lie-algebra $\left[\tau^a, \tau^b\right] = f^{abc}\tau^c$, the field carriers $A_\mu{}^a(x)$, the gauge transformation $\Omega(x) = \exp\left(-i\,g\Lambda^a(x)\,\tau^a\right)$, and the covariant derivative $D_\mu = \partial_\mu - igA_\mu{}^a(x)\tau^a$, where g is the (dimensionless) interaction constant.
The covariant derivative transforms correctly under the gauge transformation
$\left(D_\mu\Psi(x)\right)' \to \Omega(x)D_\mu\Psi(x)$
The field tensor (corresponding to the electromagnetic Maxwell tensor) is
$G^a{}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,f^{abc}A^b_\mu A^c_\nu$
For the color interaction, $A_\mu{}^a(x)$ are the eight gluon wave functions, and the Lie-algebra is the SU(3) algebra with the eight 3x3 Gell-Mann matrices λ^a as generators, $\tau^a = \lambda_a / 2$.

2 Interactions

Long-range (classical) interactions

Long range (classical) interactions are gravity and electromagnetism. Both have also a quantum version. Green's third theorem states $\int_\Sigma \nabla\varphi\, d\Sigma = \int_V \Delta\varphi\, dV$, i.e. the surface integral over the field $\nabla\varphi$ is equal to the enclosed charge *Q*, if φ satisfies the Poisson equation $\Delta\varphi = \rho$ for the charge density ρ . Therefore long-range interactions should obey the Poisson equation at infinity.

But then long range interactions at infinity *must have the form* $\varphi(r) = \frac{c_1}{r}$, because only this harmonic function is the potential of a Dirac-delta point source: $\nabla^2 \frac{1}{r} = 4\pi\delta^3(r)$.

The electrostatic potential fulfills this requirement exactly, the GR-gravity becomes the Newtonian potential in the limit $r \to \infty$.

electromagnetic classical regime

The interaction is described by the Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$ formulated with the field 3-vector electric field $\vec{E}$ and the 3-pseudovector magnetic field $\vec{B}$, $E_i = -cF^{0i}$, $B_k = -\varepsilon_{kij}F^{ij}$, which are dual in a certain sense.

Maxwell field tensor $F_{\mu\nu} = \partial_\mu A_\nu{}^\kappa - \partial_\nu A_\mu{}^\kappa$, Lagrangian $L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu$

4-vector potential $A^\mu = (\frac{\varphi}{c}, \vec{A})$ 4-current density $j^\mu = (c\rho, \vec{J})$

Maxwell equations in cgs-units $\nabla\cdot\vec{E} = 4\pi\rho \quad \nabla\cdot\vec{B} = 0 \quad \nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \quad \nabla\times\vec{B} = \frac{4\pi}{c}\left(\vec{J} + \frac{1}{4\pi}\frac{\partial\vec{E}}{\partial t}\right)$

$$\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t} \quad \vec{B} = \nabla\times\vec{A}$$

$$\varphi(\vec{x},t) = \int \frac{\rho\left(\vec{x}',t-\left(\vec{x}-\vec{x}'\right)/c\right)}{|\vec{x}-\vec{x}'|}d^3x' \quad \vec{A}(\vec{x},t) = \frac{1}{c}\int \frac{\vec{J}\left(\vec{x}',t-\left(\vec{x}-\vec{x}'\right)/c\right)}{|\vec{x}-\vec{x}'|}d^3x'$$

orbit equations $m\frac{du^\mu}{d\tau} = F^{\mu\nu}u_\nu$

Lorentz force density $\vec{f} = \rho\vec{E} + \rho\vec{v}\times\vec{B}$

Poynting vector (energy flux) $\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B}$

gravitational classical regime

field carrier graviton $A_\mu{}^\nu$ S=2

tetrad $E^{\mu\nu}$, which can be interpreted as generalized coordinate (gc) gc~1/√metric

The interaction is described by the 32 Ashtekar-Kodama-equations for graviton $A_\mu{}^\nu$ and tetrad $E^{\mu\nu}$ with the cosmological constant Λ

$H_{(\mu,\nu)}{}^\kappa \equiv F_{\mu\nu}{}^\kappa + \frac{\Lambda}{3}\varepsilon_{\mu\nu\rho}E^{\rho\kappa} = 0$ interpretation: *field= const.*asym(gc)* , i.e. antisymmetrized gc

$G^\mu \equiv \partial_\nu E^{\nu\mu} + \varepsilon^\mu{}_{\kappa\lambda}A_\nu{}^\kappa E^{\nu\lambda} = 0$ interpretation: *covariant-derivative*($E^{\mu\nu}$) $D_\nu E^{\nu\mu} = 0$

$I_\mu \equiv E^\kappa{}_\nu F_{\mu\kappa}{}^\nu = 0$ interpretation: generalized hom. Maxwell equation $E^\kappa{}_\nu F_{\mu\kappa}{}^\nu = 0$

in analogy to hom. Maxwell equation $F_\mu{}^\nu A_\nu = 0$

with field tensor $F_{\mu\nu}{}^{\kappa}$

$F_{\mu\nu}{}^{\kappa} = \partial_{\mu} A_{\nu}{}^{\kappa} - \partial_{\nu} A_{\mu}{}^{\kappa} + \varepsilon^{\kappa}{}_{\kappa_1\kappa_2} A_{\mu}{}^{\kappa_1} A_{\nu}{}^{\kappa_2}$

and boundary condition at $r \to \infty$ for tetrad and external metric $g_{\mu\nu}$

$E^{\mu\kappa} E^{\nu}{}_{\kappa} = g^{\mu\nu} / (-\det(g))^{3/4}$,

so the large-scale metric determines the physics via boundary condition

orbit equations $\dfrac{du^{\mu}}{d\tau} = -\Gamma^{\mu}_{\nu\lambda} u^{\nu} u^{\lambda}$, $E^{\mu\kappa} E^{\nu}{}_{\kappa} / (\det(E))^{3/4} = g^{\mu\nu}$

AK gravity yields full GR in the limit *$\Lambda \to 0$* , where $A_{\mu}{}^{\nu}$ *and* $E^{\mu\nu}$ decouple .

AK gravity removes the singularity of GR Black Holes with the AK correction length $l_0 = r_s{}^3 \Lambda$,

we get the corrected outer metric $\left|g_{11,c}\right| = \dfrac{1}{\left|g_{00,c}\right|} = \dfrac{1}{1 - \dfrac{r_s - l_0}{r}}$

with the sub-luminal escape velocity $v_c(r) = c\sqrt{1 - \dfrac{l_0}{r_s}} \approx c\left(1 - \dfrac{l_0}{2r_s}\right) < c$.

In the non-relativistic approximation GR-gravity becomes Newtonian gravity $E_N = -mc^2 \dfrac{r_s}{r}$

static AK-interaction: yields $A_{\mu}{}^{\nu}$*=const* , $E^{\mu\nu}$ satisifies the Einstein equations and generates the metric at infinity with the usual metric boundary condition $E \eta E^{t} = g^{-1} / \left(-\det g\right)^{3/4}$, so the field carrier is the tetrad, fully compatible with GR

wave AK-interaction: yields Ashtekar-Kodama wave-equations *AK*($A_{\mu}{}^{\nu}$,$E^{\mu\nu}$,Λ)=0 with Λ-scaled ansatz for the wave tensor $A_{\mu}{}^{\nu}$, $A_{\mu}{}^{\nu}$ carries energy and generates an *exponentially damped* metric wave $E^{\mu\nu}$.

The wave tensor $A_{\mu}{}^{\nu}$ and the tetrad $E^{\mu\nu}$ play different roles: $E^{\mu\nu}$ generates the metric and is measurable by length oscillation, $A_{\mu}{}^{\nu}$ carries energy and is measurable by absorption.

Quantum interactions (QFT)

electromagnetic: gauge-group=Lie group SU(1) (trivial) commutative, long-range, classical and quantum, scale Bohr radius $r_B = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = 5.29*10^{-11}m$, energy scale $E_{em} = \frac{\hbar c}{r_B} = 3.7keV$, the interaction-boson=photon is a vector with spin=1

charges: electrical charge *Q*+, *Q*- ; elementary (minimal) charge: e_0

covariant derivative $D_\mu = \partial_\mu + ieA_\mu$, fundamental equations Maxwell equations $\partial_\mu F^{\mu\nu} = 0$

the interaction constant is the fine structure constant $\alpha_{em} = \frac{e_0^{\ 2}}{4\pi} = \frac{1}{137}$, the elementary cross-section for electron-interaction is $\sigma_{em} = \alpha_{em}{}^2 \tilde{\lambda}_e{}^2 = \alpha_{em}{}^2 \left(\frac{\hbar}{m_e c}\right)^2$

wave: photon A_μ spin=1 one polarization, modes: spherical($l = 0$) and and all $l > 0$, angular function Y_{lm} , wave equation , radial Helmholtz equation, Lagrangian $L_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ with the Maxwell field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Dirac eq. $\left(i\hbar\gamma^\mu D_\mu - mc\right)\Psi = 0$

field eq. Maxwell $\partial_\mu F^{\mu\nu} = j^\nu$

self-field eq. $\partial_\mu F^{\mu\nu} = j^\nu = -ce\,\bar{\psi}\gamma^\nu\psi$

quantum gauge version QED is renormalizable, with 3 divergent Feynman diagrams: electron self-energy, photon self-energy, vertex correction

photon propagator in Feynman gauge is $D_{\mu\nu}\left(k\right) = \frac{-ig_{\mu\nu}}{k^2}$

quasi-classical approximation: $E_{el}(r) = \frac{e_0^{\ 2}}{4\pi\varepsilon_0\, r}$ of an electron for $r >> r_{ce} = \frac{e_0^{\ 2}}{4\pi\varepsilon_0\, m_e c^2} = 2.8*10^{-15}m$, where r_{ce} is the classical electron radius

gravitation: gauge-group=Liegroup($\varepsilon_{\kappa\lambda\mu\nu}$) with 4 generators τ_μ ,additional invariance: background independence, long-range, classical and quantum, its interaction-boson=graviton is a 2-tensor with spin=2

charges: mass , elementary (minimal) mass (minimal energy is reciprocal to the maximum length $R_\Lambda = 1/\sqrt{\Lambda}$)

$$m_\Lambda = \frac{\hbar}{cR_\Lambda} = \frac{\hbar\sqrt{\Lambda}}{c} = \frac{1.05*10^{-34}\,Js 1.052*10^{-26}\,m^{-1}}{3*10^8\,ms^{-1}} = 0.368*10^{-68}\,kg$$ energy scale $E_\Lambda = \frac{\hbar c}{R_\Lambda} = 2.07*10^{-33}eV$

scale $r_{gr} = \sqrt{l_P\sqrt{\frac{1}{\Lambda}}} = 39\mu m$, energy scale quantum $E_{gr} = \frac{\hbar c}{r_{gr}} = 5.0*10^{-3}eV$ and classical $E_{AK} = \hbar c\sqrt{\Lambda} = 2.06*10^{-33}eV$

fundamental equations: Ashtekar-Kodama equations $AK(A_\mu{}^\nu, E^{\mu\nu}, \Lambda) = 0$ for the graviton tensor $A_\mu{}^\nu$, the tetrad $E^{\mu\nu}$ and the cosmological constant Λ , in the limit $\Lambda \to 0$ the Einstein-equations are valid (except at the horizon)

and $A_\mu{}^\nu$ =const (i.e. trivial background)

Λ generated by the tensor field φ_Λ is scale dependent, classical scale $r >> r_{gr}$ $\Lambda = \Lambda_0 = 2.7\ 10^{-52} m^{-2}$

wave: graviton $A_\mu{}^\nu$ spin=2 four polarizations, modes: quadrupole and higher $l \geq 2$,

angular function $\exp(i\,l\,\theta)$, gravitational radial wave equation weq$_\nu$($E^{1\nu}, \partial_r{}^3 E^{1\nu}$) for the tetrad component $E^{1\nu}$

determines $A_\mu^{\ \nu}$,$E^{\mu\nu}$ with 4 free parameters for $E^{\mu\nu}$ and 4 free parameters for $A_\mu^{\ \nu}$, $E^{\mu\nu}$ damped $\exp(-4\sqrt{\frac{r}{3}})$

for $r \le r_{gr}$ QFT gravity,

covariant derivative $D_\mu = \partial_\mu - i\sqrt{\alpha_{gr}}\left(\tau_a A^a\right)_\mu$ $\left(D_\mu\right)^\lambda_{\ \kappa} = \partial_\mu + \varepsilon^\lambda_{\ a\kappa}\sqrt{\alpha_{gr}}A_\mu^{\ a}$

$\sqrt{\alpha_{gr}} = 0.29 \times 10^{-30}$ interaction constant

with 4 graviton column vectors $A_\mu^{\ a}$, $\tau^a = i\varepsilon_\nu^{\ a}{}_\lambda$ are the generators of the Lie(ε) Lie-algebra

graviton polarizations: 4 modes, 4 classical polarizations with real resp. imaginary amplitude

Lie(ε) Lie-algebra with 4 generators τ^a and the commutator $[\tau^a, \tau^b] = i\varepsilon_c^{\ ab}\tau^c$, a=1,2,3,4

equivalent to Lie3(ε) Lie-algebra with 3 generators τ^a and the commutator $\left[\tau_a, \tau_b\right] = i\sum_k \left(1+\varepsilon_{abk}\right)\tau_k$, a=1,2,3

which is isomorphic to SU(2)

field equations $AK(A_\mu^{\ \nu}, E^{\mu\nu}, \Lambda)$, graviton tensor $A_\mu^{\ \nu}$, tetrad $E^{\mu\nu}$

the interaction constant is, with a normalized one-graviton-wave tensor $A_\mu^{\ \nu}$, $\alpha_{gr} = \frac{l_p^{\ 2}}{\tilde{\lambda}(m)^2} = l_p^{\ 2}\left(\frac{mc}{\hbar}\right)^2$,

it depends on the graviton-generating mass m , $\tilde{\lambda}(m)$ is the reduced de-Broglie-wavelength of m .

The AK-graviton propagator is $D(q) = \frac{1}{q^3 - 3iq^2 - 2q} = \frac{1}{q(q-i)(q-2i)}$ of order 3, and is finite-integrable in d^3q

The elementary cross-section is $l_p^{\ 2}$ (independent of m_0) , which is very small.

the (dimensionally) renormalizable Lagrangian is

$$L_{gr} = L_H + L_I = \hbar c\left(\begin{array}{l} -\frac{1}{4}F_{\mu\nu}^{\ \ \kappa}F^{\mu\nu}_{\ \ \kappa} - \frac{\bar{\phi}_\Lambda\phi_\Lambda}{3}(\varepsilon^\kappa_{\ \mu\lambda}E^{\lambda\nu}\partial_\kappa A_{\mu\nu} + \alpha_{gr}\varepsilon_{\mu_2\lambda_2\kappa_2}\varepsilon^{\mu_1\lambda_1}_{\ \ \ \kappa_1}E^{\kappa_1\kappa_2}A_{\lambda_1}^{\ \lambda_2}A_{\mu_1}^{\ \mu_2}) \\ +E^{\kappa_1}_{\ \nu_1}F_{\mu\kappa_1}^{\ \ \nu_1}E^{\kappa_2}_{\ \nu_2}F^{\mu\ \nu_2}_{\ \kappa_2} \end{array}\right)$$

with the constraint $\bar{\varphi}_\Lambda\varphi_\Lambda = \Lambda$

and with the field tensor $F_{\mu\nu}^{\ \ \kappa} = \partial_\mu A_\nu^{\ \kappa} - \partial_\nu A_\mu^{\ \kappa} + \alpha_{gr}\varepsilon^\kappa_{\ \kappa_1\kappa_2}A_\mu^{\ \kappa_1}A_\nu^{\ \kappa_2}$

quasi-classical approximation: $E_{gr}(r) = \frac{r_\Lambda}{r}mc^2$ for $r \ll r_{gr}$, $r_\Lambda = 3.7 \times 10^{-49}m$

generates temperature-independent gravitational quantum noise $P_p = \frac{r_\Lambda}{r_p}m_p c^2\frac{c}{r_p} \sim 10^{-22}eV/s$

Dirac eq. $\left(i\hbar\gamma^\mu D_\mu - mc\right)\Psi = 0$

field eq.= $AK(A_\mu^{\ \nu}, E^{\mu\nu}, \Lambda)$

boundary condition at $r = r_{gr}$ $E^{\mu\kappa}E^\nu_{\ \kappa} = g^{\mu\nu}/(-\det(g))^{3/4}$, with Minkowski metric $g = \eta$

equation-of-motion: Dirac equation $\left(i\hbar\gamma^\mu g_{\mu\nu}\partial_\nu - mc\right)\Psi = 0$ with external GR-metric g ,

in quasi-classical approximation $\left(i\hbar\gamma^\mu\partial_\mu - mc\frac{r_s}{r} - mc\right)\Psi = 0$, with the Newtonian potential energy

$V_N\left(r\right) = mc^2\frac{r_\Lambda}{r}$ $r_\Lambda = 3.7 \times 10^{-49}m$

weak interaction: gauge-group=Lie-group SU(2) 3 generators Pauli matrices σ_j , corresponding 3 massive gauge bosons

Z, W^+ , W^- ,short range, pure quantum , are Yukawa bosons of hypercolor interaction

cut-off energy M_Z=*91.17GeV* ,

length scale $r_{weak} = 10^{-17}m$

energy scale $E_{weak} \approx 10^{-4} E_{em} = 7eV$

Lagrangian, $L_1 = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ where

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, f^{abc} W^b_\mu W^c_\nu \quad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

where the physical gauge fields Z, W^+ , W^- are

$$Z_\mu = \frac{g W^3_\mu + g' B_\mu}{\sqrt{g^2 + g'^2}} = \cos\theta_W W^3_\mu + \sin\theta_W B_\mu$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}} \left(W^1_\mu \pm i W^2_\mu \right)$$

covariant derivatives left and right with Pauli matrices σ_i

$$D_\mu R = \left(\partial_\mu + i\, g' B_\mu \right) R \quad D_\mu L = \left(\partial_\mu + \frac{i}{2} g' B_\mu - \frac{i}{2} g \sigma_i W^i_\mu \right) L$$

strong (color) interaction: gauge-group=Lie-group SU(3) 8 generators Gell-Mann-matrices λ_α , corresponding 8 massless spin-1 gauge bosons , 3 charges (colors) *r g b* , short range, pure quantum, asymptotically free (confinement),

The coupling from the Callan-Symanzik equation is

$$g_{hc}(m) = 4\pi \sqrt{\frac{1}{18\sqrt{\left(log(\frac{m}{\Lambda_{hc}}) \right)^2 + c_{GE1}{}^2}}}$$

energy scale $E_{col} = 220 MeV$, with the zero-shift constant in the Callan-Symantzik relation to remove the singularity $c_{GE} = 0.69$

confinement length scale $r_{col} = \frac{\hbar c}{E_{col}} = 0.89 * 10^{-15} m \approx r_{pr}$

cut-off energy $M_{col} = \frac{\hbar c}{r_{col}} = 2.33 * 10^{10} eV \approx 23 GeV$

the pion π with mass 106MeV is the Yukawa field boson of the color interaction

Lagrangian $L = \bar{\psi} \left(i\gamma^\mu D_\mu - m \right) \psi - \frac{1}{4} F^a{}_{\mu\nu} F_a{}^{\mu\nu}$

with the field tensor $F^a{}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$

covariant derivative $D_\mu = \partial_\mu - ig A^a_\mu T_a$, with Lie generators $T_a = \lambda_a / 2$

where g is the coupling constant, A^a_μ *A* is the gluon gauge field,

for eight different gluons *a* = 1 … 8 , and where λ_a λa are the eight 3x3-Gell-Mann matrices, *a* = 1 … 8 .

commutation relations with structure constants $\left[T_a, T_b \right] = i\, f^{abc} T_c$, $\left[\lambda_a, \lambda_b \right] = 2i\, f^{abc} \lambda_c$

Dirac eq. $\left(i\hbar\gamma^\mu D_\mu - mc \right) \psi = 0$, hc wave function $\psi = \left(\psi_1 \quad \psi_2 \quad \psi_3 \right)$ with three components

field eq. $\frac{1}{2} \partial_\nu \left(F^{a\mu\nu} - F^{a\nu\mu} \right) + \frac{g}{2} \left(f^{kac} A^c_\nu F^{k\mu\nu} + f^{kba} A^b_\nu F^{k\nu\mu} \right) = j^a{}_\nu = g\, \bar{\psi} \gamma^\mu T^a \psi$

quantum gauge version QCD is renormalizable, due to the renormalization group equations

gluon propagator in Feynman gauge is $D^{ab}_{\mu\nu}(k) = \delta^{ab} \left(\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{1}{k^2} + \frac{k_\mu k_\nu}{k^4} \right)$

hypercolor interaction: gauge-group= Lie-group SU(4), 15 generators massless bosons spin=1, short range, pure quantum

