Kinds of Uncertainty and Possibilities for Their Treatment during Modeling and Simulation in Engineering

Ekaterina Auer

University of Technology, Business and Design Wismar

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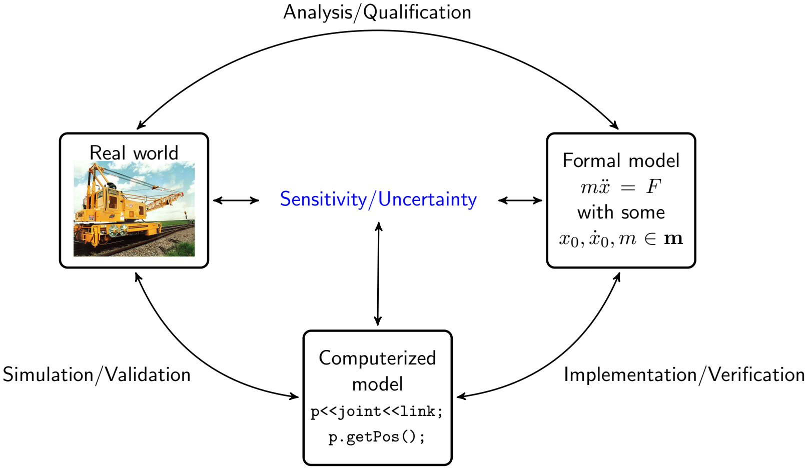
Modeling and Simulation Cycle



Uncertainty and sensitivity analyses are needed at each stage!

Imperfect Information: Philosophical Questions

Open question: Is there imprecision and uncertainty in the real world?

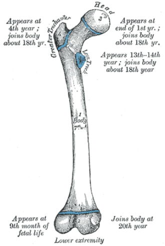
Fact: Data/information as available to an engineer are always imperfect

Modeling imperfect data: Probability theory only (until $\approx$ 1960s)

Currently: Many different possibilities for modeling, which are not equally suitable for a given situation

Classification possibility: Aspects of imperfect information*

- Imprecision
  → The length of Mr. X's femur is either 52.6cm or 53.2cm
- Uncertainty
  → The length of Mr. X's femur is probably 52.6cm
- Inconsistency, vagueness, ambiguity, error
  → The length of the femur is on average 26.74% of the height, measured 70cm on Mr. X (whose height is 190cm)



*Ph. Smets, *Imperfect information: Imprecision – Uncertainty*, 1999

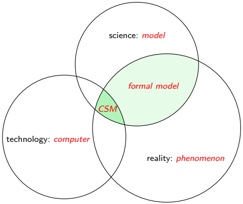
Basic Notions

Model (traditional): A set of mathematical equations along with the *computational expression* that describe a physical phenomenon

Computerized model (CM): *Code*

Accuracy: The agreement between estimated values and their true values

Credibility: The degree of trust that the CM answers a specific research question



Impreciseness: Characterized by the absence of an error component

Uncertainty: Arises from e.g. a gap in knowledge about the real system or its inherent variability

Sensitivity: A measure of the effect of a change in a particular variable on the simulation outputs

Hicks et al., Is My Model Good Enough? DOI: 10.1115/1.4029304

Errors in an Engineering Application

P — real system
V — representation of the CM solution

$$\max\{|P-M|,|M-D|,|D-L|,|L-V|\} \leq |P-V|$$
$$\leq |P-M|+|M-D|+|D-L|+|L-V|$$

Reality

1. Errors in the input data
2. Modeling error $|P-M|$
3. Discretisation error $|M-D|$
4. Truncation error $|D-L|$
5. Representation error $|L-V|$

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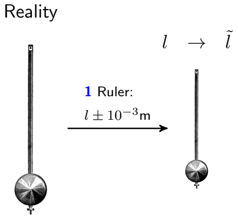

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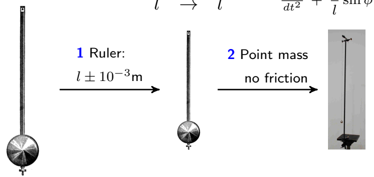
Reality

