Interval Methods for Reliable Modeling, Identification and Control of Dynamic Systems

Andreas Rauh, Luise Senkel, and Ekaterina Auer

Abstract
A large variety of real-life dynamic systems in engineering, biology, biomechanics, and medicine are significantly influenced by uncertainty. In the field of uncertainty quantification, two main sources are discerned: aleatoric (due to randomness) and epistemic (due to the lack of knowledge). Both kinds have to be taken into consideration while designing a model for the given system to deal with such tasks as reliable simulation, offline system and parameter identification, optimization and real-time (online) control, or state observation. Although it is usually possible to reduce the epistemic uncertainty by performing further experiments during system identification, this is not the case for the aleatoric one. Hence, mathematical approaches for system modeling, simulation, and design should explicitly make use of suitable uncertainty descriptions. Here, set-valued or stochastic techniques offer appropriate solutions depending on the application at hand. The combination of both approaches, which constitutes a challenging subject of current research, is less explored but promising in certain real-life situations, for example, if probabilities are not known exactly.

This workshop is focused on set-valued uncertainty representations which are described in the form of scalar intervals and multi-dimensional interval boxes. Where necessary, we touch upon polytopes, affine forms, or more general descriptions such as Taylor models to improve accuracy or to reduce the computational load. The topic of interoperability of techniques is addressed both from the theoretical (e.g., the concept of imprecise probabilities) and from the practical point of view. In the latter case, generalizations of the Itô differential operator are employed for robust variable-structure control and state estimator design of systems where both bounded and stochastic uncertainty are present.

The workshop consists of two interconnected parts, the theoretical and the application-oriented one. The topics of the first part are methodological aspects of interval analysis along with the available software, a general framework for uncertainty modeling/assessment, and the solution of initial value problems for systems of ordinary differential equations with smooth and non-smooth right-hand sides. In the second part of the workshop, engineering, biological and biomedical applications are presented to highlight the use of the theoretical contributions in the context of robust parameter identification, reliable simulation, and guaranteed stabilizing control. Considered application scenarios include the simulation and control of mechanical systems with friction and hysteresis, biological system models in wastewater treatment and human blood cell growth, as well as modeling, identification, and control of high-temperature solid oxide fuel cells. Both simulation results and experimental validation are addressed for the above-mentioned benchmark applications.

Workshop length
Full-day

List of presenters
Andreas Rauh, Luise Senkel, Ekaterina Auer
Andreas Rauh received his diploma degree in electrical engineering and information technology from the Technische Universität München, Munich, Germany, in 2001 and his PhD degree (Dr.-Ing.) from the University of Ulm, Germany, in 2008. He has published more than 130 articles and chapters in edited books, international conferences and peer-reviewed journals. His research interests are: state and parameter estimation for stochastic and set-valued uncertainties, verified simulation of nonlinear uncertain systems, nonlinear, robust, and optimal control, interval methods for ordinary differential equations as well as differential-algebraic systems. Dr. Rauh is currently with the Chair of Mechatronics, University of Rostock, Germany, as post-doctoral Research Associate. Since 2008 he is a member of the IEEE 1788 Working Group for the Standardization of Interval Arithmetic. Moreover, in October 2011, he was elected as Corresponding Member of the International Academy of Engineering, Moscow, Russia.

Luise Senkel received her Master of Science degree in mechanical engineering from the University of Rostock, Germany, in 2012. Currently, she is a PhD student at the Chair of Mechatronics of the University of Rostock and works on the development of interval methods for robust control and estimation of uncertain dynamic systems as well as on control and observer synthesis for solid oxide fuel cells and distributed parameter systems. Luise Senkel is (co-)author of more than 30 articles in international conferences and peer-reviewed journals.

Ekaterina Auer received her diplomas in mathematics and computer science from Ulyanovsk State University, Russia, in 2001 and from the University of Duisburg-Essen, Germany, in 2002. Since 2002, she has been working at the Chair for Computer Graphics and Scientific Computing at the University of Duisburg-Essen as a research assistant, receiving her Ph.D. in 2006 and the postdoctoral lecture qualification in 2014. Her main interests are scientific computing, methods with result verification accompanied by their application to problems in (bio)mechanics or engineering, and uncertainty quantification.

Workshop goals
Presentation of methodological approaches for modeling and quantification of uncertainty as well as their application to real-life systems in engineering, biology, and medicine with a focus on an efficient usage of interval arithmetic techniques.