4 charges $r_L^- \, r_L^+ \, r_R^- r_R^+$ for r-preon and $q_L^- \, q_L^+ \, q_R^- q_R^+$ for q-preon resp. with charge + or - and helicity L or R, antiparticle $C(r_L^+) = r_R^-$, $C(r_L^-) = r_R^+$

the weak-interaction is the Yukawa-limit of the hypercolor interaction

L-R-symmetry breaking $SU(4) = SU(2)_L \otimes SU(1)_R \otimes SU(1)_{em}$

covariant derivative $D_\mu = \partial_\mu - ig A_\mu^a T_a$, with Lie generators $T_a = \lambda_a / 2$

where g is the hyper-color coupling constant, A_μ^a A is the hc-boson gauge field,

for fifteen different hc-boson a = 1 … 8 , and where λ_a λa are the fifteen SU(4) 4x4-Gell-Mann matrices, a = 1 … 15 .

field tensor $F^a{}_{\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$

Lagrangian $L = \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4} F^a{}_{\mu\nu} F_a{}^{\mu\nu}$

The coupling from the Callan-Symanzik equation is

$$g_{hc}(m) = 4\pi \sqrt{\frac{3}{76\sqrt{\left(log(\frac{m}{\Lambda_{hc}})\right)^2 + c_{GE1}{}^2}}}$$

critical energy $\Lambda_{hc} = 2m(Z_0) = 180 GeV$ in analogy to the QCD , and zero-shift constant

$$c_{GE1} = \frac{1}{Log\left(\frac{m(t)}{m(d)}\right)} = 0.095$$

Dirac eq. $\left(i\hbar\gamma^\mu D_\mu - mc\right)\psi = 0$, hc wave function $\psi = \left(\psi_{L-} \quad \psi_{L+} \quad \psi_{R-} \quad \psi_{R+}\right)$

field eq. $\frac{1}{2}\partial_\nu\left(F^{a\mu\nu} - F^{a\nu\mu}\right) + \frac{g}{2}\left(f^{kac} A_\nu^c F^{k\mu\nu} + f^{kba} A_\nu^b F^{k\nu\mu}\right) = j^a{}_\nu = g\,\bar{\psi}\gamma^\mu T^a \psi$

quantum gauge version is formally renormalizable, because the interaction constant is dimensionless

GUT (approximate) *minimal SU(5)* of Georgi-Glashow:

We have for the right-handed basic particles in SM

R=(3 rgb *d*-antiquark, 2 leptons e ν)= $d(\bar{3},1) \oplus e\nu(1,\bar{2})$ or $SU(5)_R = SU(\bar{3}) \oplus SU(2)$

and for the left-handed basic particles in SM

L=(3x2 rgb *u*-quark *d*-quark, 3 rgb *u*-antiquark, 1 antilepton *e*)= $ud(3,2) \oplus \bar{u}(\bar{3},1) \oplus \bar{e}(\bar{1},1)$ or

$SU(5)_L \oplus SU(5)_L = SU(3) \oplus SU(3) \oplus SU(\bar{3}) \oplus SU(\bar{1})$

This symmetry is respected by the color and the weak interaction, but is (slightly) broken by the much stronger hc-interaction at high energies. Nevertheless, it yields an approximately correct Weinberg angle for the electro-weak interaction.

3 Statistical mechanics

Statistical mechanics describes the thermodynamic behaviour of large systems.
Key features of a thermodynamic ensemble is its *partition function* (probability distribution) and its *macroscopic function* (extremal variable of the ensemble: e.g. entropy=max, free energy=min) . They are functions of the *thermodynamic state variables*: temperature, volume pressure, number of particles, chemical potential [7].
Fundamental variable: partition function Z

$Z=\sum_i \exp\left(-\frac{E_i}{k_B T}\right)$ classical Boltzmann statistics, prob. distribution $f(E,T)=\exp\left(-\frac{E}{k_B T}\right)$

$Z=\sum_i f_{MJ}(E_i,T,\gamma)$ classical relativistic Maxwell-Jüttner statistics, prob. distribution

$$f_{MJ}(E,T,\gamma)=\gamma^2\sqrt{1-1/\gamma^2}\,\frac{E}{\gamma k_B T}\,\frac{\exp(-E/k_B T)}{K_2\left(\frac{E}{\gamma k_B T}\right)},\ \gamma=\frac{1}{\sqrt{1-v^2/c^2}}$$

$Z=\sum_i \frac{1}{\exp\left(\frac{E_i}{k_B T}\right)+1}$ quantum fermion Fermi-Dirac statistics , prob. distribution $f(E,T)=\frac{1}{\exp\left(\frac{E}{k_B T}\right)+1}$

$Z=\sum_i \frac{1}{\exp\left(\frac{E_i}{k_B T}\right)-1}$ quantum boson Bose-Einstein statistics, prob. distribution $f(E,T)=\frac{1}{\exp\left(\frac{E}{k_B T}\right)-1}$

Important thermodynamic variables U , F, S , C_v

$U=\langle E\rangle=-\frac{\partial \ln Z}{\partial \beta}=k_B T^2\frac{\partial \ln Z}{\partial T}$ mean energy

$F=\langle E\rangle-TS=-k_B T\ln Z$ Helmholtz free energy

$G=F+PV=\langle E\rangle-TS+PV=-\frac{1}{\beta}\frac{\partial(V\ln Z)}{\partial V}$ Gibbs free energy

$S=\frac{\partial}{\partial T}(k_B T\ln Z)$ entropy

$C_v=\frac{\partial\langle E\rangle}{\partial T}=\frac{1}{k_B T^2}\left\langle(\Delta E)^2\right\rangle=k_B T\left(2\frac{\partial \ln Z}{\partial T}+T\frac{\partial^2 \ln Z}{\partial T^2}\right)$ heat capacity

for $Z\approx W\exp\left(-\frac{E}{k_B T}\right)$, $S=\frac{\partial}{\partial T}(k_B T\ln Z)\approx\frac{\partial}{\partial T}(-E+k_B T\ln W)=k_B\ln W$

which gives the famous Boltzmann approximate formula $S\approx k_B\ln W$

The three most important ensembles [7]
The microcanonical ensemble describes a system with fixed energy and fixed number of particles.
The canonical ensemble describes a system of fixed number of particles that is in thermal equilibrium with a heat bath of a fixed temperature.
The grand canonical ensemble describes a system with non-fixed particle numbers that is in thermal and chemical equilibrium with a thermodynamic reservoir with fixed temperature and particle number.

Thermodynamic classical and quantum ensembles with their partition function and the macroscopic function

	Thermodynamic classical ensembles		
	Microcanonical	Canonical	Grand canonical
Fixed variables	N, V, E	N, V, T	μ, V, T
Microscopic	Number of microstates	Canonical partition function	Grand partition function

features	W, with width ω $W=\sum_k f\left(\frac{H_k-E}{\omega}\right)$ $f\left(x\right)=\exp\left(-\pi x^2\right)$	$Z=\sum_k exp\left(-\frac{E_k}{k_B T}\right)$ $Tr\left(exp\left(-\beta H\right)\right)=$ $\sum_k exp\left(-\beta E_k\right)=Z(\beta)$	$Z=\sum_k exp\left(-\frac{E_k-\mu N_k}{k_B T}\right)$ $Z\left(\beta,\mu_1,\mu_2,\ldots\right)=Tr\left(exp\left(\beta\left(\sum_k \mu_k N_k-H\right)\right)\right)$
Probability and density matrix	$\hat{\rho}=\frac{1}{W}\sum_k f\left(\frac{H_k-E}{\omega}\right)\|\psi_k\rangle\langle\psi_k\|$	$\rho=\frac{e^{-\beta H}}{Tr\, e^{-\beta H}}$ $p(E_m)=\frac{exp\left(-\beta E_m\right)}{\sum_k exp\left(-\beta E_k\right)}$	$\rho=\frac{e^{-\beta\left(H-\sum_i \mu_i N_i\right)}}{Tr\, e^{-\beta\left(H-\sum_i \mu_i N_i\right)}}$ $P(E_m)=\frac{e^{-\beta\left(E_m-\sum_i \mu_i N_i\right)}}{\sum_n e^{-\beta\left(E_n-\sum_i \mu_i N_i\right)}}$
Macroscopic function	Boltzmann entropy $S=k_B \ln W$ v.Neumann entropy $S=-k_B Tr(\rho\ln\rho)$	Helmholtz free energy $F=-k_B T\ln Z$	Grand potential $\Omega=-k_B T\ln Z$

The **macroscopic function** obeys the **minimum principle** of statistics:
the macroscopic function attains an **extremum in equilibrium**
microcanonical: entropy is maximum
canonical: free energy is minimal
grand canonical: grand potential is minimal

The **partition function** describes the statistical properties of a system in thermodynamic equilibrium .Partition functions are functions of the thermodynamic state variables .

For a canonical discrete ensemble

classical $Z=\sum_k g_k exp\left(-\beta E_k\right)$, $\beta=\frac{1}{k_B T}$, g_k=degeneracy , g_j degeneracy

relativistic classical $Z=\sum_k f_{MJ}\left(E_k,T,\gamma\right)$

quantum $Z=Tr\left(exp\left(-\beta\hat{H}\right)\right)$ or $Z=\sum_k \frac{g_k}{\exp\left(\frac{E_k}{k_B T}\right)\pm 1}$ (fermion + , boson -)

probability of state s $p_s=\frac{1}{Z}exp\left(-\beta E_s\right)$

The four **probability distributions** for the *average number in a state* ε: $N(\varepsilon)$ with degeneracy $g(\varepsilon)$ and chemical potential μ are derived from the maximization of number of states $W(\nu_i)$ with occupation numbers ν_i of N particles for states with degeneracy g_r and energy levels η_r $r=1,2,\ldots N'$ under energy condition $\sum_r \nu_r\eta_r=E$ and particle number condition $\sum_r \nu_r=N$

fermions $S=n+1/2$ (Pauli principle) Fermi-Dirac statistics $W(\nu_i)=\prod_r \frac{g_r!}{\nu_r!(g_r-\nu_r)!}$ follows from Pauli-principle $\nu_r=0,1$ because the wave function is antisymmetric in particles $\Psi(\ldots\nu_i\ldots\nu_k\ldots)=-\Psi(\ldots\nu_k\ldots\nu_i\ldots)$:

$$N(\varepsilon)=\frac{g(\varepsilon)}{\exp\left(\frac{\varepsilon-\mu}{k_B T}\right)+1}$$

bosons $S = n$ Bose-Einstein statistics $W(\nu_i) = \prod_r \frac{(g_r + \nu_r - 1)!}{\nu_r!(g_r - 1)!}$ follows from $\nu_r = 0,1,2,...$ because the wave function is symmetric in particles $\Psi(...\nu_i...\nu_k...) = \Psi(...\nu_k...\nu_i...)$:

$$N(\varepsilon) = \frac{g(\varepsilon)}{\exp\left(\frac{\varepsilon - \mu}{k_B T}\right) - 1}$$

classical Boltzmann-Maxwell statistics $N(\varepsilon) = \exp\left(-\frac{\varepsilon}{k_B T}\right)$ is derived from both FD and BM statistics for high temperature $\exp\left(\frac{\varepsilon}{k_B T}\right) >> 1$

classical relativistic Maxwell-Jüttner statistics, $N(\varepsilon) = \gamma^2 \sqrt{1 - 1/\gamma^2} \frac{\varepsilon}{\gamma k_B T} \frac{\exp(-\varepsilon / k_B T)}{K_2\left(\frac{\varepsilon}{\gamma k_B T}\right)}$, $\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$

applicability:
quantum condition :
$d \le \lambda_B = \frac{h}{mv}$ (diameter d less than de-Broglie-wavelength): fermion or boson statistics apply
$d \gg \lambda_B = \frac{h}{mv}$ Boltzmann or Maxwell-Jüttner statistics apply

photons: *m=0*, $\lambda_B = \infty$, $d = \lambda$: boson statistics applies
electrons: $d \approx 10^{-18} m$, $\lambda_B \approx 10^{-10} m$: fermion statistics applies
relativistic electrons: $d \approx 10^{-18} m$, $\lambda_B = 2.4 \times 10^{-12} m$, $v \approx c$: fermion statistics applies
thermal neutrons: $d = 1.6 \times 10^{-15} m$, $\lambda_B \approx 0.2 \times 10^{-9} m$: fermion statistics applies
relativistic neutrons: $d = 1.6 \times 10^{-15} m$, $\lambda_B = 1.3 \times 10^{-15} m$, $d \approx \lambda_B$: fermion statistics applies
bi-atomic gas (H2) thermal: $d = 0.3 \times 10^{-9} m$, $\lambda_B \approx 0.07 \times 10^{-9} m$ T=300K, Boltzmann statistics applies
bi-atomic gas (H2) relativistic: $d = 0.3 \times 10^{-9} m$, $\lambda_B \approx 0.6 \times 10^{-15} m$, $v <\approx c$, Maxwell-Jüttner statistics applies

second principle of thermodynamics states that
$\frac{dS}{dt} \ge 0$ entropy is non-decreasing in time *t*,
or (Lorentz-invariant) $\frac{dS}{d\tau} \ge 0$ entropy is non-decreasing in proper time τ

4 Models

Model of macrocosm
The macrocosmic model is the **phenomenological ΛCDM model** based on the Friedmann equations derived from GR, that govern the expansion of space in homogeneous and isotropic models of the universe and having 13 free parameters.
Basic ΛCDM model
The basic ΛCDM model is derived from the principle of isotropy under GR and yields the Robertson-Walker metric [4]

$$ds^2 = -c^2dt^2 + R(t)^2\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + sin^2\theta d\varphi^2\right)\right)$$

The fundamental Friedmann equations for the 3 variables world radius *R(t)* , density *ρ(t)* and pressure *P(t)* are

$$\dot{\rho} = -3\frac{\dot{R}}{R}(\rho + \frac{P}{c^2})$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\Lambda c^2}{3}$$

and the eos $P(\rho) = const * \rho^w$,
where *w=0 P=0* , $\rho_m R^3 = const$ for ordinary matter, *w=1* $P = \rho_r c^2/3$, $\rho_r R^4 = const$ for radiation .
From this we can derive the fundamental Friedmann-Lemaitre-Robertson-Walker (FLRW) equation for the time-dependent world radius *R(t)*

$$\dot{R}^2 + V(R)c^2 = -kc^2$$

where the effective (dimensionless) potential is $V(R) = -\frac{K}{R^2} - \frac{D}{R} + \frac{\Lambda}{3}R^2$ with the constants

$$D = \frac{8\pi G}{3c^2}\rho_m R^3 = 1.61*10^{26}m \quad K = \frac{8\pi G}{3c^2}\rho_r R^4 = 2.56*10^{48}m^2 \quad k=sign(curvature)$$

k=-1 : hyperbolic case, unbounded expansion
k=0 flat case
k=+1 bounded movement
Hubble constant $H = \frac{\dot{R}(t_0)}{R(t_0)}$, critical density $\rho_c = \frac{3H^2}{8\pi G}$, relative density $\Omega = \frac{\rho(t_0)}{\rho_c}$,
which results in the famous relation for the Hubble constant

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}$$

The basic ΛCDM model is based on six *dependent fundamental parameters* $H, k, \Lambda, \Omega, t_0, \rho_r, \rho_m$, which are based on four *independent fundamental parameters* D, K, k, Λ , which determine the fundamental FLRW equation, plus the current time=age of universe t_0 .
Perturbational ΛCDM model
The basic ΛCDM model is extended by primordial perturbations, which explain the evolution of structure in the universe [11].
The metric in Newtonian gauge becomes
$ds^2 = a^2(\eta)\left(-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right)$ in conformal time η with $d\eta = \frac{dt}{a}$,
where Ψ and Φ are primordial metric perturbations.
We introduce species of matter and radiation with relative their densities (baryon, dark matter, neutrinos, photons) Ω_b , Ω_c , Ω_ν , Ω_γ , with their respective equation-of-state $P(\rho,T) = w(T)\rho$
Finally, we introduce stochastic perturbations of density pressure, and velocity, which obey relativistic fluid equations: continuity and Euler.
In the end, we get as fundamental equations [11]:
6 Einstein equations

$$\nabla^2\Phi - 3H\left(\Phi' + H\Psi\right) = \pi G a^2 \delta\rho$$

$$\left(\Phi' + H\Psi\right) = \pi G a^2 \frac{a''}{a' H}$$

$$\partial_i \partial_j \left(\Phi - \Psi\right) = 8\pi G a^2 \Pi_{ij}\ , i < j$$

$$\Phi'' + H\Psi' + 2H\Phi' + \frac{1}{3}\nabla^2\left(\Phi - \Psi\right) + \left(2H' + H^2\right)\Psi = \pi G a^2 \delta P \qquad \text{(15a-d)}$$

4 conservation equations : continuity +Euler

$$\delta' = -\left(1 + \frac{\bar{P}}{\bar{\rho}}\right)\left(\partial^i v_i - 3\Phi'\right) - 3H\left(\frac{\delta P}{\delta\rho} - \frac{\bar{P}}{\bar{\rho}}\right)\delta$$

$$v_i' = -\left(H + \frac{\bar{P}'}{\bar{P} + \bar{\rho}}\right)v_i - \frac{1}{\bar{P} + \bar{\rho}}\left(\partial_i \delta P + \partial^j \Pi_{ij}\right) - \partial_i \Psi \qquad \text{(15ef)}$$

$q^i = \left(\bar{\rho} + \bar{P}\right)v^i \quad \delta = \dfrac{\delta\rho}{\bar{\rho}}$ decelaration conformal $q = -\dfrac{a''}{a' H}$,

for **10 variables** 4scalar $\Phi, \Psi, \delta, \delta P$, 3vector v^i , 3tensor $\Pi^i{}_j$

where δ =relative density fluctuation, δP =pressure fluctuation, v^i =velocity, $\Pi^i{}_j$ =anisotropic stress

with background parameters

Hubble $H = \dfrac{a'}{a}$, scale factor a , mean density $\bar{\rho}$, mean pressure $\bar{P}$

Parameters

For the full perturbational ΛCDM model we get 13 parameters [11] :
$\Lambda(\Omega_b , \Omega_c , t_0 , n_s, A_s , \tau , \Omega_\Lambda , w , \sum m_\nu , N_{eff}(\nu) , r , dn_s/dk , H_0)$,
with denominations:
physical baryon density parameter Ω_b; dark matter density parameter Ω_c, world age t_0 , scalar spectral index n_s; curvature fluctuation amplitude A_s ; and reionization optical depth τ , dark energy density Ω_Λ, dark energy state factor w, neutrino masses sum $\sum m_\nu$, number of neutrino generations $N_{eff}(\nu)$, spectral index gradient dn_s/dk , Hubble constant H_0 .