$l \rightarrow \tilde{l}$

$$\frac{d^2\phi}{dt^2} + \frac{g}{l}\sin\phi = 0$$



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Reality

1 Ruler: $l \pm 10^{-3}$m $\longrightarrow$

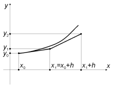
$l \rightarrow \tilde{l}$

2 Point mass, no friction $\longrightarrow$

$$\frac{d^2\phi}{dt^2} + \frac{g}{l}\sin\phi = 0$$

3 Grid: $h = t_i - t_{i-1}$ $\longrightarrow$

$$y_{i+1} = y_i + hf(t_i, y_i) + \Delta_i$$



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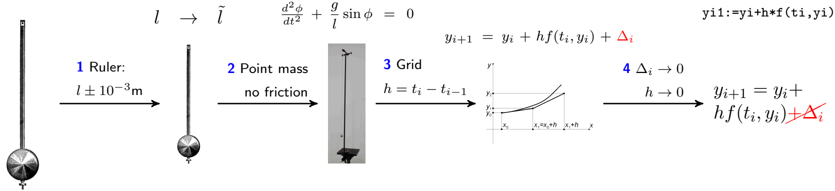

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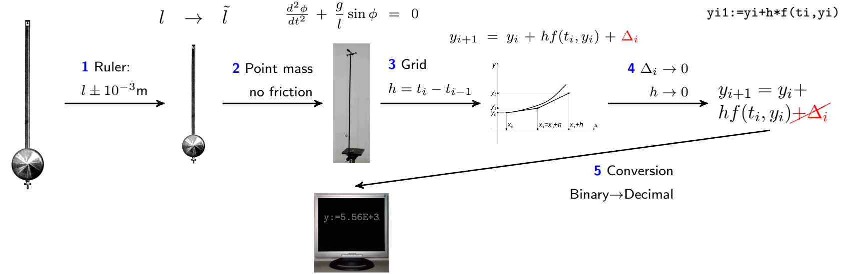

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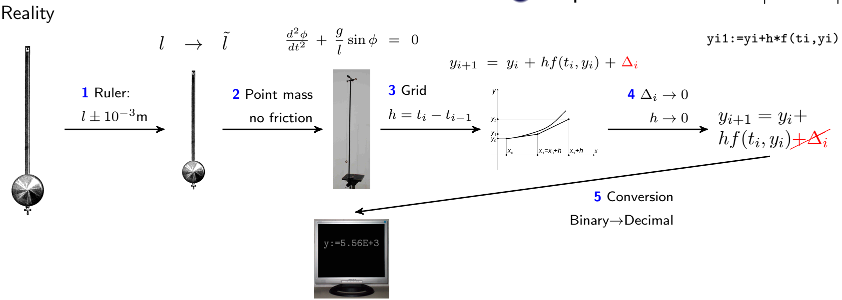


These 'errors' can be seen as *sources* of uncertainty

Outline

Uncertainty versus Sensitivity Analyses

Uncertainty analysis

Quantify the uncertainty in the the model output from the uncertainty in the input or vice versa

Direct:

- Probabilistic (e.g. Monte-Carlo, polynomial chaos expansion)
- Non-probabilistic (e.g. interval, fuzzy)

Inverse:

- Frequentist
- Bayesian

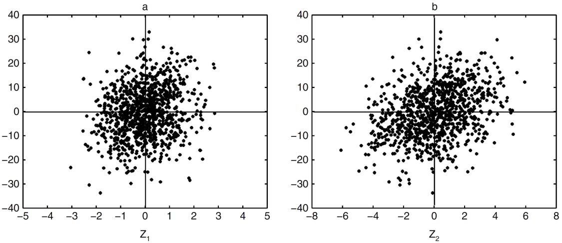
Sensitivity analysis

Apportion the uncertainty in the model output to different sources of uncertainty in the model input

- Scatter plots
- First derivative $\mathbf{s}_i = \dfrac{\partial x(\mathbf{p}_1, \ldots, \mathbf{p}_n)}{\partial p_i}$ for $p_i \in \mathbf{p}_i$

Uncertainty versus Sensitivity Analyses

Example:* $Y = \Omega(Z_1 + Z_2)$, $Z_i \sim \mathcal{N}(0, \sigma_i)$, Z_1 less uncertain $(\sigma_1 < \sigma_2)$



$$S_i = \frac{\partial Y}{\partial Z_i} = \Omega$$

$\rightsquigarrow$ Z_1, Z_2 are equal

$\rightsquigarrow$ Z_1 is less influential

- Point derivatives can lead to wrong conclusions $\rightsquigarrow$ $S^\sigma = \dfrac{\sigma_i \partial Y}{\sigma_Y \partial Z_i}$
- Another view: S^σ combines uncertainty and sensitivity!