Workshop contents
Part 1: Methods for modeling of uncertain dynamic systems, software demonstration, and mathematical background for verified simulation of dynamic systems with interval uncertainty
Part 2: Application of interval arithmetic and stochastic techniques in the framework of parameter identification, robust and nonlinear control, state estimation, and system optimization

Intended audience
Researchers and practitioners in robust and nonlinear simulation and control of dynamic systems with uncertainty

Schedule

Part 1: Morning session

1. Fundamentals of Interval Arithmetic (Andreas Rauh, Luise Senkel)
   - Presentation of the Fundamental Mathematical Concept of Interval Arithmetic for Set-Valued Computations
     Fundamental mathematical concepts for set-valued computations are reviewed. This is the basis for all theoretical approaches and applications considered in this workshop.
Software Demonstration
Fundamental examples for the use of interval arithmetic software packages (IntLab and C-XSC) are presented to strengthen the understanding of the above-mentioned theoretic background. Moreover, supplementary software tools (FADBAD++ for the automatic computation of derivatives and Taylor series expansions) are introduced which are valuable extensions for an efficient implementation of interval algorithms in real-life environments.

2. Kinds of Uncertainty and Possibilities for Their Treatment during Modeling and Simulation in Engineering (Ekaterina Auer)
This presentation gives an overview of kinds of uncertainty an engineer has to deal with along with the existing methods for modeling them (which include imprecise probabilities and methods with result verification). Working with such models is explained using many illustrative examples ranging from textbook ones to close-to-life applications and is supplemented with actual software suggestions.

3. Verified Simulation of Dynamic Systems (Andreas Rauh)
An overview of the fundamental properties of interval arithmetic approaches and related techniques for the verified simulation of systems of smooth ordinary differential equations and discrete-time difference equations is given. The contribution is focused on the main algorithmic properties and addresses the compromise between tightness of computed enclosures and the necessary computational effort while choosing appropriate simulation techniques for specific application scenarios. An overview of novel approaches for an application-specific detection of overestimation in the computed intervals concludes this talk. Examples include simplified models for human blood cell growth and wastewater treatment.

In many applications, there is a need to choose mathematical models that depend on non-smooth functions. The task of simulation becomes especially difficult if such functions appear on the right-hand side of an initial value problem. In this talk, the audience learns about current simulation techniques for such problems relying on the so-called verified methods (which guarantee the correctness of the result mathematically).

— break —

Part 2: Afternoon session

1. Control-Oriented Applications of Simulation Techniques for Non-Smooth Dynamic Systems (Andreas Rauh)
Non-smooth dynamic systems arise, e.g., during modeling of mechanical systems with friction and hysteresis. Illustrative simulations and optimal control results are presented for this class of applications, while experimental results are discussed for the reliable, interval-based identification of parameters of a prototypic test rig for the investigation of the uncertain longitudinal dynamics of a vehicle.

2. Interval-Based Design of Sliding Mode Control and State Estimation Procedures (Luise Senkel)
Many real applications require control strategies that are able to cope with switching between several operating points and still guarantee the system’s stability. These requirements can be fulfilled by applying sliding mode approaches, that furthermore are not restricted to a special system representation and have the feature of finite-time convergence despite uncertainty and noise. In this talk, sliding mode techniques are, on the one hand, applied to control tasks in order to reach a desired trajectory and, on the other hand, to estimation tasks of non-measurable system states in combination with an identification of not exactly known system parameters using interval arithmetics and the Itô differential operator. These approaches are applied to a drive train test rig in simulation and experiment.

Interval-based approaches for modeling and identification of dynamic system models for the thermal behavior of high-temperature fuel cell systems are described. They are validated both by means of offline simulations and by applying real-time capable online techniques for a sensitivity-based state and
parameter estimation. An explanation of how to achieve a significant performance improvement by a GPU implementation of offline identification algorithms concludes this talk.

4. **Solid Oxide Fuel Cell Systems — Interval-Based Sliding Mode Control** (Andreas Rauh)
Nonlinear control approaches for the thermal behavior of high-temperature fuel cell systems have to guarantee that upper temperature limits are not violated. This has to hold despite uncertain parameters and a-priori unknown disturbances (resulting e.g. from time-varying electric power demands). A novel sliding mode control approach, employing barrier Lyapunov functions, is used for the guaranteed stabilization of the system dynamics and for preventing overshoots over the maximum admissible operating temperatures.

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