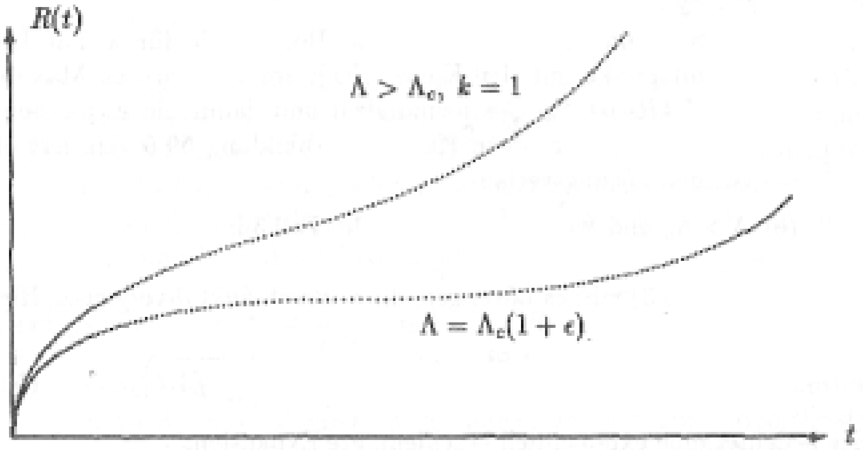
The Planck-satellite measurements of the CMB background in 2015 have yielded the following values

Description	Symbol	Value
Physical baryon density parameter	$\Omega_b h^2$	0.02230 ± 0.00014
Physical dark matter density parameter	$\Omega_c h^2$	0.1188 ± 0.0010
Scalar spectral index	n_s	0.9667 ± 0.0040
Curvature fluctuation amplitude, $k_0 = 0.002$ Mpc^{-1}	$\Delta_R{}^2$	$2.441 \pm 0.092 \times 10^{-9}$
Reionization optical depth	τ	0.066 ± 0.012
Age of the universe	t_0	$13.799 \pm 0.021 \times 10^9$ years
Reduced Hubble constant	h	0.704 ± 0.025
Type of curvature	k	uncertain
Cosmological constant	Λ	$1.1\ 10^{-52}$ m^{-2}
Density of matter	ρ_m	$0.3089\ \rho_c$
Density of radiation	ρ_r	$3.79\ 10^{-5}\ \rho_c$
Relative density	Ω	$1.01 \pm 0.05\ \rho_c$
Critical density	ρ_c	$3.79\ 10^{-26}$ kg/m^3

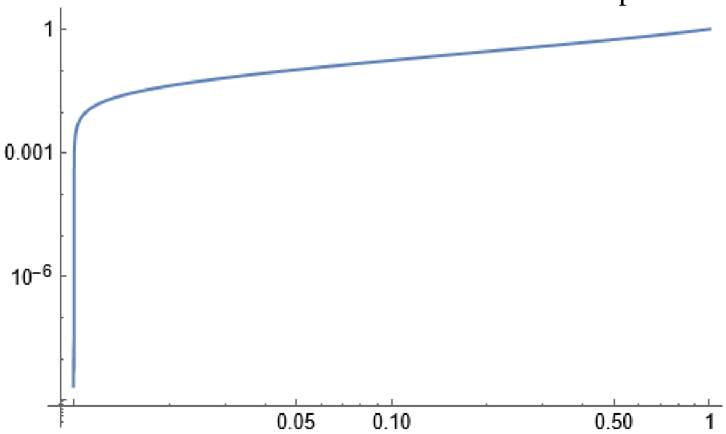
Possible scenarios

The decisive parameters here are curvature sign k=*-1, 0 ,+1* and the critical Lambda-value $\Lambda_c = \frac{4}{9D^2} = 0.17\, 10^{-52}\, m^{-2}$, $\Lambda = 6.5\Lambda_c$

Schematic scenario for k=-1, 0, +1 , $\Lambda = \Lambda_c\left(1+\varepsilon\right)$: scale factor a in expanding universe (k=0 valid in our world. lower curve) [4]



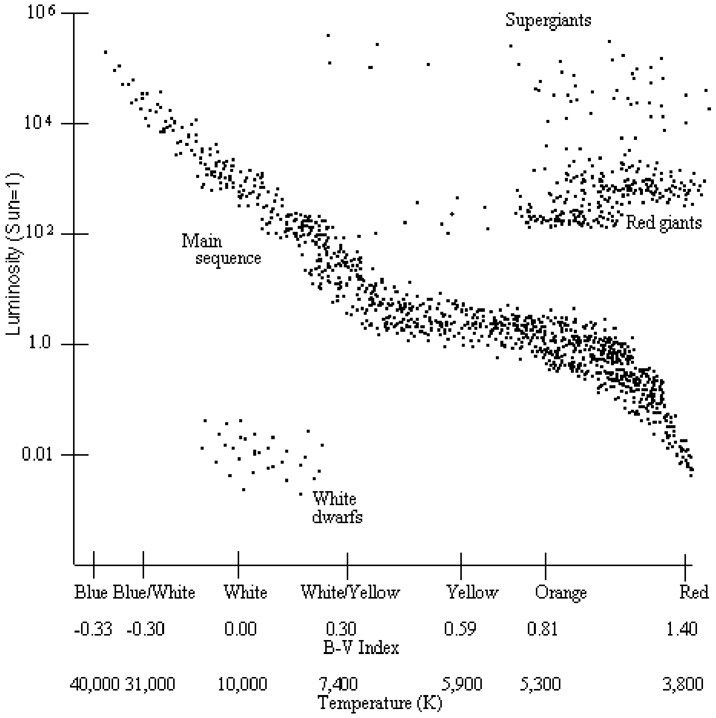
The relative scale factor a calculated for Planck parameters [11]:

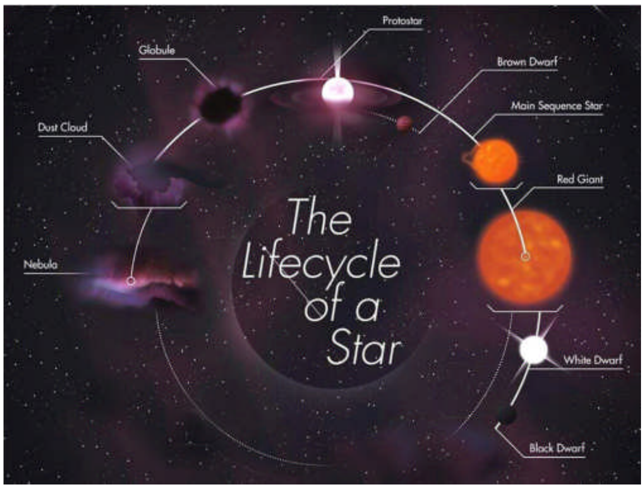


scale factor *a[x]* in dependence of relative time $x = \frac{tc}{R_H}$

Star models

[15]





Hertzsprung-Russell diagram

Sun-like stars

mass $M \sim M_{sun} = 1.99 \times 10^{30}\, kg$

radius $R \sim R_{sun} = 0.7 \times 10^{6}\, km$

density $\rho \sim \rho_{sun} = 1.4 g\, cm^{-3}$

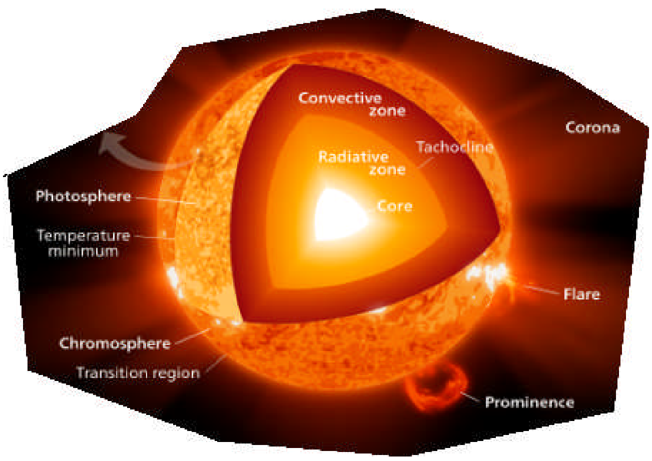
age $t_0 \sim t_{sun} = 4.6 \times 10^{9}\, y \quad t_0 \le 10 \times 10^{9}\, y$

eos **ideal gas** $PV = n_m RT \quad P = nk_B T = \rho \dfrac{k_B T}{m_0}$

temperature $T = \begin{cases} 15.7 \times 10^{7}\, K\ core \\ 5.8 \times 10^{3}\, K\ surface \end{cases}$

gravitational acceleration $g \sim 275 m\, s^{-2}$

energy production p-p-chain $4H_1^{+} + 2e^{-} \to He_4^{2+} + 2\overline{\nu}_e + 26.7 MeV$



White-dwarf stars

mass $0.17M_{sun} \leq M \leq 1.33M_{sun}$, Chandrasekhar mass limit $M < M_{max} = 1.4M_{sun}$

radius $R \propto M^{1/3}$ $6000km \leq R \leq 12000km$ calc. $R\left(M = 0.6M_{sun}\right) = 9000km$

density calc. $\rho\left(0.6M_{sun}\right) = 1.6\times10^{6}\,g\,cm^{-3}$

typical age $t_0 \sim 6\times10^{9}\,y$

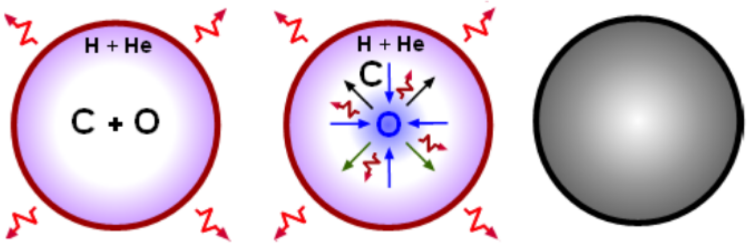
magnetic field 0.2 T to 100 kT $0.2T \leq B \leq 100kT$

eos pressure Fermi **electron gas**

low-density approximation $P = 8\pi P_0 \dfrac{x_F^{\;5}}{15} K_1 \rho^{5/3}$,

$x_F = \dfrac{p_F}{m_e c} = \dfrac{h}{2m_e c}(3\pi)^{1/3} n^{1/3}$, where p_F is the Fermi-angular-momentum, n the particle density

temperature core $T \sim 10^{7}\,K$



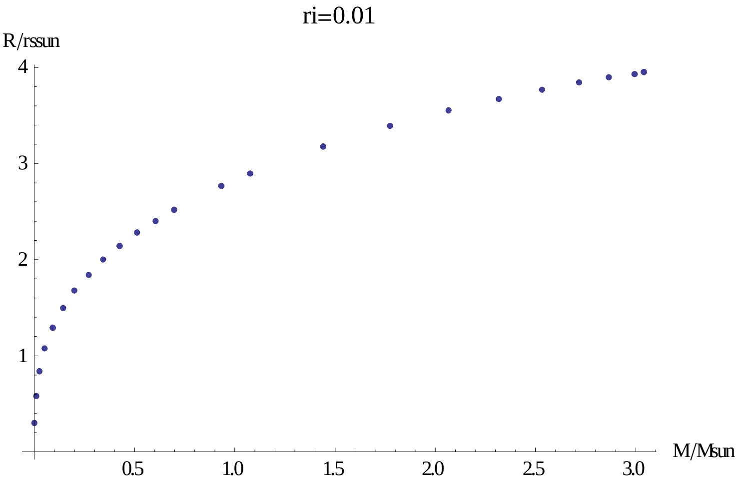
left :newly formed white dwarf, center: a cooling and crystallizing white dwarf, right: a black dwarf.

Neutron stars

mass calc. $0.14M_{sun} \leq M \leq 3.04M_{sun}$

radius $R \propto M^{1/3}$ calc. $4.5km \leq R \leq 12km$ $R\left(M = 0.5M_{sun}\right) = 6.9km$

measured in sun-units Schwarzschild-radius $r_{ssun} = 3km$, and Schwarzschild-density $\rho_{ssun} = 0.74\times10^{17}\,g\,cm^{-3}$



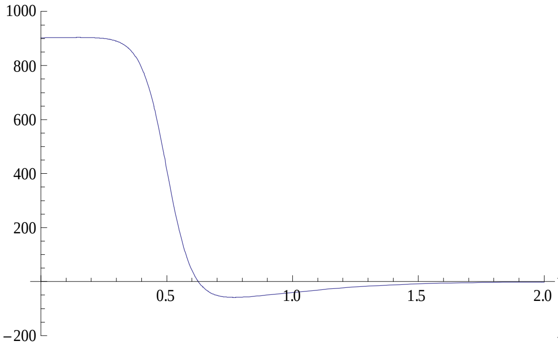
density calc. $3.5\times10^{14} \leq \rho \leq 4\times10^{14}\,g\,cm^{-3}$

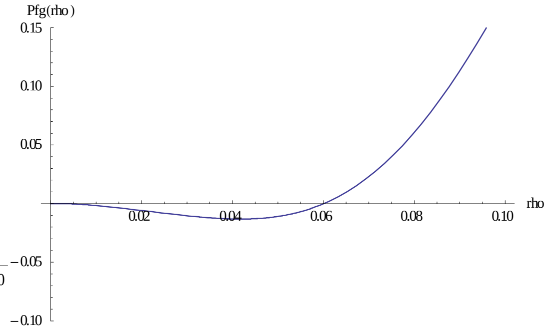
typical age $t_0 \sim 6\times10^{9}\,y$

magnetic field 0.2 T to 100 kT $0.2T \leq B \leq 100kT$

eos pressure Fermi **neutron fluid**

$P_1(r_1) = c_1 \rho_1(r_1) V_{nn}((E_n / \rho_1(r_1))^{1/3})$

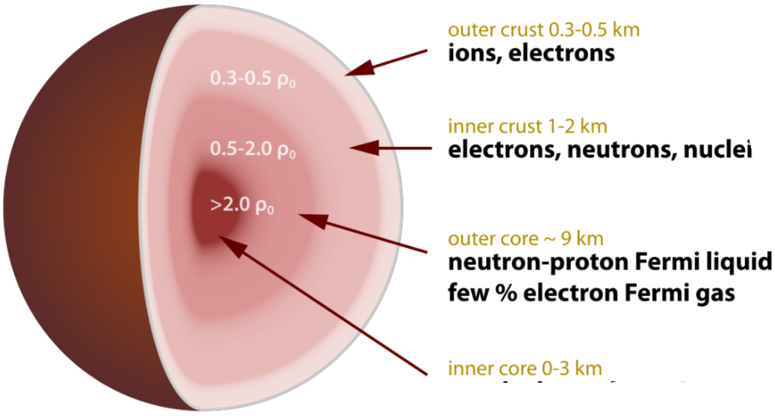




$V_{nn}(r)$ with energy(MeV), r(fm) total pressure $P_1(\rho)$

temperature $T \sim 10^6 K$

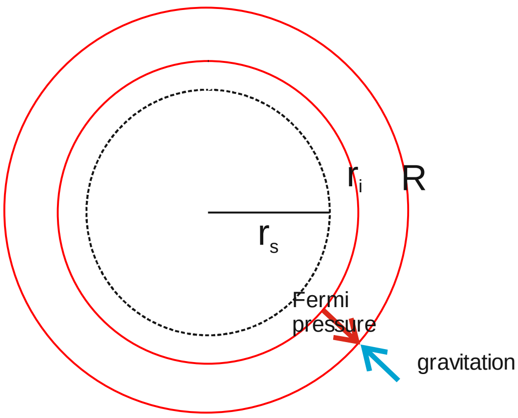
gravitational acceleration $g \sim 2 \times 10^{12} ms^{-2}$



Stellar black holes

shell star with mass M, outer radius R, inner radius $r_i > r_s(M)$

measured in sun-units Schwarzschild-radius $r_{ssun} = 3km$, and Schwarzschild-density $\rho_{ssun} = 0.74 \times 10^{17}\, g\, cm^{-3}$

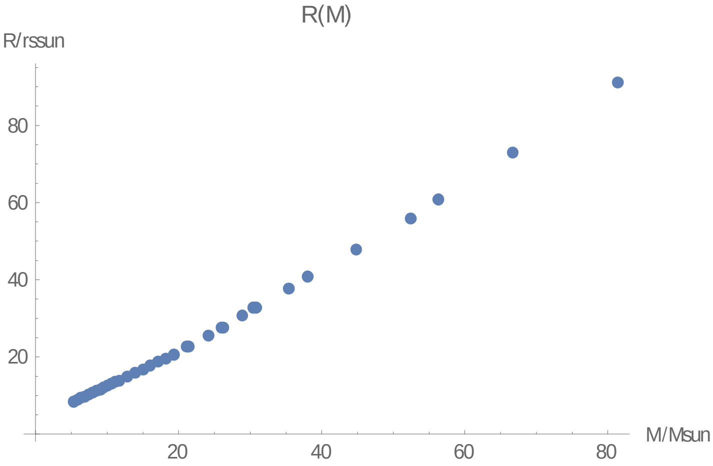


mass calc. $5.35 M_{sun} \le M \le 100 M_{sun}$

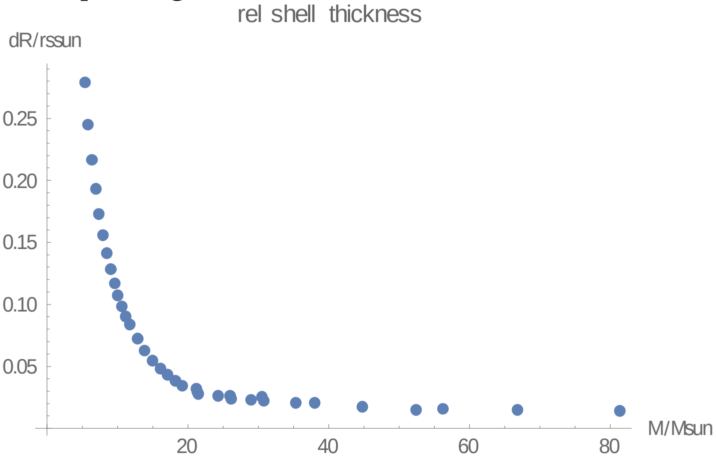
radius $R \propto M$ calc. $25.5km \le R \le 336km$

inner radius calc. $21km \le r_i \le 330km$

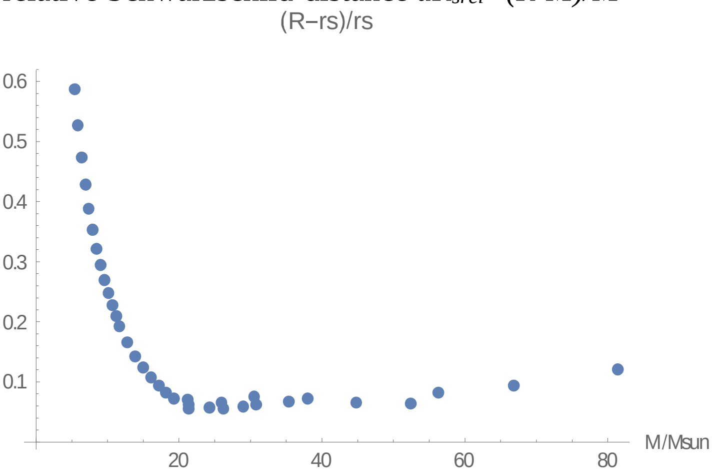
R-M-relation is practically linear



corresponding relative shell thickness $dR_{rel}=dR/M$ is



relative Schwarzschild-distance $dR_{srel}=(R-M)/M$



relative flight velocity deviation from 1 is $\frac{\Delta v}{c} \approx d_{Rsrel} = \frac{R-M}{M} \sim 0.07$

the redshift is $z \approx \frac{1}{d_{Rsrel}} \sim 14$

eos pressure Fermi **neutron gas**

low-density approximation $P = 8\pi P_0 \frac{x_F{}^5}{15} K_1 \rho^{5/3}$,

$x_F = \frac{p_F}{m_n c} = \frac{h}{2m_n c}(3\pi)^{1/3} n^{1/3}$, where p_F is the Fermi-angular-momentum, *n* the particle density

magnetic field probably like neutron stars $B = 1kT \ldots 100kT$ (form relativistic jets at poles)

temperature probably like neutron stars $T \sim 10^6 K$

entropy of a shell star is $S = \dfrac{k_B\, A}{\pi L_P^{\ 2}} log\, 2$, which approximately equal to the Hawking- Bekenstein entropy of an abstract black-hole $S = \dfrac{k_B\, A}{4 L_P^{\ 2}}$

Supermassive (galactic) black holes

supermassive shell star with mass *M*, outer radius *R*, inner radius $r_i > r_s\left(M\right)$

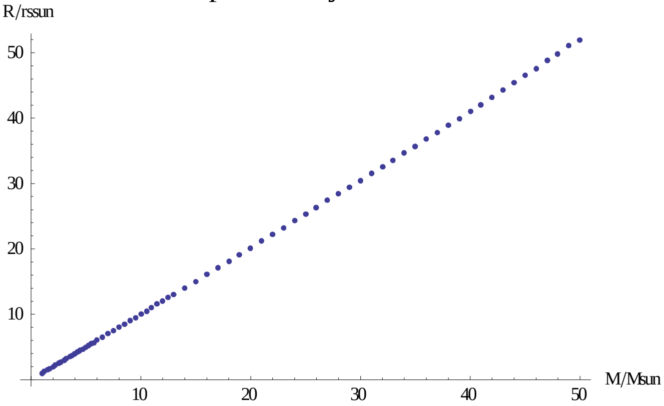
measured in mea-sun-units, mea sun-mass $M_{Msun} = 10^6 M_{sun}$ mega-Schwarzschild-radius $r_{Mssun} = 3\times10^6 km$, and mega-Schwarzschild-density $\rho_{Mssun} = 10^{-12} \rho_{ssun} = 0.74\times10^5 g\, cm^{-3}$ (comparable to white-dwarf stars)

mass calc. $2\times10^6 M_{sun} \le M \le 50\times10^6 M_{sun}$

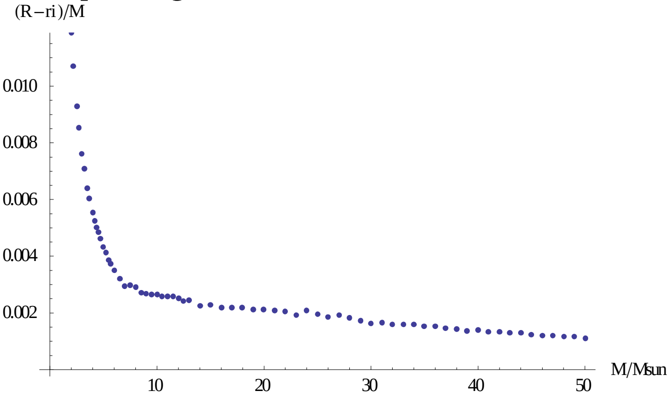
radius $R \approx r_s\left(M\right)$ calc. $6\times10^6 km \le R \le 150\times10^6 km$

inner radius $r_i \approx r_s\left(M\right)$

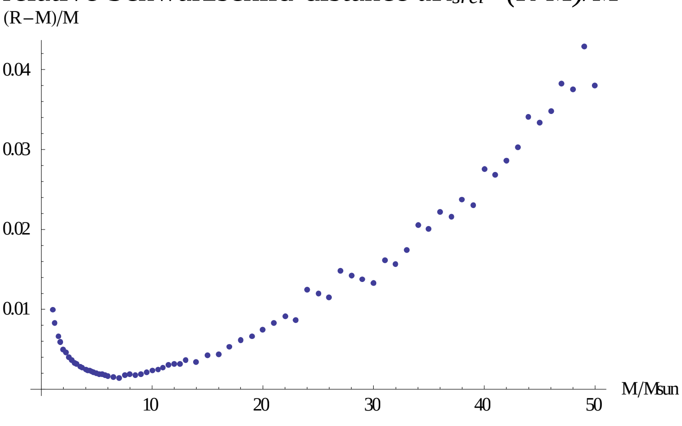
R-M-relation is practically linear



corresponding relative shell thickness $dR_{rel}=dR/M$ is



relative Schwarzschild-distance $dR_{srel}=(R-M)/M$



relative flight velocity deviation from 1 is $\frac{\Delta v}{c} \approx d_{Rsrel} = \frac{R-M}{M} \sim 0.002$

the redshift is $z \approx \frac{1}{d_{Rsrel}} \sim 500$

eos pressure Fermi **electron gas** like for a white-dwarf

low-density approximation $P = 8\pi P_0 \frac{x_F^{\;5}}{15} K_1 \rho^{5/3}$,

$x_F = \frac{p_F}{m_e c} = \frac{h}{2m_e c}(3\pi)^{1/3} n^{1/3}$, where p_F is the Fermi-angular-momentum, *n* the particle density

temperature probably in x-ray region, i.e. $T \sim 100keV = 10^8 K$

magnetic field probably scaling with redshift $B_{sm}(M) \approx \frac{z_{sm}}{z_{st}} B_{st} \approx 35 B_{st}(M)$,