* A. Saltelli et al., *Global Sensitivity Analysis: The Primer*, John Wiley & Sons, 2008

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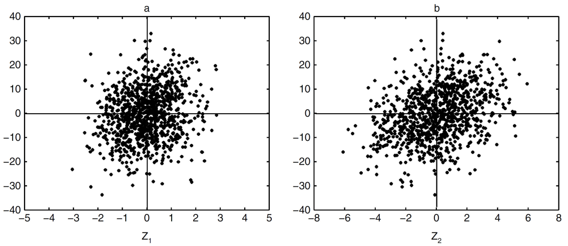
Example:* $Y = \Omega(Z_1 + Z_2)$, $Z_i \sim \mathcal{N}(0, \sigma_i)$, Z_1 less uncertain $(\sigma_1 < \sigma_2)$



$\rightsquigarrow$ scatter plots propagate the uncertainty

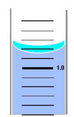
$$S_i = \frac{\partial Y}{\partial Z_i} = \Omega$$

$\rightsquigarrow$ the model response is indeed equal

- Sensitivity is *the response of the model to the changes in parameters*
- Uncertainty is quantified by *propagating* it from input to output
- S^σ or similar notions *combine* both in one indicator

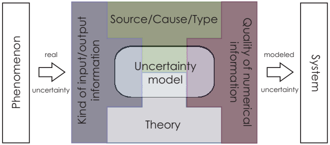
* A. Saltelli et al., *Global Sensitivity Analysis: The Primer*, John Wiley & Sons, 2008

Uncertainty Modeling: Choice of the Method

Uncertainty 

→ Uncertainty model
Interval $[1.2, 1.6]$ → Observer 

1. Kind/source/cause of uncertainty
   lack of info/complexity/conflict/belief/ambiguity
2. Type of input information
   numerical/interval/linguistic/symbolic
3. Quality of numerical data
   nominal/ordered/metric/precise/interval/absolute
4. Required output information
   numerical/interval/linguistic/symbolic



Uncertainty models require a certain scale level of numerical information

Scale of method's operations $\leq$ Scale of provided information

Example: Frequentist Kolmogorov probability theory: $(LoI, Num, Cardinal, Num)$

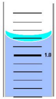
Sources of Uncertainty

Two major kinds of uncertainty

Aleatory (irreducible)

environmental stochasticity
(as in games of chance)

Epistemic (reducible)

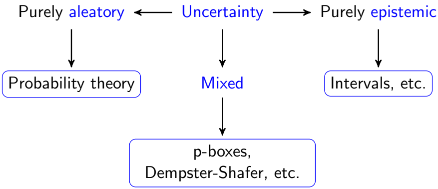


lack of knowledge:
measurement uncertainty,
unobservability, censoring

Sources of uncertainty

1. Uncertainty in the model itself (e.g., due to simplifications or parameter/dimension reduction)
2. Possible numerical discretization
3. Uncertainty in parameters (e.g., due to physical reasons or measurement errors)
4. Errors due to the finite nature of floating-point arithmetics

Main Types of Methods



Fuzzy methods handle impreciseness

1. A.C. Cullen, H.C. Frey, *Probabilistic techniques in exposure assessment*, 1999
2. H.M. Wadsworth, *Handbook of statistical methods for engineers and scientists*, 1998
3. S. Ferson et al, *Constructing Probability Boxes and Dempster-Shafer Structures*, 2003
4. V. Kreinovich et al, *Monte-Carlo-type techniques for processing interval uncertainty...*, 2004
5. H.-J. Zimmermann, *Fuzzy Set Theory and its Applications*, 2003

	Model	Discretization	Parameters	Arithmetic
PT	1		2	✓
IP	3		4	
F	5		5	
VR	✓	✓	✓	✓

PT=probability theory, IP=p-boxes or Dempster-Shafer, VR= methods with result verification, F=fuzzy

Types of Algorithms with Uncertain Numbers

Rigor-preserving (≈ with result verification): the result is guaranteed to enclose the uncertainty completely, if inputs enclose it completely

Best possible (≈ inner enclosure): the result cannot get any tighter without more information

Statistical confidence: guarantee of the type "in x percent of the trials, the result is sure to enclose the uncertainty completely"