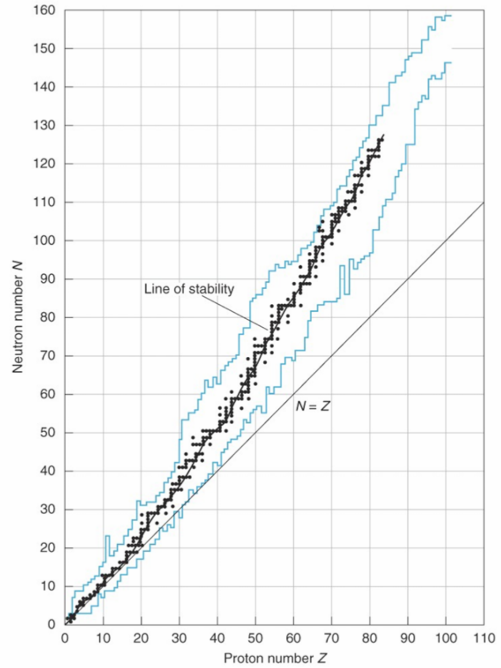
so $B = 30kT \ldots 3MT$

Models of conglomerates

The generic denomination “conglomerate physics” encompasses conglomerates of molecules and atoms in solid, fluid, and gaseous state, on the intermediate scale above the quantum regime (>39um) and below the asteroid scale (~1km). This is the scale region, which our human cognition is adapted to, and which is directly accessible by experiment.

Structural base I: nuclei

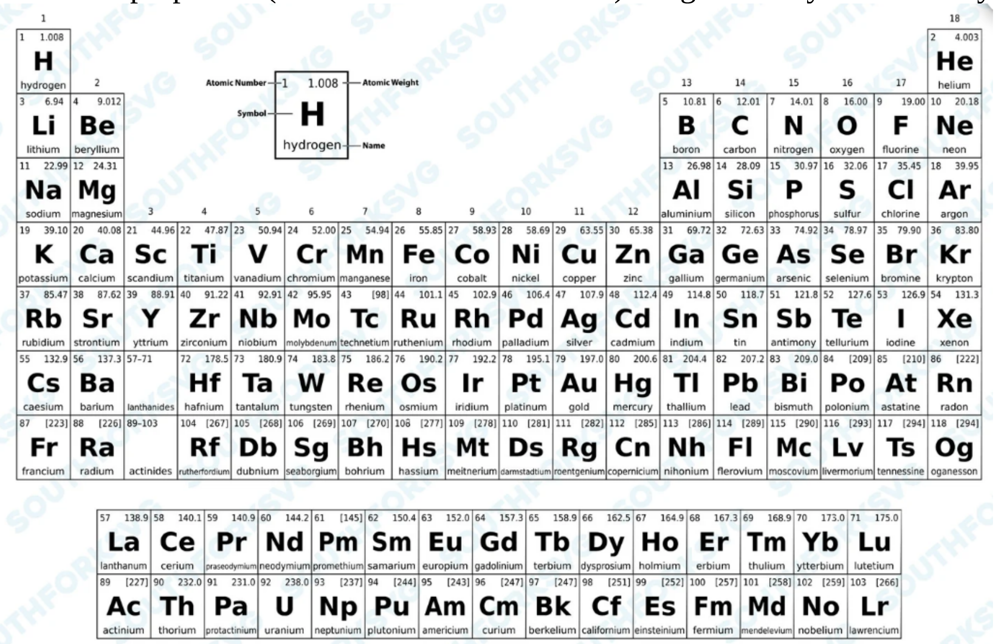
nuclei 80 elements from 1H to 82Pb with 251 stable isotopes [16]



Structural base II : atoms

80 atoms with characteristic electronic orbital configuration

Chemical properties (electronic orbital structure) are generically described by the Periodic Table [17]



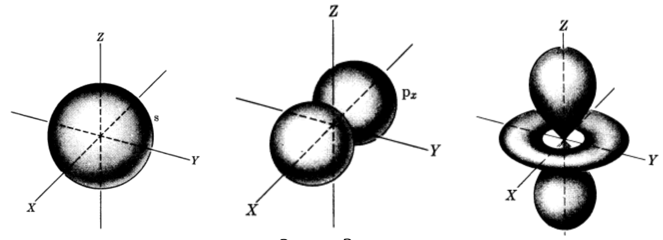
Orbitals are wave functions of external (valence) electrons with the generic form

$$\Psi_{n_A, n_I, l_1, l_2}(\vec{r})$$

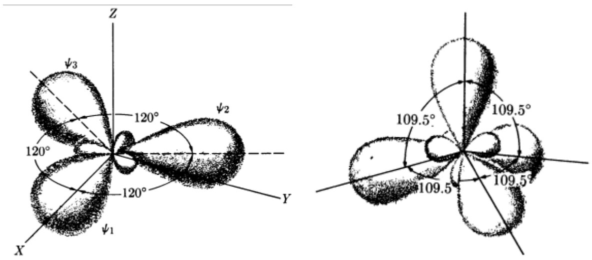
where n_A= atomic number, n_I= ionization number (positive for positive ions, negative for negative ions), l_1 orbit angular momentum , l_2 second orbit angular momentum when hybridized.

Orbitals *determine the material constants* involved in the chemical reactions, and dynamic and static (equilibrium) behavior in conglomerates.

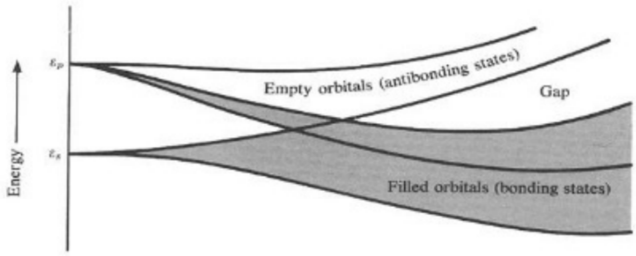
orbitals: *s* (l=0), *p* (l=1), *d* (l=2) [18]



hybridized orbitals: sp^2 , sp^3



Interaction energy of orbitals $E\left(O_1,O_2,r\right)$ [19]

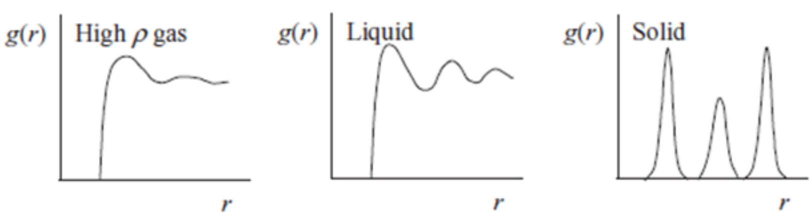


Structural base III: molecules
These are simple molecules like metal oxides, silicates, alcohols, aldehydes/ketons, organic acids; complex organic molecules like proteins, fats, sugars; molecule chains like organic polymers.

Chemical reactions
Chemical reaction rate is described by the law of mass action and the Arrhenius law [19]
reaction $\nu_A A+\nu_B B\leftrightarrow \nu_X X+\nu_Y Y$
rate $r=k_{for}\left[A\right]^{\nu_A}\left[B\right]^{\nu_B}\quad k_{for}=A'T\,\exp\left(-E_{act}/RT\right)$
where E_{act} is the activation energy, and R is the gas constant, and ν_A and ν_B are partial orders in [A] and [B], respectively

Phases of matter in first-order solid-fluid-gas transitions
[26]
Phases of matter differ in their number $N\left(r\right)$ of particles at radius *r*, described by pair correlation function *g(r)*

$$N\left(r\right)=n_0\int_0^r g\left(r\right)4\pi r^2 dr$$



and in their temperature range: solid $T<T_{freeze}$, fluid $T_{freeze}<T<T_{boil}$, gas $T>T_{boil}$
-Fluid-gas eos
Peng-Robinson eos describes the fluid-gas phase

$$p=p\left(\beta,v,p_c,\beta_c\right)=\frac{1}{\beta\left(v-b_1\right)}-\frac{\left(1+\left(0.480+1.574\omega\left(p_c,\beta_c\right)-0.176\,\omega\left(p_c,\beta_c\right)^2\right)\left(1-\sqrt{\beta/\beta_c}\right)\right)^2 a_1}{v\left(v+b_1\right)+b_1\left(v-b_1\right)},$$

with vdWaals parameters *a, b,* in terms of critical parameters p_c, inverse thermal energy $\beta_c = \frac{1}{k_B T}$,

$a_1 = \frac{0.45723}{p_c \beta_c^{\,2}}$, $b_1 = 0.077796 \frac{1}{p_c \beta_c}$,

ω = acentric factor $\omega = -\log_{10}\left(\frac{p_{sat}(0.7T_c)}{p_c}\right) = \omega(p_c, \beta_c)$

-Solid phase eos

Mie-Grueneisen equation describes the solid phase

$$p(\eta,\beta) = p_0 + \frac{(y_1\eta / v_0 - y_2)(\eta-1)\left(\eta - \frac{\Gamma_0}{2}(\eta-1)\right)}{\eta(\eta - s(\eta-1))^2} + 3\frac{\Gamma_0}{v_0}\left(\frac{1}{\beta} - \frac{1}{\beta_0}\right),$$

where $\eta = \frac{v_0}{v}$, $\Gamma_0 \approx 2$, $s \approx 3/2$

$\beta_0 = \beta_t$, $p_0 = p_t$, $v_0 = v_t$ are the triple-point values

Characteristic points of first-order solid-fluid-gas transitions

[26]

- Fluid-gas curve

Maxwell condition: equal pressure and equal Gibbs energy

$p = p_{PR}(v_f, \beta)$, $p = p_{PR}(v_g, \beta) \rightarrow v_f = v_{PR,1}(p,\beta)$, $v_g = v_{PR,2}(p,\beta)$ cubic roots of *p*-equation

Maxwell-Gibbs equation $G_{PR}(v_f(p,\beta),\beta) - G_{PR}(v_g(p,\beta),\beta) = 0 \rightarrow p_{fg} = p(\beta)$ fluid-gas curve

where $G_{PR}(v,\beta)$ and $G_{MG}(v,\beta)$ is the Gibbs energy of Peng-Robinson resp. Mie-Grueneisen eos.

- Solid-fluid curve

Maxwell condition: equal pressure and equal Gibbs energy

$p = p_{PR}(v_f, \beta)$, $p = p_{MG}(v_s, \beta) \rightarrow v_f = v_{PR,1}(p,\beta)$, $v_s = v_{MG,1}(p,\beta)$ cubic roots of *p*-equation

Maxwell-Gibbs equation $G_{PR}(v_f(p,\beta),\beta) - G_{MG}(v_s(p,\beta),\beta) = 0 \rightarrow p_{sf} = p(\beta)$ solid-fluid curve

where $G_{PR}(v,\beta)$ and $G_{MG}(v,\beta)$ is the Gibbs energy of Peng-Robinson resp. Mie-Grueneisen eos.

- Triple point

At the triple point, the saturation curve ends, fluid-->gas , fluid-->solid , gas-->solid curves meet,
i.e. $p_{PR}(v_g, T) = p_{PR}(v_f, T) = p_{MG}(v_s, T)$

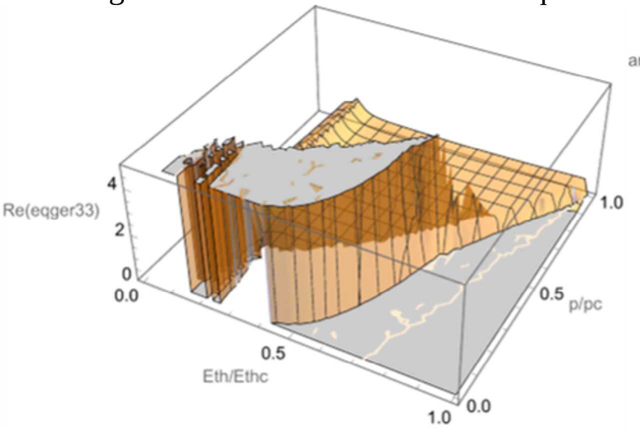
The triple point can be found as the branching point of saturation curve in the $p(\lambda,\beta)$ diagram.

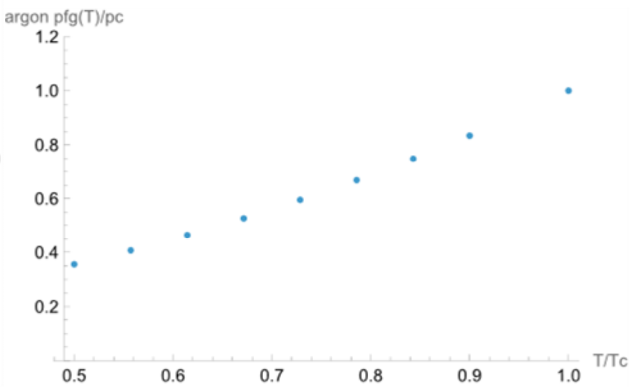
- Critical point

At the critical point, only one phase exists, the isotherm has a turning point there.

$\frac{\partial p}{\partial v} = 0$, $\frac{\partial^2 p}{\partial v^2} = 0$

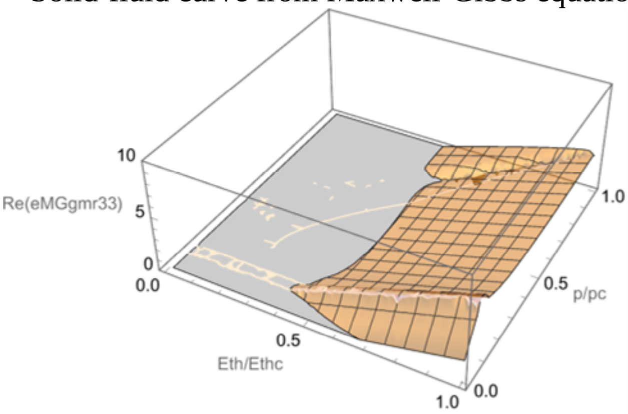
- Fluid-gas curve from Maxwell-Gibbs equation

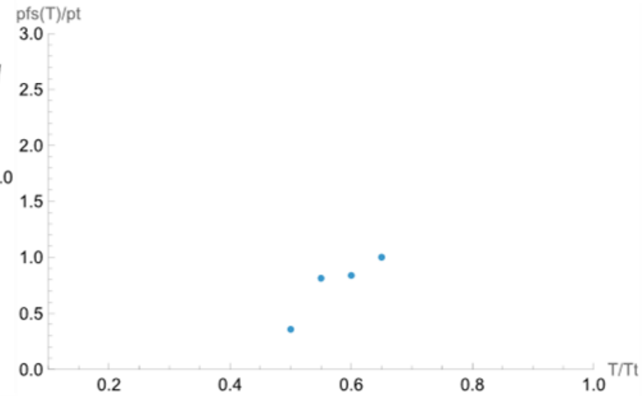




fg-curve: $G_{PR}\left(v_f\left(p,\beta\right),\beta\right)-G_{PR}\left(v_g\left(p,\beta\right),\beta\right)=0 \qquad p_{fg}=p\left(\beta\right)$

- Solid-fluid curve from Maxwell-Gibbs equation





sf-curve: $G_{PR}\left(v_f\left(p,\beta\right),\beta\right)-G_{MG}\left(v_s\left(p,\beta\right),\beta\right)=0 \qquad p_{sf}=p\left(\beta\right)$

Landau theory of phase transitions

We distinguish first-order and second-order transitions [23].

First-order transitions are discontinuous in the first derivative of free energy F, second-order are continuous in the first derivative and discontinuous in the second derivative.

- First-order phase transitions

These apply mostly to the solid-fluid-gas transitions, where the discontinuous variable is the volume $V=\dfrac{\partial G}{\partial p}$, or equivalently the average distance λ .

Here the average distance λ, the density ρ, specific volume $v=\dfrac{V}{N}$ and heat capacity C_v are discontinuous.

Transitions happen at the melting temperature T_m, and at the boiling temperature T_b , there is a latent heat ΔC at the transition temperature.

- Second-order (continuous) phase transitions

These apply mostly to the phase transitions in magnetic materials (ferromagnetic, antiferromagnetic), where the magnetization $M=\left(\dfrac{\partial F}{\partial B}\right)_T$ is continuous at Curie temperature T_C : in ferromagnetic phase transtion at T_C spontaneous magnetization $M=\left(\dfrac{\partial F}{\partial B}\right)_T$ becomes 0, and there is no latent heat.

The generalized Landau ansatz for the partition function is a functional integral [23]

$Z=\int D\varphi\, dV \exp\left(-E\left(\vec{r},\varphi,\beta\right)\right)$, where $E\left(\vec{r},\varphi,\beta\right)$ is the thermal Hamiltonian.

The functional integration $D\varphi$ has here a precise mathematical meaning: $D\varphi=\sum_i \dfrac{\partial\varphi}{\partial p_i}dp_i$ and is perfectly well-defined. The function variable φ is a function of location $\vec{r}$ and molecular distribution parameters p_i (like molecular diameter σ, average distance λ, correlation length l_c , local distribution periods l_{ak}) $\varphi=\varphi\left(r;p_i\right)$.

The actual function φ_0 , which yields the valid partition function of a thermodynamic system, is found by minimization of the free energy $F=-\dfrac{1}{\beta}\log Z$ in the parameters p_i in a value range Ω , which yield optimal parameters p_{i0} and the corresponding function $\varphi_0=\varphi\left(r;p_{i0}\right)$

$F\left(\varphi\left(p_{i0}\right)\right)=min\left(F\left(\varphi\left(p_i\right)\right);p_i\in\Omega\right)$.