We will focus (mostly) on parametric uncertainty, direct case

Probabilistic methods →	Monte-Carlo, polynomial chaos exp.
Set-based methods →	Result verification
Mixed methods →	p-boxes, Dempster-Shafer

Monte-Carlo Simulation

Applications in connection with uncertainty

→ Properties of random variables with unknown distributions

→ Uncertainty propagation, failure analysis

Advantage: simple to implement **Disadvantage:** no proof of correctness

Random numbers

$z_1, z_2, \ldots, z_N$

↓

Model with N variables

(metric $\eta(z) = y$)

↓

System response

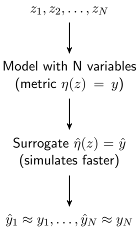
$y_1, y_2, \ldots, y_N$

1. Use the existing data to create a CDF for each input
2. Create an empty Frequency Distribution Histogram
3. The ith iteration step ($i = 1 \ldots 50000$):
   1. Loop over each input variable; calculate a weighted random number for inputs
   2. Use the weighted value of all input variables in the metric to calculate a representative answer
   3. Adjust the FDH appropriately
4. Repeat Step 3 if the final FDH is not complete
5. Normalize the FDH into DPDF; interpret the results

`www.drjfwright.com/c/montecarlosimulation.html`

Polynomial Chaos Expansion

Monte-Carlo can become computationally prohibitive for complex metrics



Goal: Quantify uncertainty (e.g. in differential equations)

Method: Represent stochastic quantities as spectral expansions of orthogonal polynomials; $X = f(\Xi)$, Ξ with a given distribution, $f \approx$ polynomial expansion

Example: $X \sim \chi^2_1$, $\Xi \sim \mathcal{N}(0,1)$, then $X = \Xi^2$, Hermite polynomials $\phi_0(\xi) = 1$,$\phi_1(\xi) = \xi$,$\phi_2(\xi) = \xi^2 - 1$,...

Generally: F_x, F_ξ the CDFs of X, Ξ, then $X = F_x^{-1}(F_\xi(\Xi)) = f(\Xi)$

Polynomial chaos expansion 1D: $X \approx \sum_{j=0}^{p} x_j \phi_j(\Xi)$, $x_j = \frac{<f,\phi_j>}{<\phi_j,\phi_j>}$ (truncated)

Propagation: $Y \approx \sum_{j=0}^{p} y_j \phi_j(\Xi) \approx \eta(\sum_{j=0}^{p} x_j \phi_j(\Xi))$ (e.g. by Galerkin projection)

Non-intrusive: Solve $y_k = \frac{<\eta(\sum_{j=0}^{p} x_j \phi_j(\Xi)),\phi_k>}{<\phi_k,\phi_k>}$, $k = 0 \ldots p$

A. O'Hagan, *Polynomial Chaos: A Tutorial and Critique*, 2013, www.tonyohagan.co.uk/academic/pdf/Polynomial-chaos.pdf

Methods with Result Verification

Idea: Use set-based methods, if uncertainty can be bounded

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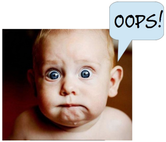

Not that easy....

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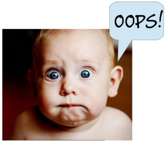

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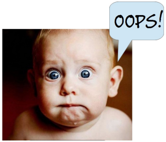

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Derivatives can be easily (if not really quickly) obtained by algorithmic differentiation, if the expressions for the model are known

Some CA System ⟶ Expressions ⟶ Verification block ⟶ Verified results

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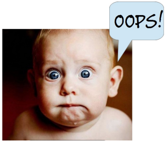

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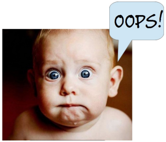

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Some CA System → Expressions → Verification block → Verified results

Many simulation tools do not produce models as expressions!

Nonetheless possible, if simulation tools are open-source

Tools: SmartMOBILE for (Bio)Mechanics

1. Verified kinematics/dynamics + uncertainty management
2. Free choice of the underlying arithmetic: templates + solvers

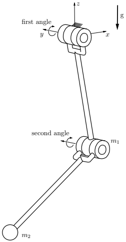
Type	Integrator	Purpose
`MoReal`	`MoAdams,...`	nonverified dynamics

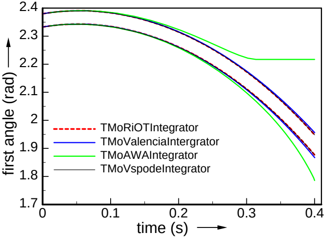
`TMoInterval`
`TMoFInterval`
`TMoTaylorModel`
`TMoTaylorModel`
	`TMoAWA`
	`TMoValencia`
	`TMoRiOT`
	`TMoVSPODE`
		verified dynamics of ODE based systems

`RDAInterval`	`---`	Taylor model based kinematics
`MoFInterval`	`MoIGradient`	verified equilibria kinematics with constraints
`MoSInterval`	`TMoValenciaS`	verified sensitivity
...	...	...

3. Converters MOBILE ⟶ SmartMOBILE

Example: Dynamics of a Double Pendulum





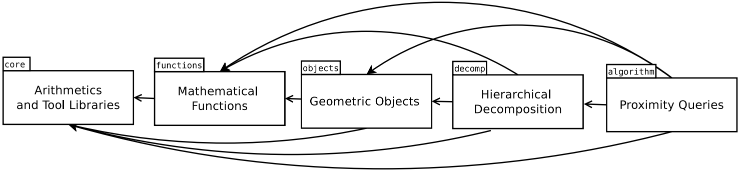

```
# define TMoInterval t;
TMoFrame<t> K0, K1, K2, K3, K4;
TMoAngularVariable<t> psi1, psi2;
// transmission elements
TMoVector<t> l1(0,0,-1), l2(0,0,-1);
TMoElementaryJoint<t> R1(K0,K1,psi1,xAxis);
TMoElementaryJoint<t> R2(K2,K3,psi2,xAxis);
TMoRigidLink<t> rod1(K1,K2,l1),rod2(K3,K4,l2);
t m1(1),m2(1);
TMoMassElement<t> Tip1(K2,m1),Tip2(K4,m2);
// the complete system
TMoMapChain<t> Pend;
Pend << R1<<rod1<<Tip1<<R2<<rod2<<Tip2;
// dynamics
TMoVariableList<t> q; q << psi1<<psi2;
TMoMechanicalSystem<t> S(q,Pend,K0,zAxis);
TMoAWAIntegrator I(S,0.0001,ITS_QR,15);
I.doMotion();
```

Strategy	TMoAWA (variable h)	TMoRiOT ($0.0002 \leq h \leq 0.2$)	TMoValencia ($h = 10^{-4}$)	TMoVSPODE (variable h)
Break–down	0.420	0.801	0.531	0.656
CPU Time*	5	285	22	10

* computed on 8 × Intel Xeon CPU 2.00GHz under Linux 2.6.25.14-69.fc8

UniVerMeC Instead of Templates

Unified Framework for Verified GeoMetric Computations



Relaxed layered structure:

core	*Adapter for underlying arithmetic libraries*
functions	*Uniform representation for functions*
objects	Implicit surfaces, CSG models, polyhedrons
decomp	Spatial decomposition, Multisection schemes
algorithms	Distance computation, Global optimization, ...

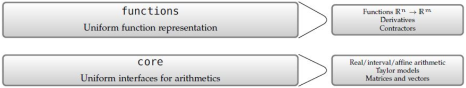
S. Kiel, UniVerMeC – *A Framework for Development, Assessment and Interoperable Use of Verified Techniques*, 2014

UniVerMeC: Function Specification

Important: Interoperability

The capability to communicate, execute programs, or transfer data among various functional units in a manner that requires the user to have little or no knowledge of the unique characteristics of those units

Necessary: Formalizations for arithmetics, types of enclosures, etc.



$f : \mathbb{R}^n \mapsto \mathbb{R}^m$, tools for user-defined functions (inductive), analytical expressions or C++ code blocks

Function extensions: Evaluated with all arithmetics supported by `core`

$\rightarrow$ e.g. natural interval extension (replace everything with interval versions)

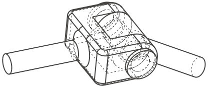
Features: set of functionalities associated with f (e.g., differentiability)

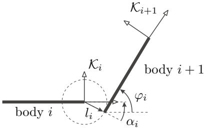
FR object: Tuple $F_{f,n,m} = (\mathcal{I}, \mathcal{F})$ where $\mathcal{I}$ is the set of inclusion functions, $\mathcal{F}$ is a choice out of r supported features

If $\mathcal{I}$ and $\mathcal{F}$ are defined appropriately, e.g. probabilistic arithmetics can be used!