For first-order transitions, we make the generalized Landau ansatz for the partition function as a functional integral , where the Boltzmann-exponential contains only the *intermolecular potential* u

$Z=\int D\varphi\, dV \exp\left(-\beta\, u\left(\vec{r}\right)\right)$

$$\varphi\left(r,\sigma,\lambda,l_c,\alpha_k,l_{a,k},\alpha_{0k}\right)=\Theta\left(r,\sigma\right)\left(1+\exp\left(-r/l_c\right)\left(\sum_k \alpha_k \cos\left(2\pi\frac{r-\sigma}{l_{a,k}}\right)\right)\right)$$

with the soft-step-up function

$$\Theta\left(r,\sigma,\Delta r\right)=\left(1-\frac{1}{1+\exp\left(\dfrac{r-\sigma}{\Delta r\ \sigma}\right)}\right)$$

For second-order transitions, like in magnetic systems, we make the generalized Landau ansatz for the partition function as a functional integral with a *φ-generated "kinetic" energy* in the exponential.

$$Z=\int D\varphi\, dV \exp\left(E(\varphi)-\beta\, u\left(\vec{r},\varphi\right)\right)$$

$E(\varphi)$ is the energy generated by φ , and is a generalization of the kinetic energy $\frac{\mu}{2}\varphi^2$, in Landau's original ansatz $E\left(\varphi,\beta\right)=\frac{\mu\left(\beta\right)}{2}\varphi^2+\frac{\lambda\left(\beta\right)}{4!}\varphi^4$

For $u=0$ (without interaction), the free energy is

$$F\left(T,V,N\right)=-\frac{1}{\beta}\log Z\approx-\frac{1}{\beta}E\left(\varphi,\beta\right)=-\frac{1}{\beta}\left(\frac{\mu\left(\beta\right)}{2}\varphi^2+\frac{\lambda\left(\beta\right)}{4!}\varphi^4\right),$$

In this way, we obtain the original Landau ansatz

$$F\left(T,\varphi\right)=F_0-a\left(T\right)\varphi^2+\frac{b\left(T\right)}{2}\varphi^4\ ,\ a\left(T\right)=\frac{1}{\beta}\frac{\mu\left(\beta\right)}{2}\ ,\ b\left(T\right)=-\frac{\lambda\left(\beta\right)}{12\beta}$$

Transport phenomena

- Diffusion

Fick's law $\frac{\partial c(x,t)}{\partial t}=D\frac{\partial^2 c(x,t)}{\partial x^2}$, where $c(x,t)$ is the concentration, D diffusion constant [20]

- Thermal conduction

for heat current density $\vec{j}\left(\vec{x}\right)$

$\vec{j}\left(\vec{x}\right)=-\kappa\nabla T$ Fourier's law , κ=thermal conductivity [18]

- Electrical conduction

current density $\vec{j}\left(\vec{x}\right)=\sigma\ \nabla U$, where U=electrical potential, σ=conductivity [18]

Mass and energy

specific weight $\rho=\frac{M}{V}$

specific heat $c_Q=\frac{Q}{m\Delta T}$, where Q=heat, m=mass, ΔT=temperature difference

Phase gaseous and fluid

- Fundamental material parameters

geometric parameters of the participating atoms/molecules

interaction energy of orbitals $E\left(O_1,O_2,r\right)$

- Eos real gas $\left(P+a\rho^2\right)\left(1-b\rho\right)=\rho RT$ (van der Waals), material constants a,b [19]
- Eos real fluid $\frac{1}{V_0}\frac{dV}{dP}=-\frac{A}{B+P}$ (Tait) [19]

where V_0 represents the molar volume at "zero" pressure P_0, while A and B are (positive) material-specific parameters.

Molecular dynamics: Boltzmann transport equation
[25]

$$\frac{\partial f}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla f+\vec{F}\cdot\frac{\partial f}{\partial \vec{p}}=\left(\frac{\partial f}{\partial t}\right)_{coll}$$

$\vec{F}\left(\vec{r},t\right)$ is the force field acting on the particles, and m is the mass of the particles.
The term on the right hand side describes the collisions between particles.
The probability density function is $f\left(\vec{r},\vec{p},t\right)$, where $dN=f\left(\vec{r},\vec{p},t\right)d^3r\,d^3p$, where N is particle number.

Continuum dynamics equation: Navier-Stokes
for the velocity distribution $\vec{u}\left(\vec{x},t\right)$: incompressible Navier-Stokes equation [20]

$\rho\frac{D\vec{u}}{Dt}=\rho g-\nabla P+\eta\nabla^2\vec{u}$, substantial derivative $\frac{D\vec{u}}{Dt}=\frac{\partial\vec{u}}{\partial t}+\vec{u}\nabla\bullet\vec{u}$
where η is dynamic viscosity

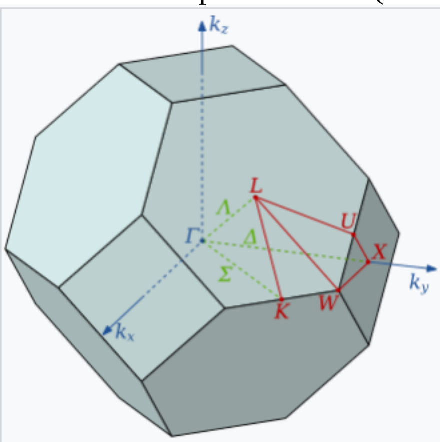
Derivation of the Boltzmann equation and Navier-Stokes equation
[24] gives a rigorous derivation of Boltzmann's kinetic theory starting from hard-core-potential , on a microscopic system formed of N particles of diameter ε undergoing elastic collisions, by taking the kinetic limit $N\rightarrow\infty$, $\varepsilon\rightarrow 0$, and shows that the one-particle density $f\left(\vec{r},\vec{p},t\right)$ obeys the Boltzmann equation

where the collision term is $\left(\frac{\partial f\left(\vec{r},\vec{p},t\right)}{\partial t}\right)_{coll}=\alpha\,Q_{n,n}\left(\frac{p}{m},r\right)$, $\alpha=N\varepsilon^2$

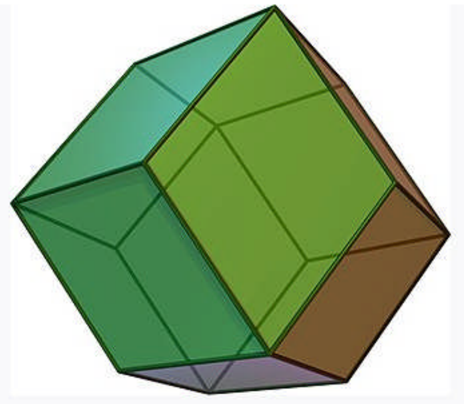
$$Q_{n,n}\left(v,x\right)=\int_{R^d}\int_{S^{d-1}}\left(\left(v-v_1\right)\cdot\omega\right)\Big|_{>0}\left(n\left(x,v'\right)n\left(x,v_1'\right)-n\left(x,v\right)n\left(x,v_1\right)\right)d\omega\,dv_1$$

where $v'=v-\left(\left(v-v_1\right)\cdot\omega\right)\omega$, $v_1'=v_1+\left(\left(v-v_1\right)\cdot\omega\right)\omega$
In a second step, in the limit $\alpha\rightarrow\infty$, the incompressible Newton-Stokes equation is derived.

Phase solid
-Fundamental material parameters
geometric parameters of the Wigner-Seitz cell: medium vertical vectors $\vec{r}_i$, $i=1,\ldots,n_{WS}$,of the n_{WS} surfaces of the Wigner-Seitz cell
interaction energy of orbitals within the Wigner-Seitz cell $E\left(O_{k_1},O_{k_2},r\right)$ $k_1,k_2=1,\ldots,n_{WS}$
-Crystal structure [21]
Unit cell in reciprocal lattice (momentum-space): Brillouin zone



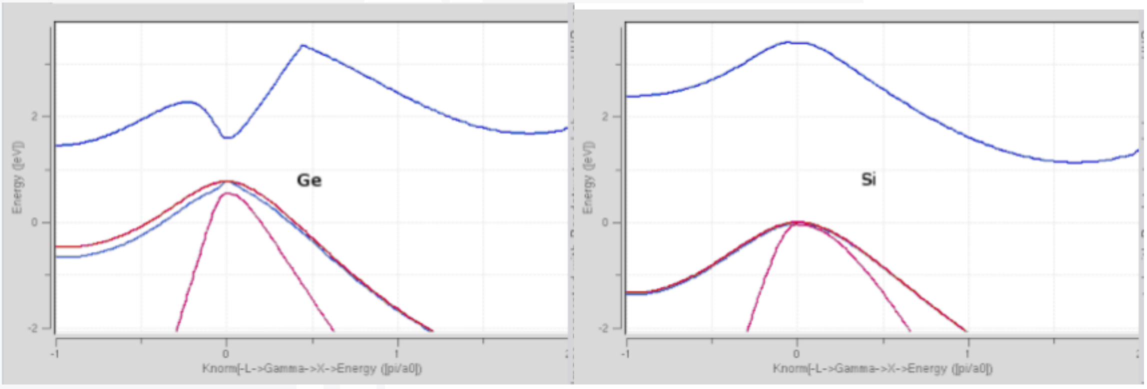
Brillouin zone of Fcc lattice showing labels for special symmetry points , in dependence of wavenumbe k
Unit Wigner-Seitz lattice cell



Wigner-Seitz cell of Fcc lattice

-Electronic band structure [21]

Energy in momentum space $E(k)$ in dependence of wavenumber k



Band structure plot for Si, Ge.

-Elastic [18]

stress $\sigma = E\varepsilon$, with strain ε, where E = Young's modulus

shear stress $\tau = G\,\gamma$, with shear modulus $G = \dfrac{E}{2(1+\nu)}$, shear strain γ, ν=Poisson's ratio

friction $F = \mu F_n$, where μ=friction coefficient, F_n=normal force

-Electromagnetic [18]

magnetization $\vec{M} = \chi_m \vec{H}$, with magnetic field strength H, magnetic susceptibility χ_m

electric polarization $\vec{P} = \varepsilon_0 \chi_e \vec{E}$ with electric field strength E, electric susceptibility χ_e

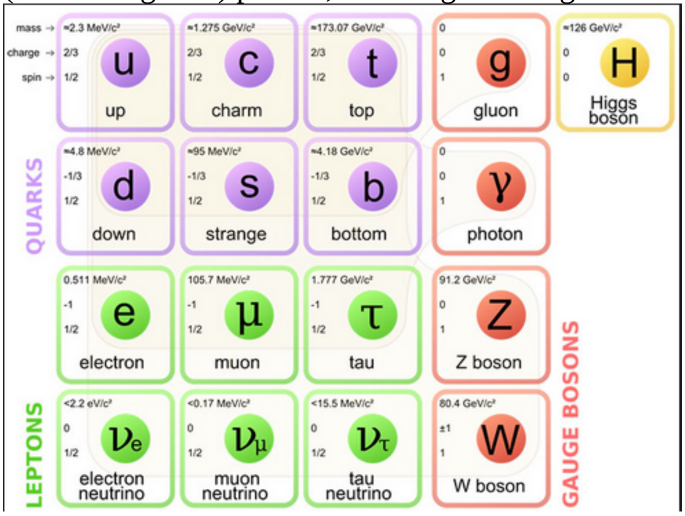
-Optical [18]

refraction $n_i \sin\theta_i = const$ Snell's law

absorption: intensity $I = I_0 \exp(-\mu x)$, where x=length, μ=absorption coefficient

Model of microcosm

The **Standard Model** of particle physics unites the three fundamental electromagnetic, weak, and strong (color) interactions, which mediate the dynamics of the known subatomic particles. It is based on the QFT for these interactions and describes 24 particles: 3x2 quarks, 3x2 leptons, 8 (color) gluons, 2 weak bosons , 1 (electromagnetic) photon, 1 mass-generating scalar higgs.



Parameters of the Standard Model

The model has 28 parameters + elementary charge e_0

Parameters of the Standard Model

Symbol	Description	Renormalization scheme (point)	Value
m_e	Electron mass		511 keV
m_μ	Muon mass		105.7 MeV
m_τ	Tau mass		1.78 GeV
m_u	Up quark mass	$\mu_{\underline{MS}}$ = 2 GeV	1.9 MeV
m_d	Down quark mass	μ_{MS} = 2 GeV	4.4 MeV
m_s	Strange quark mass	μ_{MS} = 2 GeV	87 MeV
m_c	Charm quark mass	$\mu_{MS} = m_c$	1.32 GeV
m_b	Bottom quark mass	$\mu_{MS} = m_b$	4.24 GeV
m_t	Top quark mass	On-shell scheme	172.7 GeV
θ_{12}	CKM 12-mixing angle	q flavor mixing	13.1°
θ_{23}	CKM 23-mixing angle		2.4°
θ_{13}	CKM 13-mixing angle		0.2°
δ_{13}	CKM CP-violating Phase		0.995
θ_{12}	PMNS 12-mixing angle	ν flavor mixing	33.6±0.8°
θ_{23}	PMNS 23-mixing angle		47.2±4°
θ_{13}	PMNS 13-mixing angle		8.5±0.15°
δ_{13}	PMNS CP-violating Phase		4.1±0.75
g_1 or g'	U(1) gauge coupling	$\mu_{MS} = m_Z$	0.357
g_2 or g	SU(2) gauge coupling	$\mu_{MS} = m_Z$	0.652
g_3 or g_s	SU(3) gauge coupling	$\mu_{MS} = m_Z$	1.221
Λ	crit. energy in SU(3)		220MeV
c_{gE0}	additional log in col-coupling		0.69
θ_{QCD}	QCD vacuum angle		~0
v	Higgs vacuum expectation value		246 GeV
m_H	Higgs mass		125.36±0.41 GeV
$m_{\nu e}$	electron neutrino mass		$\le$ 0.12 eV
$m_{\nu\mu}$	mu neutrino mass		$\le$ 0.12 eV
$m_{\nu\tau}$	tau neutrino mass		$\le$ 0.12 eV

additional: elementary charge resp. electromagnetic fine-structure constant $\alpha_{em} = \frac{e_0^{\,2}}{4\pi} = \frac{1}{137}$

Lagrangian of the Standard Model of Particle Physics

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{gauge}/\psi}$$

with the field tensors (color G, weak W and Z, electromagnetic Maxwell F)

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_3 f^{abc} A^b_\mu A^c_\nu \\ W^\pm_{\mu\nu} &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \ , \end{aligned}$$

and the coupling constants g_1 (R-weak). g_2 (SU(2)-L-weak) , g_3 (SU(3) color)

Dirac term

$$\mathcal{L}_{\text{Dirac}} = i\bar{e}^i_{\text{L}} \partial\!\!\!/ e^i_{\text{L}} + i\bar{\nu}^i_{\text{L}} \partial\!\!\!/ \nu^i_{\text{L}} + i\bar{e}^i_{\text{R}} \partial\!\!\!/ e^i_{\text{R}} + i\bar{u}^i_{\text{L}} \partial\!\!\!/ u^i_{\text{L}} + i\bar{d}^i_{\text{L}} \partial\!\!\!/ d^i_{\text{L}} + i\bar{u}^i_{\text{R}} \partial\!\!\!/ u^i_{\text{R}} + i\bar{d}^i_{\text{R}} \partial\!\!\!/ d^i_{\text{R}}$$

mass term

$$\mathcal{L}_{\text{mass}} = -m_e{}^i\, \bar{e}^i e^i - m_u{}^i\, \bar{u}^i u^i - m_d{}^i\, \bar{d}^i d^i - M_W^2 W^+_\mu W^{-\mu} - \frac{M_W^2}{2\cos^2\theta_W} Z_\mu Z^\mu$$

gauge term (color, weak, electromagnetic)

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}(G^a_{\mu\nu})^2 - \frac{1}{2} W^+_{\mu\nu} W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{WZA} \ ;$$

where the field-boson-interaction term is

$$\begin{aligned} \mathcal{L}_{WZA} = \ & ig_2 \cos\theta_{\text{W}} \left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu \right) \partial^\mu Z^\nu + W^+_{\mu\nu} W^{-\mu} Z^\nu - W^-_{\mu\nu} W^{+\mu} Z^\nu \right] \\ & + ie \left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu \right) \partial^\mu A^\nu + W^+_{\mu\nu} W^{-\mu} A^\nu - W^-_{\mu\nu} W^{+\mu} A^\nu \right] \\ & + g_2^2 \cos^2\theta_{\text{W}} \left(W^+_\mu W^-_\nu Z^\mu Z^\nu - W^+_\mu W^{-\mu} Z_\nu Z^\nu \right) \\ & + g_2^2 \left(W^+_\mu W^-_\nu A^\mu A^\nu - W^+_\mu W^{-\mu} A_\nu A^\nu \right) \\ & + g_2 e \cos\theta_{\text{W}} \left[W^+_\mu W^-_\nu \left(Z^\mu A^\nu + Z^\nu A^\mu \right) - 2 W^+_\mu W^{-\mu} Z_\nu A^\nu \right] \\ & + \frac{1}{2} g_2^2 \left(W^+_\mu W^-_\nu \right) \left(W^{+\mu} W^{-\nu} - W^{+\nu} W^{-\mu} \right) \ ; \end{aligned}$$

current interaction term

$$\mathcal{L}_{\text{gauge}/\psi} = -g_3 A^a_\mu J^{\mu a}_{(3)} - g_2 \left(W^+_\mu J^\mu_{W+} + W^-_\mu J^\mu_{W-} + Z_\mu J^\mu_Z \right) - e A_\mu J^\mu_A$$

with the currents

color quark current

$$J^{\mu a}_{(3)} = \bar{u}^i \gamma^\mu T^a_{(3)} u^i + \bar{d}^i \gamma^\mu T^a_{(3)} d^i$$

weak charged lepton and Cabibbo-rotated weak charged quark current

$$\begin{aligned} J^\mu_{W+} &= \frac{1}{\sqrt{2}} \left(\bar{\nu}^i_{\text{L}} \gamma^\mu e^i_{\text{L}} + V^{ij} \bar{u}^i_{\text{L}} \gamma^\mu d^j_{\text{L}} \right) \\ J^\mu_{W-} &= (J^\mu_{W+})^* \end{aligned}$$

and its conjugated current

neutral lepton and quark current

$$\begin{aligned} J^\mu_Z = \ & \frac{1}{\cos\theta_{\text{W}}} \left[\frac{1}{2} \bar{\nu}^i_{\text{L}} \gamma^\mu \nu^i_{\text{L}} + \left(-\frac{1}{2} + \sin^2\theta_{\text{W}} \right) \bar{e}^i_{\text{L}} \gamma^\mu e^i_{\text{L}} + (\sin^2\theta_{\text{W}}) \bar{e}^i_{\text{R}} \gamma^\mu e^i_{\text{R}} \right. \\ & + \left(\frac{1}{2} - \frac{2}{3} \sin^2\theta_{\text{W}} \right) \bar{u}^i_{\text{L}} \gamma^\mu u^i_{\text{L}} + \left(-\frac{2}{3} \sin^2\theta_{\text{W}} \right) \bar{u}^i_{\text{R}} \gamma^\mu u^i_{\text{R}} \\ & \left. + \left(-\frac{1}{2} + \frac{1}{3} \sin^2\theta_{\text{W}} \right) \bar{d}^i_{\text{L}} \gamma^\mu d^i_{\text{L}} + \left(\frac{1}{3} \sin^2\theta_{\text{W}} \right) \bar{d}^i_{\text{R}} \gamma^\mu d^i_{\text{R}} \right] \end{aligned}$$

electromagnetic current

$$J^\mu_A = (-1) \bar{e}^i \gamma^\mu e^i + \left(\frac{2}{3} \right) \bar{u}^i \gamma^\mu u^i + \left(-\frac{1}{3} \right) \bar{d}^i \gamma^\mu d^i \ .$$

higgs term $g=2g_2$

$$\mathcal{L}_{\textbf{higgs1}} = -\frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2}{m_h}^2 H^2 - \left(g\alpha_h H^3 + \frac{1}{8} g^2 \alpha_h H^4 \right) - \beta_h \frac{2M_W}{g} H$$

$$\mathcal{L}_{\textbf{higgs2}} = -gM_W W^+{}_\mu W^{-\mu} H - \frac{g\,M_W}{2\cos^2\theta_W} Z_\mu Z^\mu H - \frac{g}{2M_W} H\left({m_e}^i\,\bar{e}^i e^i + {m_u}^i\,\bar{u}^i u^i + {m_d}^i\,\bar{d}^i d^i \right)$$

The extended **SU(4) preon model** of the Standard Model (SU4PM)

The basic particle families are
-leptons L=r⊗r being a hc-tetra-spinor of a doublet of two r-preons, fermions with total spin S=1/2
-quarks Q=r⊗q being a hc-tetra-spinor of a doublet of an r- and a q-preon, colored fermions with color Q_c=1 with total spin S=1/2
-(hypothetical) strong neutrinos N_c=q⊗q being a hc-tetra-spinor of a doublet of two q-preons, colored fermions with color Q_c=0 with total spin S=1/2
-weak bosons B_w=r±r being a linear combinations of two or more r-preons, with total spin S=0 (scalar like higgs H) or S=1 (vector like W and Z)
-(hypothetical) strong bosons B_c=q±q being a linear combinations of two or more q-preons, with color Q_c=0 and total spin S=0 (scalar like higgs H_q) or S=1 (vector like Z_q)

A a hc-tetra-spinor is a hc-quadruplet with the hc-charges {L- , L+, R-, R+} .
Both preons can carry all four charges of SU(4), i.e. there are {rL-, rL+, rR-, rR+} and {qL-, qL+, qR-, qR+}, where the spinor-anti-spinor pairs are {rL-, rR+} and {rL+, rR-} .
The r-q-doublets, i.e. the quarks, have one more degree of freedom, as they consist of different fermions, and are therefore chiral-neutral, which is energetically more favorable.