Bounded Uncertainty at the Analysis Stage

`TMoSloppyJoint:`

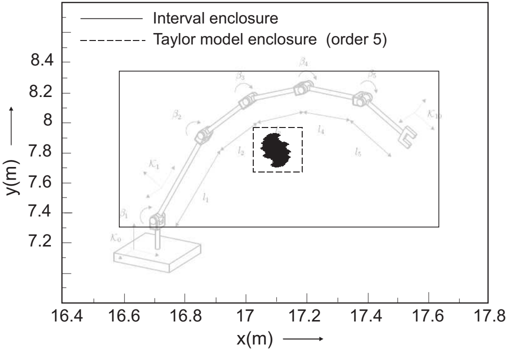




Parameter:

Lengths	$\pm 1\%$
Slackness	± 2mm
Angle	$\pm 0.1°$

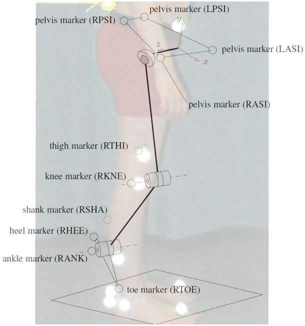
Results:



	x	y	Time (s)
`TMoInterval`	1.047	1.041	0.02
`TMoTaylorModel`	0.163	0.290	0.14

Purely Parametric Bounded Uncertainty

Body segment motion



Parameters (mm):

knee width	120 ± 10
ankle width	80 ± 10
displacements	tangential/soft tissue ± 10
	normal ± 5

Femur length (mm):

	TMoRDA	INTERVAL
Knee, ankle	[377.6; 396.7]	$[0;\infty]$
Skin displacement	[0.000; 621.4]	no answer

Point sensitivity of femur wrt.

Knee	Ankle	Tangential	Normal	Soft
0.4	-0.3	-2	0.7	1.4

Knee, Ankle: ±7mm; Tangential, Normal, Soft: ±37.5mm

Reference $\mathbf{r} = \sum_{i=1}^{n} s_i \cdot \mathbf{p}_i$:

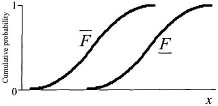
Probability Boxes

Given: $\overline{F}$ and $\underline{F} : \mathbb{R} \mapsto [0,1]$ nondecreasing, $\underline{F}(x) \leq \overline{F}(x)$, $\forall x \in \mathbb{R}$

$[\overline{F}, \underline{F}]$: Set of all nondecreasing functions $F : \mathbb{R} \mapsto [0,1]$ with $\underline{F}(x) \leq F(x) \leq \overline{F}(x)$

Definition: $[\overline{F}, \underline{F}]$ is called a p-box (probability box) when $\underline{F}$ and $\overline{F}$ circumscribe an imprecisely known probability distribution

Meaning: If X is a random variable with the unknown distribution $F \in [\overline{F}, \underline{F}]$ then $\underline{F}(x)$ is a lower bound on $F(x) = P(X \leq x)$



$$\overline{F}(x) = 1 - \underline{P}(X > x),\ \underline{F}(x) = \underline{P}(X \leq x)$$

$\underline{P}$ is a lower probability for an event A (the maximum rate one would be willing to pay for the gamble that pays 1 unit of utility if A occurs)

Bounds on the result of $+, -, \cdot, /$ of random variables defined using only bounds on their input distributions can be given

S. Ferson et al, *Constructing Probability Boxes and Dempster-Shafer Structures*, 2003

(Finite) Dempster-Shafer Structures

Discrete distributions

Probability density

$$f(x) = \begin{cases} \underbrace{P(X = x_i)}_{\approx\text{probability mass}} & \text{for } x = x_i \\ 0 & \text{for } x \neq x_i \end{cases}$$

$i = 1, 2, \ldots$

Idea Dempster-Shafer

$$\begin{cases} \underbrace{P(X = x_i)}_{\text{probability mass}} & \text{for } x \in \text{Set}_i \\ 0 & \text{otherwise} \end{cases}$$

$i = 1, 2, \ldots, n$

Interpretation: Classical probability theory in a topologically coarser space (where each focal element is identified as a point)

Focal elements $a_i \subseteq \mathbb{R}$: Sets associated with nonzero mass; may overlap one another