Generations (flavor) of SM
A hc-doublet occupies two positions in a hc-tetra-spinor with indices (i, j), e.g the e-neutrino with the configuration {rL-, rL+, 0, 0} has the hc-indices $(1,\bar{2})$, the bar over 2 signifies the anti-spinor.
One can show, that for two hc-indices {i, j} there are three field-boson configurations, which are compatible with the SU(4) symmetry:
●one hc-boson A_{ij} (corresponding to the non-diagonal hc-matrix $\tilde{\lambda}_{ij}$ interchanging i with j , e.g. for $(i,j)=(1,2)$ $\tilde{\lambda}_{ij}=\lambda_1$) : generation 1 with lowest mass m~0.5-6MeV
●four hc-bosons $A_{ij}, \bar{A}_{ij}, A_{kl}, \bar{A}_{kl}$ (interchanging resp. (i,j), $(i,\bar{j})$, and the dual index pairs (k,l), $(k,\bar{l})$) : generation 2 with medium-range mass m~100MeV-1.3GeV
●all 15 hc-bosons as the third configuration m~1.8-170GeV
The maximum generation mass scales approximately with boson number N_b^{κ} with $\kappa=3.8$

This is in analogy to the color SU(3) interaction, where 3 gluon configurations form 3 families of hadrons:
$Ag=\{Ag_1,...,Ag_8\}$ all gluons for nucleons
$Ag=\{Ag_1,Ag_2,Ag_4,Ag_5,Ag_6,Ag_7\}$ 6 non-diagonal gluons for vector-mesons
$Ag=\{Ag_2,Ag_5,Ag_7\}$ 3 quark-antiquark gluons for for pseudo-scalar mesons

Basic parameters of SU4PM
We have 4 parameters for SU4PM: 2 preon masses, and for the coupling constant the critical energy Λ_{hc} and the peak height constant c_{GE1} in the Callan-Symanzik equation

$$g_{hc}(m)=4\pi\sqrt{\frac{3}{76\sqrt{\left(log(\frac{m}{\Lambda_{hc}})\right)^2+c_{GE1}{}^2}}}$$

we set $\Lambda_{hc}=2m(Z_0)=180GeV$ in analogy to the QCD , and $c_{GE1}=\dfrac{1}{Log\left(\dfrac{m(t)}{m(d)}\right)}=0.095$

Parameters of the SU4 preon model (6 parameters+ elementary charge e_0)
Description Value

r mass	0.033meV
q mass	0.77 MeV
char. SU(4) en. Λ	180GeV
zero corr. c_{GE}	0.095
char . SU(3) en. Λ_g	220MeV
zero corr. c_{gE0}	0.69

additional: elementary charge resp. electromagnetic fine-structure constant $\alpha_{em} = \frac{e_0^{\ 2}}{4\pi} = \frac{1}{137}$

Preon data
wave function $\Psi = (u_{L-}, u_{L+}, u_{R-}, u_{R+})$
r-preons $(r_{L-}, r_{L+}, r_{R-}, r_{R+})$
Q(r)=-1/2, m(r)=0.033 meV
q-preons $(q_{L-}, q_{L+}, q_{R-}, q_{R+})$
Q(q)=+1/6, m(q)=0.77MeV

Fundamental rules
There is a **fundamental duality** of particles :
• **massive particles**= constituents of matter
massive particles have mass m>0, velocity v<c,
- basic spinors in 3 generations: spin S=1/2 (3x2 quarks u,d,c,s,t,b; 3x2 leptons e,ν_e,μ, ν_μ,τ, ν_τ)
- Yukawa bosons of the extended weak SU4-hypercolor interaction: spin S=1 (W/Z boson), S=0 (Higgs H)
• **field particles**=massless field carriers of the four fundamental interactions
field particles have m=0, v=c, spin S=1 (photon, 8 gluons, 15 hc-bosons), spin S=2 (graviton 4x4 tensor, 2x2 polarizations, four rows/columns)
a field is generated by a Lie-group-algebra
-quantum-electromagnetic: SU(1)
-color: SU(3) , 8 generators, corresponding 8 gluons
-hypercolor: SU(4) 15 generators, corresponding 15 hc-bosons
-quantum gravity: Lie(ε) Lie-group-algebra (of the antisymmetric tensor ε), 4 generators, corresponding 4 polarizations

Configuration of the SM in the SU4PM

charged leptons {e, mu, tau}

$$x=\left(\begin{pmatrix} rL- \\ 0 \end{pmatrix},0,\begin{pmatrix} rR- \\ 0 \end{pmatrix},0\right)$$

$e = x + A13$ flavor F=1 one boson m=0.511MeV

$mu = x + A13 + \overline{A}13 + A24 + \overline{A}24$ F=2: four bosons m=105.7MeV

$tau = x + A$ F=3: all bosons m=1.78GeV

lepton neutrinos {nue, num, nut}

$$x=\left(\begin{pmatrix} rL- \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ rL+ \end{pmatrix},0,0\right)$$

$nue = x + A12$ m=0.170meV

$num = x + A12 + \overline{A}12 + A34 + \overline{A}34$ m=10meV

$nut = x + A$ m=42meV

u-quarks {u,c,t}

$$x=\left(0,\begin{pmatrix} (rL++qL+)/\sqrt{2} \\ (rL++qL+)/\sqrt{2} \end{pmatrix},0,\begin{pmatrix} (rL++qR+)/\sqrt{2} \\ (rL++qR+)/\sqrt{2} \end{pmatrix}\right)$$

$u = x + A24$ m=2.3MeV

$c = x + A24 + \overline{A}24 + A13 + \overline{A}13$ m=1.34GeV

$t = x + A$ m=171GeV

d-quarks {d, s, b}

$$x=\left(\begin{pmatrix} (rR-+qL+)/\sqrt{2} \\ (rR-+qL+)/\sqrt{2} \end{pmatrix},0,\begin{pmatrix} (rR-+qR+)/\sqrt{2} \\ (rR-+qR+)/\sqrt{2} \end{pmatrix},0\right)$$

$d = x + A13$ m=4.8MeV

$s = x + A13 + \overline{A}13 + A24 + \overline{A}24$ m=100 MeV

$b = x + A$ m=4.2 GeV

weak massive bosons {W, Z, ZL, H}
F=3, all A

$$W^- = \left(0,0,\begin{pmatrix} u1 \\ -u1 \end{pmatrix},0\right)\sqrt{2} \quad u1 = ((rR-)-(rR-))/\sqrt{2}, \ (W^-)^\dagger = W^+ \ \text{m=80.4GeV}$$

$$Z0 = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},0,0,\begin{pmatrix} u1 \\ u1 \end{pmatrix}\right)/\sqrt{2} \quad u1 = ((rL-)+(rR+))/\sqrt{2}, \ (Z0)^\dagger = Z0 \ \text{m=91.2GeV}$$

participate in neutron decay and similar weak decays

$$ZL = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},0,0\right)/\sqrt{2} \quad u1 = ((rL-)+(rL+))/\sqrt{2} \ \text{m=91GeV}$$

$$H = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix}\right)/2 \quad u1 = ((rL-)+(rL+)+(rR-)+(rR+))/2 \ \text{m=125GeV}$$

SU(4) mass generator

strong neutrinos {qnue, qnum, qnut} self-interaction

$$x=\left(\begin{pmatrix} qL- \\ 0 \end{pmatrix},0,0,\begin{pmatrix} 0 \\ qR+ \end{pmatrix}\right)$$

$qnue = x + A14$ m=23.2MeV

$qnum = x + A14 + \overline{A}14 + A23 + \overline{A}23$ m=205MeV

$qnut = x + A$ m=2.4GeV

strong massive bosons {Zq, Hq}
F=3, all A
neutral strong exchange boson

$$Zq = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) / \sqrt{2} \quad u1 = \left((qL-) + (qR-) \right) / \sqrt{2} \,,\ (Zq)^{\dagger} = Zq \ \text{m=644GeV}$$

SU(3) mass generator

$$Hq = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix} \right) / 2 \quad u1 = \left((qL-) + (qL+) + (qR-) + (qR+) \right) / 2 \ \text{m=637GeV}$$

Ashtekar-Kodama gravitation classical

field carrier graviton $A_\mu{}^\nu$ S=2

32 AK-equation for graviton $A_\mu{}^\nu$ and tetrad $E^{\mu\nu}$

$H_{(\mu,\nu)}{}^\kappa \equiv F_{\mu\nu}{}^\kappa + \dfrac{\Lambda}{3}\varepsilon_{\mu\nu\rho}E^{\rho\kappa} = 0$, $G^\mu \equiv \partial_\nu E^{\nu\mu} + \varepsilon^\mu{}_{\kappa\lambda}A_\nu{}^\kappa E^{\nu\lambda} = 0$, $I_\mu \equiv E^\kappa{}_\nu F_{\mu\kappa}{}^\nu = 0$

orbit equations $\dfrac{du^\mu}{d\tau} = -\Gamma^\mu_{\nu\lambda}u^\nu u^\lambda$, $E^{\mu\kappa}E^\nu{}_\kappa/(\det(E))^{3/4} = g^{\mu\nu}$

with field tensor $F_{\mu\nu}{}^\kappa$

$F_{\mu\nu}{}^\kappa = \partial_\mu A_\nu{}^\kappa - \partial_\nu A_\mu{}^\kappa + \varepsilon^\kappa{}_{\kappa_1\kappa_2}A_\mu{}^{\kappa_1}A_\nu{}^{\kappa_2}$

boundary cond. at $x \to \infty$ for tetrad and external metric $g_{\mu\nu}$

$E^{\mu\kappa}E^\nu{}_\kappa = g^{\mu\nu}/(-\det(g))^{3/4}$

for $\Lambda \to 0$

GR with Einstein equations

$R_{\mu\nu} - \dfrac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$

with $\kappa = \dfrac{8\pi G}{c^4}$

at the horizon $r = r_s$ $E^{\mu\nu} >> 1$ and $\Lambda|E| \sim 1$

singularity becomes a finite peak

AK quantum $r < r_{gr}$

min .length $r_{gr} = \sqrt{l_P\sqrt{\dfrac{1}{\Lambda}}} = 39\mu m$, energy scale $E_{gr} = \dfrac{\hbar c}{r_{gr}} = 5.04*10^{-3}eV$

max length $R_\Lambda = 1/\sqrt{\Lambda} = 0.95*10^{26}m$ energy scale $E_\Lambda = \dfrac{\hbar c}{R_\Lambda} = 3.31*10^{-33}eV$

Lagrangian $L_{gr} = L_H + L_I = \hbar c\begin{pmatrix} -\dfrac{1}{4}F_{\mu\nu}{}^\kappa F^{\mu\nu}{}_\kappa - \dfrac{\overline{\varphi}_\Lambda\varphi_\Lambda}{3}(\varepsilon^\kappa{}_{\mu\lambda}E^{\lambda\nu}\partial_\kappa A_{\mu\nu} + \varepsilon_{\mu_2\lambda_2\kappa_2}\varepsilon^{\mu_1\lambda_1}{}_{\kappa_1}E^{\kappa_1\kappa_2}A_{\lambda_1}{}^{\lambda_2}A_{\mu_1}{}^{\mu_2}) \\ + E^{\kappa_1}{}_{\nu_1}F_{\mu\kappa_1}{}^{\nu_1}E^{\kappa_2}{}_{\nu_2}F^\mu{}_{\kappa_2}{}^{\nu_2} \end{pmatrix}$

with the constraint $\overline{\varphi}_\Lambda\varphi_\Lambda = \Lambda$, Feynman propagator $D_g(q,m) = \dfrac{m}{q(q-im)(q-2im)}$

where *m*=mass of source fermion

quasi-classical approximation: $E_{gr}(r) = \dfrac{r_\Lambda}{r}mc^2$ for $r \ll r_{gr}$, $r_\Lambda = 3.7\times10^{-49}m$

Lie group Lie(ε)
cov. derivative $D_\mu = \partial_\mu - iA_\mu{}^a \tau^a$, where $\tau^a = i\varepsilon_\nu{}^a{}_\lambda$ a=1,2,3,4
generators of Lie(ε) commutator $[\tau^a, \tau^b] = i\varepsilon_c{}^{ab}\tau^c$
equivalent to Lie3(ε) Lie-algebra with 3 generators τ^a and the commutator
$[\tau_a, \tau_b] = i\sum_k (1+\varepsilon_{abk})\tau_k$, a=1,2,3 which is isomorphic to SU(2)

electromagnetic classical
Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$
field carrier photon A^μ S=1 , charges (+, -)
orbit equations $m\frac{du^\mu}{d\tau} = F^{\mu\nu}u_\nu$

quantum electrodynamics
length scale Bohr radius $r_B = 5.29*10^{-11}m$
energy scale $E_{em} = \frac{\hbar c}{r_B} = 3.7keV$
cov. derivative $D_\mu = \partial_\mu - ieA_\mu$
Lagrangian $L_{em} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
quasi-classical approximation: $E_{el}(r) = \frac{e_0{}^2}{r}$ of an electron for
$r >> r_{ce} = \frac{e_0{}^2}{m_e c^2} = 2.8*10^{-15}m$, where r_{ce} is the classical electron radius

Lie group SU(1)
={real numbers}

color (strong) interaction
field carrier gluon $A^a{}_\mu$ a=1..8 S=1
charge color=(r,g,b)
energy scale $E_{col} = 220MeV$, confinement length scale
$r_{col} = \frac{\hbar c}{E_{col}} = 0.89*10^{-15}m \approx r_{pr}$
cov. derivative $D_\mu \equiv \partial_\mu - igA^a_\mu\lambda_a/2$
$[\lambda_a, \lambda_b] = 2i f^{abc}\lambda_c$
field tensor $G^a{}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}A^b_\mu A^c_\nu$
Lagrangian $L_{col} = -\frac{1}{4}G^a{}_{\mu\nu}G^{a\mu\nu}$
Yukawa-boson π^0, π^+, π^-

Lie group SU3)
generators Gell-Mann matrices λ_a
a=1...8

hyper-color interaction
4 charges (L-, L+, R-, R+)
15 hc-bosons $A^a{}_\mu$ a=1...15, S=1
cov. derivative $D_\mu \equiv \partial_\mu - igA^a_\mu \lambda_a / 2$
$\Psi = \left(u_{L-}, u_{L+}, u_{R-}, u_{R+}\right)$, e.g. electron $e^- = \left(r_{L-}, 0, r_{R-}, 0\right)$
generations $f_1 \simeq 1A_h$ $f_2 \simeq 4A_h$ $f_3 \simeq 15A_h$
$\left[\lambda_a, \lambda_b\right] = 2i\, f^{abc} \lambda_c$
field tensor $G^a{}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$
Lagrangian $L_{hc} = -\frac{1}{4} G^a{}_{\mu\nu} G^{a\mu\nu}$
coupling Callan-Symanzik $g_{hc}(\mu) = 4\pi \sqrt{3 / \left(76 \sqrt{\left(\log(\frac{\mu}{\Lambda_{hc}})\right)^2 + c_{GE1}{}^2}\right)}$

critical energy $\Lambda_{hc} = 2m(Z_0) = 180GeV$
$r_{hc} = \frac{\hbar c}{\Lambda_{hc}} = 1.08 * 10^{-18} m$ $c_{GE1} = \frac{1}{Log\left(\frac{m(t)}{m(d)}\right)} = 0.095$
Yukawa bosons: W boson, Z-boson

extended weak
hyper-color Lie group SU(4)
generators SU(4)-matrices λ_a
a=1...15

symmetry breaking $SU(4) = SU(2)_L \otimes SU(1)_R \otimes SU(1)_{em}$
field Yukawa boson Z_0, W^+, W^-
interaction $L(W^\pm) \otimes Y(Z) \otimes em(A^\mu)$
doublet $SU(2)_L$ interaction $L(W^\pm) = W^\mu J_W{}^\mu$
current $J_W{}^\mu = \bar{\nu}_L \gamma^\mu e_L + \ldots$
singlet $SU(1)_R$ interaction $Y(Z) = Z^\mu J_Z{}^\mu$
current $J_Z{}^\mu = c_{eL} \bar{e}_L \gamma^\mu e_L + c_{eR} \bar{e}_R \gamma^\mu e_R + \ldots$
singlet $SU(1)_{em}$ interaction $em(A^\mu) = A^\mu J_A{}^\mu$
em-current $J_A{}^\mu = q(e) \bar{e} \gamma^\mu e + \ldots$
SU(1) right-handed derivative
$D_\mu R = \left(\partial_\mu + i\, g' B_\mu\right) R$
SU(2) left-handed derivative
$D_\mu L = \left(\partial_\mu + \frac{i}{2} g' B_\mu - \frac{i}{2} g \sigma_i W^i_\mu\right) L$

Fermi-Pauli weak interaction
Lie group SU(2)
generators Pauli-matrices $i\sigma_k$
k=1...3

Comparison of QFT's

electromagnetic: gauge-group=Lie group SU(1) (trivial) commutative, long-range, classical and quantum, scale Bohr radius $r_B = 5.29 * 10^{-11} m$, energy scale $E_{em} = \frac{\hbar c}{r_B} = 3.7 keV$, the interaction-boson=photon is a vector with spin=1

charges: electrical charge Q+, Q- ; elementary (minimal) charge: e_0

covariant derivative $D_\mu = \partial_\mu - \frac{ie}{\hbar} A_\mu$, fundamental equations Maxwell equations $\partial_\mu F^{\mu\nu} = 0$ resp. $\partial_\mu F^{\mu\nu} = j^\nu$, where $A_\mu = (\frac{\varphi}{c}, \vec{A})$ is the 4-vector potential, $\vec{B} = \nabla \times \vec{A}$ the magnetic field, and $j^\mu = (c\rho, \vec{j})$ the 4-current density.

The interaction constant is the fine structure constant $\alpha_{em} = \frac{e_0^{\,2}}{4\pi} = \frac{1}{137}$, the elementary cross-section for electron-interaction is $\sigma_{em} = \alpha_{em}^{\,2} \tilde{\lambda}_e^{\,2} = \alpha_{em}^{\,2} \left(\frac{\hbar}{m_e c} \right)^2$

The field carrier is photon A_μ spin=1 one polarization, modes: spherical($l = 0$) and and all $l > 0$, angular function Y_{lm} , wave equation , radial Helmholtz equation, Lagrangian $L_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ with the Maxwell field tensor $F_{\mu\nu} = \partial_\mu A_\nu^{\ \kappa} - \partial_\nu A_\mu^{\ \kappa}$

gravitation: gauge-group= Lie(ε) with 4 generators τ_μ , additional invariance: background independence, long-range, classical and quantum, its interaction-boson=graviton is a 2-tensor with spin=2

charges: mass , elementary (minimal) mass (minimal energy is reciprocal to the maximum length R_Λ)

$$m_\Lambda = \frac{\hbar}{cR_\Lambda} = \frac{\hbar\sqrt{\Lambda}}{c} = \frac{1.05 * 10^{-34} Js 1.052 * 10^{-26} m^{-1}}{3 * 10^8 ms^{-1}} = 0.368 * 10^{-68} kg$$

scale $r_{gr} = \sqrt{l_P \sqrt{\frac{1}{\Lambda}}} = 39 \mu m$, energy scale $E_{gr} = \frac{\hbar c}{r_{gr}} = 5.04 * 10^{-3} eV$

fundamental equations: Ashtekar-Kodama equations $AK(A_\mu^{\ \nu}, E^{\mu\nu}, \Lambda)=0$ for the graviton tensor $A_\mu^{\ \nu}$, the tetrad $E^{\mu\nu}$ and the cosmological constant Λ , in the limit $\Lambda \to 0$ the Einstein-equations are valid (except at the horizon) and $A_\mu^{\ \nu}$ =const (i.e. trivial background)

Λ generated by the tensor field ϕ_Λ is scale dependent, classical scale $r >> r_{gr}$ $\Lambda = \Lambda_0 = 2.7\ 10^{-52} m^{-2}$

wave: graviton $A_\mu^{\ \nu}$ spin=2 two polarizations, modes: quadrupole and higher $l \geq 2$, angular function $\exp(i\, l\, \theta)$, gravitational radial wave equation weq$_\nu$($E^{1\nu}, \partial_r^{\ 3} E^{1\nu}$) for the tetrad component $E^{1\nu}$ determines $A_\mu^{\ \nu}$,$E^{\mu\nu}$ with 4 free parameters for $E^{\mu\nu}$ and 4 free parameters for $A_\mu^{\ \nu}$, $E^{\mu\nu}$ damped $\exp(-4\sqrt{\frac{r}{3}})$ for $r \leq r_{gr} = 39 \mu m$ QFT gravity,

Λ becomes a running constant $\Lambda(k) = \left(\frac{2\sqrt{2}\, k^2}{M^2 + w_* 2k^2} \right)^{4/3} l_p^{-2}$

In the **low energy limit** $k = \sqrt{\Lambda_0}$, $\Lambda\left(k = \sqrt{\Lambda_0}\right) = \Lambda_0 = 1.1 * 10^{-52} m^{-2}$ so $M^2 = 0.45 * 10^{40} m^{-2}$

In the **high energy limit** $k = \infty$ $\Lambda(k = \infty) = \Lambda_{GUT} = 0.14 \times 10^{55} m^{-2}$ start of the cosmological inflation, so $w_* = \sqrt{2} \left(\Lambda_{GUT}\, l_p^{\,2} \right)^{-3/4} = 0.54 \times 10^{12}$

with the **quantum-scale limit**

$$\Lambda_q = \Lambda\left(k = 1/r_{gr}\right) = \left(\frac{2\sqrt{2}}{M^2 r_{gr}^{\,2} + w_* 2} \right)^{4/3} l_p^{-2} = 2. \times 10^{39} m^{-2}$$

covariant derivative $D_\mu = \partial_\mu - i\sqrt{\alpha_{gr}}\left(\tau_a \bullet A^a\right)_\mu$,

where $\tau^a = i\varepsilon_\nu{}^a{}_\lambda$ are the generators of the Lie(ε)-algebra with 4 generators τ^a and the commutator $[\tau^a, \tau^b] = i\varepsilon_c{}^{ab}\tau^c$ a=1,2,3,4

equivalent to Lie3(ε) Lie-algebra with 3 generators τ^a and the commutator $[\tau_a, \tau_b] = i\sum_k (1+\varepsilon_{abk})\tau_k$, a=1,2,3

which is isomorphic to SU(2)

interaction via $\left(D_\mu\right)^\lambda{}_\kappa = \partial_\mu + \varepsilon^\lambda{}_{a\kappa}\sqrt{\alpha_{gr}}\,A_\mu{}^a$ covariant derivative,

interaction constant $\sqrt{\alpha_{gr}} = 0.29\times10^{-30}$

the (renormalizable) Lagrangian is

$$L_{gr} = L_H + L_I = \hbar c\begin{pmatrix} -\frac{1}{4}F_{\mu\nu}{}^\kappa F^{\mu\nu}{}_\kappa - \frac{\overline{\varphi}_\Lambda\varphi_\Lambda}{3}(\varepsilon^\kappa{}_{\mu\lambda}E^{\lambda\nu}\partial_\kappa A_{\mu\nu} + \varepsilon_{\mu_2\lambda_2\kappa_2}\varepsilon^{\mu_1\lambda_1}{}_{\kappa_1}E^{\kappa_1\kappa_2}A_{\lambda_1}{}^{\lambda_2}A_{\mu_1}{}^{\mu_2}) \\ +E^{\kappa_1}{}_{\nu_1}F_{\mu\kappa_1}{}^{\nu_1}E^{\kappa_2}{}_{\nu_2}F^\mu{}_{\kappa_2}{}^{\nu_2} \end{pmatrix}$$

with the constraint $\overline{\varphi}_\Lambda\varphi_\Lambda = \Lambda$

and with the field tensor $F_{\mu\nu}{}^\kappa = \partial_\mu A_\nu{}^\kappa - \partial_\nu A_\mu{}^\kappa + \varepsilon^\kappa{}_{\kappa_1\kappa_2}A_\mu{}^{\kappa_1}A_\nu{}^{\kappa_2}$

quasi-classical approximation: $E_{gr}(r) = \frac{r_\Lambda}{r}mc^2$ for $r \ll r_{gr}$, $r_\Lambda = 3.7\times10^{-49}m$

weak interaction: gauge-group=Lie-group SU(2) 3 generators Pauli matrices σ_j , corresponding 3 massive gauge bosons
Z, W^+ , W^- ,short range, pure quantum
cut-off energy M_Z=*91.17GeV* ,
length scale $r_{weak} = 10^{-17}m$
energy scale $E_{weak} \approx 10^{-4}E_{em} = 7eV$

Lagrangian $L_1 = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$,

where $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,f^{abc}W^b_\mu W^c_\nu$ $\quad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

where the physical gauge fields Z, W^+ , W^- are

$$Z_\mu = \frac{gW^3_\mu + g'B_\mu}{\sqrt{g^2+g'^2}} = \cos\theta_W W^3_\mu + \sin\theta_W B_\mu$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \pm iW^2_\mu\right)$$

covariant derivatives left and right

$$D_\mu R = \left(\partial_\mu + i\,g'B_\mu\right)R \quad D_\mu L = \left(\partial_\mu + \frac{i}{2}g'B_\mu - \frac{i}{2}g\sigma_i W^i_\mu\right)L$$

strong (color) interaction: gauge-group=Lie-group SU(3) 8 generators Gell-Mann-matrices λ_α , corresponding 8 massless spin-1 gauge bosons , 3 charges (colors) *r g b* , short range, pure quantum, asymptotically free (confinement),

confinement length scale $r_{col} = r_{pr} = 0.84*10^{-15}m$, energy scale $E_{col} = 210MeV$

cut-off energy $M_{col} = \frac{\hbar c}{r_{col}} = 2.33*10^{10}eV \approx 23GeV$

Lagrangian $L_{col} = -\frac{1}{4}G^a{}_{\mu\nu}G^{a\mu\nu}$

with the field tensor $G^a{}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,f^{abc}A^b_\mu A^c_\nu$

covariant derivative $D_\mu \equiv \partial_\mu - igA^a_\mu\lambda_a/2$, where *g* is the coupling constant, *A* is the gluon gauge field, for eight different gluons α = 1 … 8 , and where λ_α is one of the eight Gell-Mann matrices, α = 1 … 8 .

Dirac eq. $\left(i\hbar\gamma^{\mu}D_{\mu}-mc\right)\Psi=0$

Dirac wave-function $\Psi=\left(u_1,u_2,u_3\right)$, e.g. proton $p=\left(u,u,d\right)$

particle types: baryon $b:8A_g$ meson $m:2A_g$ v-meson $m:4A_g$

(preon [6]) **hypercolor interaction**: gauge-group= Lie-group SU(4), 15 generators massless bosons spin=1, short range, pure quantum

4 charges $r_L{}^{+}\, r_R{}^{+}\, r_L{}^{-}\, r_R{}^{-}$ for r-preon and $q_L{}^{+}\, q_R{}^{+}\, q_L{}^{-}\, q_R{}^{-}$ for q-preon resp. with charge + or - and helicity L or R, antiparticle $C(r_L{}^{+})=r_R{}^{-}$, $C(r_L{}^{-})=r_R{}^{+}$

the weak-interaction is the Yukawa-limit of the hypercolor interaction

the angular momentum $l=1$ determines the flavor $l_z=0$ for flavor 0, $l_z=+1$ for flavor 1 and $l_z=-1$ for flavor 2

Dirac eq. $\left(i\hbar\gamma^{\mu}D_{\mu}-mc\right)\Psi=0$

$\Psi=\left(u_{L-},u_{L+},u_{R-},u_{R+}\right)$, e.g. electron $e^{-}=\left(r_{L-},0,r_{R-},0\right)$

generations $f_1:1A_h$ $f_2:4A_h$ $f_3:15A_h$

L-R-symmetry breaking $SU(4)=SU(2)_L\otimes SU(1)_R\otimes SU(1)_{em}$

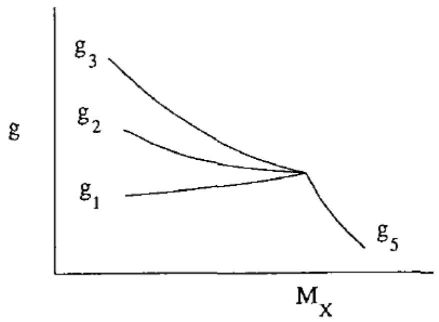
GUT (approximate) SU(5) :

R=(3 rgb *d*-antiquark, 2 leptons e ν)= $d(\overline{3},1)\oplus e\nu(1,\overline{2})$

L=(3x2 rgb *u*-quark *d*-quark, 3 rgb *u*-antiquark, 1 antilepton *e*)= $ud(3,2)\oplus\overline{u}(\overline{3},1)\oplus\overline{e}(\overline{1},1)$

GUT scale

For the three vector Yan-Mills interactions QED, WI, QCD , the coupling constants converge to a single value at the GUT scale $M_X\approx10^{15}GeV$ [3].



Callan-Symanzik relation for Yang-Mills running coupling constant is

$$\beta\left(g\right)=-\frac{g^3}{16\pi^2}\left(\frac{11}{3}N-\frac{2}{3}N_f\right)+\dots\qquad\beta=E\frac{g\left(E\right)}{\partial E}$$

$\beta_i\left(g\right)=b_i\frac{g_i^{\,3}}{16\pi^2}$ *i=1* QED, *i=2* weak interaction (WI), *i=3* QCD

$$g\left(E\right)=\frac{g\left(E_0\right)^2}{1-2bg\left(E_0\right)^2 log\left(\frac{E}{E_0}\right)}$$

QED N=1, Nf=6 $\beta=+\frac{2}{3}\frac{g^3}{16\pi^2}$ $b_1=-\frac{2}{3}$, $\beta\left(E\right)$ increases with E

WI N=2, Nf=3 $\beta=-\frac{16}{3}\frac{g^3}{16\pi^2}$ $b_2=\frac{16}{3}$, asymptotic freedom $\beta\left(E\right)\rightarrow0$

QCD N=3, Nf=3 $\beta=-9\frac{g^3}{16\pi^2}$ $b_3=9$, asymptotic freedom, $\beta\left(E\right)\rightarrow0$

at GUT scale $M_X \approx 10^{15} GeV$ we have a single coupling constant

For $\alpha_i = \frac{g_i^{\,2}}{4\pi^2}$ we have

$$\frac{1}{\alpha_i(E)} = \frac{1}{\alpha_i(E_0)} + \frac{b_i}{2\pi} log\left(\frac{M_X}{E}\right)$$

Comparison of gravity and electromagnetism

Electromagnetism

gauge-group=Lie group SU(1) (trivial) commutative, 1 generator $\tau^1 = 1$
long-range, classical and quantum,

one scale Bohr radius $r_B = 5.29*10^{-11} m$, energy scale $E_{em} = \frac{\hbar c}{r_B} = \frac{1.96*10^{-7} eVm}{5.29*10^{-11} m} = 3.7keV$,

the interaction-boson=photon is a vector with spin *S=1* (obeys the Bose-Einstein statistics)
charge: electrical charge Q+, Q- ; elementary (minimal) charge: e_0

fundamental constants: fine structure constant $\alpha_{em} = \frac{{e_0}^2}{4\pi} = \frac{1}{137}$, classical electron radius r_e

covariant derivative $D_\mu = \partial_\mu + \frac{ie}{\hbar} A_\mu$,

field equations= Maxwell equations $\partial_\mu F^{\mu\nu} = 0$ or with current $\partial_\mu F^{\mu\nu} = j^\nu$

wave: photon A_μ spin=1 one polarization, modes: spherical($l = 0$) and and all $l > 0$, scale : wave number k
wave equation:

radial Helmholtz equation for the wave factor *fs(r)* from the ansatz $fs(t,r,\theta) = \frac{fs(r)}{r} Y_{lx,m}(\theta,\varphi) \exp(-ik(r-t))$:

$- lx(1+lx)\, fs(r) - 2ikr^2\, fs'(r) + r^2\, fs''(r)$

solution: $c_1\sqrt{r}\, \exp(ikr) J_{lx+1/2}(kr) + c_2\sqrt{r}\, \exp(ikr) Y_{lx+1/2}(kr)$

Lagrangian $L = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ with the Maxwell field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

duality: the 3-vector E and the axial 3-vector B are dual, they form together the 6 parameters of the field tensor $F_{\mu\nu}$

electromagnetic classical regime

The interaction is described by the Maxwell equations $\partial_\mu F^{\mu\nu} = 0$ formulated with the field 3-vector electric field $\vec{E}$ and the 3-pseudovector magnetic field $\vec{B}$, $E_i = -cF^{0i}$, $B_k = -\varepsilon_{kij} F^{ij}$, which are dual in a certain sense. The Maxwell field tensor $F_{\mu\nu}$ has 6 independent components, which correspond exactly to the 3 components E_i and 3 components B_i , from this result the Poynting vectors of energy flux $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ and the energy density $u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$, where $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, in natural units $\varepsilon_0 = 1, \mu_0 = \frac{1}{c^2}$

The electrical variables like voltage *U* and capacitance *C* are derived from $\vec{E}$, while magnetic variables like current *I* and inductance *L* are derived from $\vec{B}$.

interaction with matter

Electromagnetic waves are diffracted (Huyghens principle of elementary waves), refracted (material refraction index *n* , where *n*=*c*/v) and absorbed (exponential decline by the imaginary part of the refraction index *n*). The light velocity in matter is slowed down to v=*c/n* by interaction with matter.

Refraction is governed by Snellius's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Absorption, transmission and reflection of electromagnetic waves are governed by the Fresnel equations, power reflection coefficient for s-polarized and p-polarized wave (normal resp. parallel to the plane of incidence)

$$R_s = \left|\frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}\right|^2 = \left|\frac{n_1\cos\theta_i - n_2\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_i\right)^2}}{n_1\cos\theta_i + n_2\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_i\right)^2}}\right|^2,$$

$$R_p = \left|\frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i}\right|^2 = \left|\frac{n_1\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_i\right)^2} - n_2\cos\theta_i}{n_1\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_i\right)^2} + n_2\cos\theta_i}\right|^2.$$

Wave pressure is exerted by the Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

Electromagnetic waves interact with gravitation via Einstein equations, they *undergo a redshift*: wave energy density is part of the energy density tensor $T\mu\nu$ on the right side of the equations, from these equations originates the redshift from time dilation

$$d\tau = \sqrt{g_{00}(x)}\ dt$$

electromagnetic quantum regime:

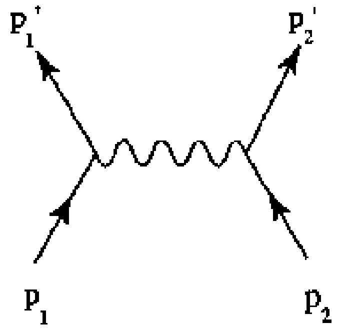
the interaction with fermions are described via the covariant derivative $D_\mu = \partial_\mu + \frac{ie}{\hbar}A_\mu$ ($D_\mu = \partial_\mu + ieA_\mu$ in natural units) in the Dirac equation, the self-interaction of the photon is described by the Maxwell equations $\partial_\mu F^{\mu\nu} = 0$, the photon is transverse (Lorentz gauge is used $\partial_\mu A^\mu = 0$) and has zero mass $k_\mu k^\mu = 0$ and one linear polarization ,

Dirac equation $\left(i\hbar\gamma^\mu D_\mu - mc\right)\Psi = 0$

self-field coupling with the fermion Dirac wavefunction ψ is described by the inhomogeneous Maxwell equations (self-field current generates the Lamb-shift and the spontaneous emission [22])

self-field eq. $\partial_\mu F^{\mu\nu} = j^\nu = -ce\,\bar{\psi}\gamma^\nu\psi$

In Feynman diagrams, the electromagnetic interaction happens by exchange of photons, e.g. Möller scattering of two electrons with momenta p_1, p_2



The interaction constant is the fine structure constant $\alpha_{em} = \frac{e_0^{\,2}}{4\pi} = \frac{1}{137}$, the elementary cross-section for electron-interaction is $\sigma_{em} = \alpha_{em}^{\,2}\tilde{\lambda}_e^{\,2} = \alpha_{em}^{\,2}\left(\frac{\hbar}{m_e c}\right)^2$

The photon Feynman propagator is

$$D(q) = \frac{-1}{q^2 + i\varepsilon}$$

Gravitation

gauge-group= Lie(ε) with 4 generators τ_μ corresponding to 4 polarization modes and 4 row vectors A_μ of the full graviton tensor $A_{\mu\nu}$

additional invariance: background independence, long-range, classical and quantum,
its interaction-boson=graviton is a 2-tensor with spin=2

scale $r_{gr} = \sqrt{l_P \sqrt{\frac{1}{\Lambda}}} = 39\mu m$, energy scale $E_{gr} = \frac{\hbar c}{r_{gr}} = 5.04 * 10^{-3} eV$

charge: mass , elementary (minimal) mass (minimal energy $E_\Lambda = \frac{\hbar c}{R_\Lambda}$ is reciprocal to the maximum length R_Λ)

$$m_\Lambda = \frac{\hbar}{cR_\Lambda} = \frac{\hbar\sqrt{\Lambda}}{c} = \frac{1.05*10^{-34} Js 1.052*10^{-26} m^{-1}}{3*10^8 ms^{-1}} = 0.368*10^{-68} kg$$

fundamental constants: r_{gr} (depends on l_P, which is derived from the gravitational constant G)´and Λ

fundamental equations: 32 Ashtekar-Kodama equations $AK(A_\mu^\nu, E^{\mu\nu}, \Lambda)=0$ for the graviton tensor A_μ^ν, the tetrad $E^{\mu\nu}$ and the cosmological constant Λ , in the limit $\Lambda\text{->}0$ the Einstein-equations are valid (except at the horizon) and A_μ^ν =const (i.e. trivial background)

Λ generated by the tensor field ϕ_Λ is scale dependent, classical scale $r>>r_{gr}$ $\Lambda = \Lambda_0 = 2.7\ 10^{-52} m^{-2}$

wave: graviton A_μ^ν spin $S=2$ and two polarizations, two scales: r_s and wave-number k

modes: quadrupole and higher $l_x \geq 2$, angular function $\exp(i\, l_x\, \theta)$,

gravitational radial wave equation $weq_\nu(E^{1\nu}, \partial_r{}^3 E^{1\nu})$ for the tetrad component $E^{1\nu} = fs(r)$

$$\begin{aligned} & r\left(\mathrm{fs}'(r)\left(\mathrm{lx}\left(-6k^2r^2+3ikr+1\right)+kr\left(-2k^2r^2+ikr+2\right)+\mathrm{lx}^2(-5kr+2i)-\mathrm{lx}^3\right)+\right. \\ & \qquad \left. r\left(r\,\mathrm{fs}^{(3)}(r)(kr+\mathrm{lx})-i\,\mathrm{fs}''(r)\left(5k\,\mathrm{lx}\,r+kr(3kr-i)+2\,\mathrm{lx}^2\right)\right)\right)+ \\ & \mathrm{fs}(r)\left(\mathrm{lx}^2\left(3ik^2r^2+4kr-2i\right)+\mathrm{lx}\left(2ik^3r^3+4k^2r^2-5ikr-1\right)+\right. \\ & \qquad \left. \mathrm{lx}^3(1+ikr)+kr(-2-ikr)\right) \end{aligned}$$

At infinity:

$$2ik^2 lx\, fs(r) - 2k^2 r\, fs'(r) - 3ikr\ fs''(r) + r\, fs'''(r)$$

solution: determines $A_\mu^\nu, E^{\mu\nu}$ with 4 free parameters for $E^{\mu\nu}$ and 4 free parameters for A_μ^ν , $E^{\mu\nu}$ damped $\exp(-4\sqrt{\frac{r}{3}})$

for $r \leq r_{gr} = 31\mu m$ QFT gravity, $\Lambda_q = \Lambda\left(k = 1/r_{gr}\right) = 1.2\times10^{29} m^{-2}$, $\sqrt{\alpha_{gr}} = 0.3\times10^{-30}$

The covariant derivative of the AK-gravitation is $D_\mu = \partial_\mu - i\sqrt{\alpha_{gr}}\left(\tau_a \bullet A^a\right)_\mu$, $\left(D_\mu\right)^\lambda{}_\kappa = \partial_\mu + \varepsilon^\lambda{}_{a\kappa}\sqrt{\alpha_{gr}} A_\mu^a$

acting on a tensor $D_\mu t^{\nu\lambda} = \partial_\mu t^{\nu\lambda} + \varepsilon^\lambda{}_{\kappa_1\kappa_2} A_\mu^{\kappa_1} t^{\nu\kappa_2}$,

where $\tau^a = i\varepsilon_\nu{}^a{}_\lambda$ are the generators of the Lie(ε) algebra with 4 generators τ^a

and the commutator $[\tau^a, \tau^b] = i\varepsilon_c{}^{ab}\tau^c$ -

equivalent to Lie3(ε) Lie-algebra with 3 generators τ^a and the commutator $\left[\tau_a, \tau_b\right] = i\sum_k\left(1+\varepsilon_{abk}\right)\tau_k$, a=1,2,3

which is isomorphic to SU(2)

(renormalizable) Lagrangian

$$L_{gr} = L_H + L_I = \hbar c\left(\begin{array}{l} -\frac{1}{4}F_{\mu\nu}{}^\kappa F^{\mu\nu}{}_\kappa - \frac{\overline{\varphi}_\Lambda\varphi_\Lambda}{3}\left(\varepsilon^\kappa{}_{\mu\lambda}E^{\lambda\nu}\partial_\kappa A_{\mu\nu} + \varepsilon_{\mu_2\lambda_2\kappa_2}\varepsilon^{\mu_1\lambda_1}{}_{\kappa_1}E^{\kappa_1\kappa_2}A_{\lambda_1}{}^{\lambda_2}A_{\mu_1}{}^{\mu_2}\right) \\ + E^{\kappa_1}{}_{\nu_1}F_{\mu\kappa_1}{}^{\nu_1}E^{\kappa_2}{}_{\nu_2}F^\mu{}_{\kappa_2}{}^{\nu_2} \end{array}\right)$$

with the constraint $\overline{\varphi}_\Lambda\varphi_\Lambda = \Lambda$ and with the field tensor $F_{\mu\nu}{}^\kappa = \partial_\mu A_\nu{}^\kappa - \partial_\nu A_\mu{}^\kappa + \varepsilon^\kappa{}_{\kappa_1\kappa_2}A_\mu{}^{\kappa_1}A_\nu{}^{\kappa_2}$

duality: graviton tensor $A_\mu^{\ \nu}$ represents the generalized momentum, tetrad $E^{\mu\nu}$ represents the metric, i.e. the generalized coordinates, the 3-tensor $A_a^{\ i}$ and the 3-tensor E^{ai} satisfy with the operator replacement

$E^{ai} \Rightarrow \frac{1}{l_P}\frac{\delta}{\delta A_{ai}}$ in the 3-dimensional Ashtekar gravity

the momentum-position commutation relations $\{A_a^{\ i}(x), E^b{}_j(y)\} = 8\pi\iota l_P{}^2 \delta_a^b \delta_j^i \delta(x,y)$

The gravitational field tensor $F_{\mu\nu}{}^{\kappa} = \partial_\mu A_\nu{}^\kappa - \partial_\nu A_\mu{}^\kappa + \varepsilon^\kappa{}_{\kappa_1\kappa_2} A_\mu{}^{\kappa_1} A_\nu{}^{\kappa_2}$ depends on the 16 independent components of the graviton tensor $A_\mu^{\ \nu}$, and it cannot be reduced to two 3–vectors as in the electrodynamics, there is also no duality $\vec{E}$ $\vec{B}$ as in the case of electrodynamics.

gravitational classical regime

static interaction: $A_\mu^{\ \nu}=const$, $E^{\mu\nu}$ satisifies the Einstein equations and generates the metric at infinity with the usual metric boundary condition $E\,\eta\,E^t = g^{-1}/(-\det g)^{3/4}$, so the field carrier is the tetrad, fully compatible with GR

wave behavior:Ashtekar-Kodama equations $AK(A_\mu^{\ \nu}, E^{\mu\nu}, \Lambda)=0$ with Λ-scaled ansatz for the wave tensor $A_\mu^{\ \nu}$, $A_\mu^{\ \nu}$ carries energy and generates an *exponentially damped* metric wave $E^{\mu\nu}$.

The wave tensor $A_\mu^{\ \nu}$ and the tetrad $E^{\mu\nu}$ play different roles: $E^{\mu\nu}$ generates the metric and is measurable by length oscillation, $A_\mu^{\ \nu}$ carries energy and is measurable by absorption.

The field tensor $F_{\mu\nu}{}^{\kappa}$ depends on 16 independent $A_\mu^{\ \nu}$, so it has 16 independent components, a dissection into 8+8 electric-magnetic components is not possible, because the resulting 3x3-tensors have together 18 components, they would not be independent.

planar and spherical waves

Gravitational planar and spherical waves obey the wave component relations

$Es_1 = 0 = Es_0\,, Es_2 = 3i\,k\left(As_0 - As_1 + As_2\right), Es_3 = -3i\,k\,As_2$

$As_3 = \left(As_0 - As_1 + As_2\right)$, i.e. the tetrad Es and the metric have only transversal components.

A planar wave in x-direction has the form

$$As = \begin{pmatrix} As_0 \\ As_1 \\ 0 \\ 0 \end{pmatrix} \quad Es = \begin{pmatrix} 0 \\ 0 \\ Es_2 = 3i\,k\left(As_0 - As_1 + As_2\right) \\ Es_3 = -3i\,k\,As_2 \end{pmatrix}$$

Gravitational waves are quadrupole or higher multipole, have two linear polarization modes, are diffracted by Huyghens-principle and absorbed in matter like light waves. Refraction of gravitational waves would require quantum-interaction with matter in order to “slow down” the wave, but classical gravitational waves have wavelength in km-range, so $\lambda >> r_{gr}$, the wavelength is far outside the quantum regime, and therefore $n=1$: there is no refraction.

The amplitude reflection and absorption ratio for planar waves is approximately

$\frac{\delta A_r}{A} = \frac{r_s(M)}{2r_s}$, $\frac{\delta A_a}{A} = \sqrt{\frac{r_s(M)}{r_s}}$, where $r_s(M)$ =Schwarzschild radius of the interacting matter M

The generated metric wave has also only transversal components and the double frequency: $\omega_g = 2\omega_E$

$$Es \bullet \eta \bullet Es^t = gs$$

$$gs = \begin{pmatrix} 0 & 0 \\ 0 & gs_{22} \end{pmatrix}$$

$$gs_{22} = \begin{pmatrix} Es_2{}^2 & Es_2 \bullet Es_3 \\ Es_2 \bullet Es_3 & Es_3{}^2 \end{pmatrix} = \begin{pmatrix} As_2{}^2 & -\left(As_0 - As_1\right)^2/2 \\ -\left(As_0 - As_1\right)^2/2 & -As_2{}^2 \end{pmatrix},$$

which is exactly the standard form of a GR metric wave $\left(h_{\mu\nu}\right)=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&e_{11}&e_{12}\\0&0&e_{12}&-e_{11}\end{pmatrix}\exp(2ik(x^1-ct))$

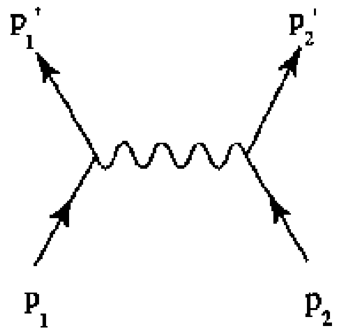
i.e. the generated metric wave satisfies the linearized Einstein equations, as required in GR.
It has 4 modes: 2 polarizations and two signs of amplitude
$e_{11}=\pm h$, $e_{12}=0$
$e_{12}=\pm h$, $e_{11}=0$

interaction with matter

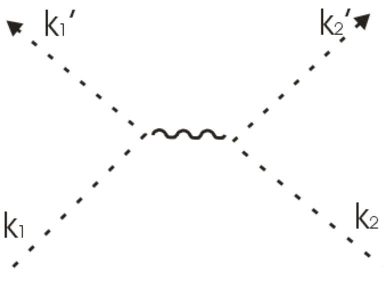
Planar waves generate a distortion of the static tetrad in the form
$dEb_{sol}(As_0)=\{dEb_{00}, dEb_{11}, dEb_{12}, dEb_{13}\}$, the radial components dEb_{00}, dEb_{11} represent a pressure in x-direction, dEb_{12}, dEb_{13} represent shear-stress in *xz* and *xy* direction.
Gravitational waves undergo *no gravitational redshift*, as opposed to the electromagnetic waves, they interact via the AK-equations, and the generated metric wave stands on the left side of the linearized Einstein equations, and is not part of the energy density tensor $T\mu\nu$.

gravitational quantum regime

In the quantum regime $r << r_{gr}$, the graviton is a particle-wave like a gamma-photon, interaction with fermions works via the covariant derivative $D_\mu=\partial_\mu-i\sqrt{\alpha_{gr}}\left(\tau_a\bullet A^a\right)_\mu$ in the Dirac-equation, like in the electromagnetic case.
The self-interaction works via the Ashtekar-Kodama equations $AK(A_\mu^{\ \nu}, E^{\mu\nu}, \Lambda)=0$ with the metric boundary condition $E\,\eta\,E^t=\eta$, with Minkowski metric η .
The graviton has spin $S=2$, is transverse like the photon (momentum perpendicular to wave front) and has zero mass $k_\mu k^\mu=0$ and two linear polarizations.
In Feynman diagrams, the gravitational interaction happens by exchange of (real or virtual) gravitons, e.g. scattering of two neutrons with momenta p_1, p_2 via exchange of a graviton



Massless bosons, like photons (dotted line) can interact gravitationally via exchange of a graviton (curved line), due to the photon energy.



interaction via $\left(D_\mu\right)^\lambda{}_\kappa=\partial_\mu+\varepsilon^\lambda{}_{a\kappa}\sqrt{\alpha_{gr}}A_\mu{}^a$ covariant derivative,
interaction constant $\sqrt{\alpha_{gr}}=0.29\times10^{-30}$
The graviton Feynman propagator is
$D_g(q,m)=\dfrac{m}{q(q-i\,m)(q-2i\,m)}$ where m=mass of source fermion

The quasi-classical approximation is: $E_{gr}(r) = \frac{r_\Lambda}{r} mc^2$ for $r \ll r_{gr}$, $r_\Lambda = 3.7 \times 10^{-49} m$

6 Solved problems

1. Problems in GR

-Black-Hole singularity

Within GR itself, non-compact solutions of the Tollman-Oppenheimer-Volkov (TOV) equation with equation-of –state of a neutron-Fermi-gas describe shell-stars with radius R>r_s(Schwarzschild radius), which have no singularity, but mimic stellar Hawking-Penrose-Black-Holes with an escape velocity only ~1% below c .
AK gravity removes the singularity of GR Black Holes with the AK correction length $l_0 = 64\Lambda r_s^3$,

we get the corrected outer metric $\left|g_{11,c}\right| = \frac{1}{\left|g_{00,c}\right|} \approx \frac{r_s}{l_0}$

with the sub-luminal escape velocity $v_c(r) = c\sqrt{1-\frac{l_0}{r_s}} \approx c\left(1-\frac{l_0}{2r_s}\right) < c$, e.g. for central Black Hole of the Milky Way $v_c(r) = c\left(1-5\times10^{-31}\right)$.

-well-defined gravitational waves

As is well-known, a gravitational wave (gw) in GR satisfies only the approximate linearized Einstein equations for small amplitudes, not the full Einstein equations.
This problem is solved by the classical AK gravity, as gw's satisfy the full AK wave equations without restriction, satisfy for small amplitudes the linearized Einstein equations, and classical AK gravity gives a description of wave reflection and absorption for planar and spherical gw's, as well as the correct Einstein formula for gw power.

2. Problems in Standard Model

-exceptional weak force

The weak force is the only one from the 4 fundamental interactions, which has massive field carriers (W- and Z-boson). The hc-SU(4) interaction explains this exception by stating that *W and Z are its Yukawa-bosons*, in the same sense as the pion is the Yukawa boson of the (strong) color SU(3) interaction.
With this correction, we can establish the principle: field carriers are massless bosons moving with speed of light (graviton A_μ^ν spin S=2, photon A_μ S=1 , 8 gluons A^a_μ S=1 , 15 hc-bosons A^a_μ S=1).
Higgs H is the *mass-generator* in SM.
Higgs H is the *Yukawa-scalar of self-interaction of basic spinors* (three generations) of SM: e-leptons , neutrinos,
u-quarks, d-quarks. H is the only field-carrier , which is scalar, with electric, color, and hypercolor charge equal 0, therefore it can form a closed loop in a Feynman diagram.

-generations

The three generations of the SM are explained by three SU(4)-invariant configurations of hc-bosons (hcb):
1 hcb e.g. $A13$, 4 hcb e.g. $A13+\overline{A}13+A24+\overline{A}24$, all 15 hcb
Their masses scale approximately with boson number N_b^{κ} with *κ=3.8* , which explains the large mass ratios.

-mass hierarchy of SM

The massive particles of SM are built from two preons
r : mass=0.033meV, hc-charges {L- , L+, R-, R+} , electric charge –1/2
where L=left-chiral, R=right-chiral, electric charge+ -
q : mass=0.77 MeV, hc-charges {L- , L+, R-, R+}, color-charges (r,g,b), electric charge +1/6
The basic particle families are
-leptons L=r⊗r being a hc-tetra-spinor of a doublet of two r-preons, fermions with total spin S=1/2
-quarks Q=r⊗q being a hc-tetra-spinor of a doublet of an r- and a q-preon, colored fermions with color Q_c=1 with total spin S=1/2

-weak bosons B_W=r±r being a linear combinations of two or more r-preons, with total spin S=0 (scalar like higgs H) or S=1 (vector like W and Z)
All masses of SM are calculated by minimization of the corresponding Lagrangian with error<10%

-chiral split of weak interaction
The weak interaction has two chiral covariant derivatives
$$D_\mu R = \left(\partial_\mu + ig' B_\mu\right) R$$
$$D_\mu L = \left(\partial_\mu + \frac{i}{2} g' B_\mu - \frac{i}{2} g \sigma_i W^i{}_\mu\right) L$$
where $W^a{}_\mu$ is the weak SU(2) boson, B_μ is the photon, L and R are left-chiral and right-chiral components of a particle.
In the original SM, there is no explanation for this.
The weak SU(4) interaction explains the weak chirality by the simple fact that there are four charges: +, -, L, R .
So the weak SU(4) acts differently on the L- and R-components (preons) of the basic particle , e.g. electron.
As the SU(4) interaction is spontaneously broken
$$SU(4) = SU(2)_L \otimes SU(1)_R \otimes SU(1)_{em}$$
this results in the split of the covariant derivative into left- and right-chiral components

-parameters of SM
The 28 parameters of SM are reduced by the SU4PM-model to 6 parameters:
mass(r), mass(q), char. energy Λ of the running-constant-relation, and zero correction c_{GE} of the running-constant-relation (analogous to the color Callan-Symanzik relation).

3. Problems in quantum gravity and quantum mechanics

-renormalizable quantum gravity as a quantum extension of GR
AK gravity yields in its quantum regime $r < r_{gr} = 39\mu m$ a complete renormalizable quantum-field-theory (QFT) with the Lie group Lie(ε) with 4 generators., and Feynman propagator $D_g(q,m) = \dfrac{m}{q(q - i\,m)(q - 2i\,m)}$
where m=mass of source fermion
AK-gravity becomes full GR in the classical regime $r > r_{gr} = 39\mu m$ in the limit Λ→0, everywhere except at the horizon of a Black-Hole.

-wavefunction collapse and decoherence
The **wavefunction collapse** in the quantum regime is described by the *objective collapse theory* of Ghirardi-Rimini-Weber (GRW) , the wave function collapse is characterized by the localization distance r_c and by the decay rate λ .
The GRW theory is supported by AK quantum gravity with GRW parameters
$r_C = r_{gr} = \sqrt{l_P \sqrt{\dfrac{1}{\Lambda}}} = 3.9 * 10^{-5} m = 39\mu m$ and $\lambda_\Lambda = \dfrac{E_\Lambda}{\hbar} = \dfrac{c}{R_\Lambda} = 0.33 \times 10^{-13} s^{-1}$
The values $r_C = r_{gr}$ and $\lambda = \lambda_\Lambda$ lie in the experimentally allowed area.

For an ensemble of N particles the decoherence rate is $\Lambda_d = N\,\lambda\, e^{r/r_c}$, decoherence time $\tau = \dfrac{1}{N\,\lambda_\Lambda\, e^{r/r_{gr}}}$,

$\lambda_\Lambda = \dfrac{E_\Lambda}{\hbar} = \dfrac{c}{R_\Lambda} = 0.33 \times 10^{-13} s^{-1}$, quantum gravitational limit $r_{gr} = \sqrt{l_P \sqrt{\dfrac{1}{\Lambda}}} = 3.9 \times 10^{-5} m$, r is the average distance in the ensemble.

The *transition from the quantum to the classical regime with distances happens by ever faster decoherence*, which becomes effectively momentary, when the *decoherence time is below the signal time* of the object $\tau = \frac{1}{\Lambda_d} = \frac{1}{N\,\lambda\, e^{r/r_c}} < t_{obj} = \frac{r_{obj}}{c}$ (r_{obj} is the object diameter).

Example Schrödinger's cat
$\tau(S-cat) = 1.1\times 10^{-12}s$. signal time $t_s = \frac{d}{c}$, $t_s(S-cat) = 0.17\times 10^{-8}s$, is purely classical $\tau < t_s$.

4. New systematics
There are now **2 classical long-range forces** (AK-gravitation , electrodynamics), which act over the classical regime, and have a quantum version, and have a renormalizable gauge theory with a Lie-algebra (AK-gravitation: Lie(ε) with 4 generators , electrodynamics: SU(1) with 1 generator).
There are **2 pure quantum forces** (color with Lie-algebra SU(3) , hypercolor with Lie-algebra SU(4)), which act only in the quantum regime, are **short-range** (exponentially decreasing), and are asymptotically free (have confinement quality).
The two classical forces have in the classical regime in the static case (v=0) the form of a **Coulomb potential** $E = \frac{const}{r}$: for the electrodynamics exactly, for the gravitation approximately in the Newtonian approximation.

In the quantum regime, **all field-carriers are massless** and move with v=c (AK), all other particles are massive and interact via field-carriers. The two massive field-bosons of the weak interaction are Yukawa-bosons of the hc-force.

The **gravitation is the only tensor force** (spin S=2). Its wave is at least quadrupole-type, it satisfies a wave-equation, which in the limit equals the normal wave equation, but differs from it in the near-regime. The graviton $A_\mu{}^\nu$ depends on the **general location tetrad** $E^{\mu\nu}$ (on the location vector x^μ) , which is basically square-root of the inverse metric tensor $\left(g_{\mu\nu}\right)^{-1}$, and satisfies in the limit r→∞ (significant for r>1.5 r_S) the Einstein equation: **it becomes GR in the limit**.
The three other forces are **vector forces** (S=1) and depend only on the location vector x^μ , e.g. the Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$ in the electrodynamics depends on the location x^μ and contains differentials $\frac{\partial}{\partial x^\mu}$.

All forces have a characteristic energy E_c and the corresponding **characteristic scale** $r_c = \frac{\hbar}{E_c}$. Gravitation is a tensor force and has **two scales**: the quantum range limit $r_{gr} = 39\mu m$ and the Λ-scale $R_\Lambda = \frac{1}{\sqrt{\Lambda}} = 0.95\,10^{26}m$.

For the two pure quantum forces, the corresponding energy-mass spectrum is divided into distinctive **groups characterized by number of field bosons** allowed by the gauge group symmetry. In the case of the **color force**, the hadrons split into **3 groups** with different mass range: heavy nucleons with all 8 gluons, middle-ranged vector-mesons with 6 gluons, and light pseudo-scalar mesons with 3 gluons. In the case of the hypercolor force, there are **3 generations**, which differ sharply in their masses: generation 1 with 1 hc-boson, generation 2 with 4 hc-boson, the heaviest generation 3 with all 15 hc-bosons.

References
[1] J. Helm, Mathematica-program PhysExt.nb, Researchgate, 2021
[2] A. Davydov, Quantum mechanics, Neo Press, 1966
[3] M. Kaku, Quantum Field Theory, Oxford University Press 1993
[4] T. Fliessbach, Allgemeine Relativitätstheorie, Bibliographisches Institut 1990
[5] J. Helm, A covariant formulation of the Ashtekar-Kodama quantum gravity and its solutions, vixra 1807.0034, www.researchgate.net, www.janhelm-works.com
[6] J. Helm, Calculation of the Standard Model parameters based on a SU(4) preon model, vixra.org/1076.6882, www.researchgate.net, www.janhelm-works.com
[7] R. Becker, Theorie der Wärme, Springer, 1966
[8] H. Goldstein, Classical mechanics, Pearson education, 2014
[9] L.D. Landau & E.M. Lifshitz, Fluid Mechanics – Course of Theoretical Physics, Pergamon Press, 1966
[10] William S. Slaughter, The Linearized Theory of Elasticity, Birkhäuser, 2001
[11] J. Helm, A new version of the ΛCDM cosmological model, Researchgate, 2023
[12] M. Carlesso, & A. Bassi , Current tests of collapse models, Conference Quantum Information and Measurement, Rome, April 2019
[13] G. Ghirardi & A. Rimini & T. Weber, Unified dynamics for microscopic and macroscopic systems, Physical Review D. 34 (2) 470, 1986
[14] E. Joos et al., Decoherence and the Appearance of a Classical World in Quantum Theory, Springer 2003, ISBN 3-540-00390-8
[15] J. Helm, Neutron stars and non-singular black-holes as solutions of the TOV-equation, Researchgate 2018
[16] G. Braunstein, Nuclear Physics, University of Central Florida, https://physics.ucf.edu/~gabriel , 2024
[17] J.G. Stark, Modern Chemistry, Penguin Books, 1970
[18] M. Alonso & E.J. Finn, Physics, Addison-Wesley, 1995
[19] G. de With, Liquid state physical chemistry, Wiley-VCH, 2013
[20] J. Helm, Chemical thermodynamics, Researchgate, 2020
[21] C. Albert et al. , Advanced Solid State Physics, TU Graz, 2011
[22] A.O. Barut & J.P.Dowling, Self-field quantum electrodynamics: The two-level atom, Physical Review A Vol.41 Nr.5 , 03/1990
[23] J. Helm, Statistical mechanics of phase transitions, Researchgate 2025
[24] Yu Deng et al., Derivation of fluid equations via Boltzmann's kinetic theory, arXiv:2503.01800v1 [math.AP], 03/2025
[25] R.J. DiPerna & P.L. Lions, On the Cauchy Problem for Boltzmann Equations, Annals of Mathematics 130 (2) p321–366, 1989
[26] J. Helm, Thermodynamics of phase transitions for pure substances and mixtures, Researchgate 2026
[27] J. Helm, Lie Algebras, Researchgate 2026