Basic probability assignment: Correspondence of probability masses associated with the focal elements; $m : 2^{\mathbb{R}} \mapsto [0, 1]$, $m(\emptyset) = 0$, $m(a_i) = p_i$, $i = 1, 2, \ldots, n$, $p_i > 0$, $\sum_i p_i = 1$

Dempster-Shafer Structures: Plausibility and Belief

Assumption: Let a_i be closed intervals, $b \subseteq \mathbb{R}$

Plausibility function: $Pl : 2^{\mathbb{R}} \mapsto [0, 1]$, $Pl(b) = \sum_{a_i \cap b \neq \emptyset} m(a_i)$

Belief function: $Bel : 2^{\mathbb{R}} \mapsto [0, 1]$, $Pl(b) = \sum_{a_i \subseteq b} m(a_i)$

Property: $Bel(b) \leq Pl(b)$

Arithmetic operations generalize the notion of convolution between distribution functions

The upper bound for the distribution function: $\sum_{\inf(a_i) \leq z} p_i$, $z \in \mathbb{R}$

(step function with n discontinuities)

The lower bound: $\sum_{\sup(a_i) \leq z} p_i$

P-boxes Versus Dempster-Shafer Structures

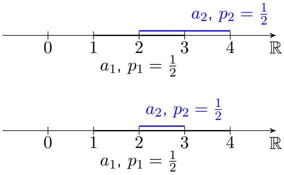
Dempster-Shafer: Uncertainty in the x-value, certainty in the p-value

P-box: Uncertainty about probabilities, certainty about events

Nonetheless: Dual, can be converted into each other

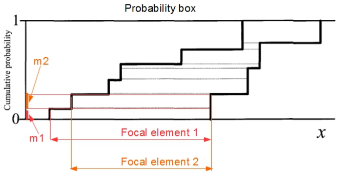
DS → p-box: $([x_i, y_i], p_i) \rightarrow \overline{F}(z) = \sum_{x_i \leq z} p_i,\ \underline{F}(z) = \sum_{y_i \leq z} p_i$

Relationship: Many Dempster-Shafer structures, a single p-box



$$\underline{F}(z) = \begin{cases} 0, & z < 3 \\ \frac{1}{2}, & z \in [3,4) \\ 1, & z \geq 4 \end{cases} \qquad \overline{F}(z) = \begin{cases} 0, & z < 1 \\ \frac{1}{2}, & z \in [1,2) \\ 1, & z \geq 2 \end{cases}$$

P-box → DS:



When to Apply?

1. Imprecisely specified distributions
2. Poorly known or even unknown dependencies
3. Non-negligible measurement uncertainty
4. Non-detects or other censoring in measurements
5. Small sample size
6. Inconsistency in the quality of input data
7. Model uncertainty
8. Non-stationarity (non-constant distributions)

Tools: Codes, References, Links for DS-Structures and P-Boxes

P-boxes: RiskCalc (commercial)

- → http://www.ramas.com/riskcalc.htm
- → probability bounds analysis, standard fuzzy arithmetic, and classical interval analysis

DS-structures: IPPToolbox (in MATLAB or R, open-source)

- → www.uni-due.de/informationslogistik/ipptoolbox.php

DS-structures with verified intervals: DSI Toolbox (MATLAB, open-source)

- → www.scg.inf.uni-due.de/forschung/software/dsi-toolbox.php

Applications, comparisons: www.lix.polytechnique.fr/~bouissou/

Stochastic arithmetic and the CADNA software

Stochastic arithmetic: Model for exact computation on imprecise data $\sim \mathcal{N}(\mu, \sigma)$

Origin: Cestac method by J. Vignes and J. M. Chesneaux, 1992

Idea: Interpret imprecise data as stochastic numbers
($\mathbb{S}$ is the set of Gaussian random variables)

Therefore: $X \in \mathbb{S}$ is characterized by μ and σ

Property: $\exists \lambda_\nu$: $P(X \in [\mu - \lambda_\nu \sigma, \mu + \lambda_\nu \sigma]) = 1 - \nu$

Confidence interval of μ with the probability $1 - \nu$: $[\mu - \lambda_\nu \sigma, \mu + \lambda_\nu \sigma]$,
(e.g. $\lambda_\nu = 1.96$ for $\nu = 0.05$)

Significant decimal digits on μ: $\log_{10}\left(\frac{|m|}{\lambda_\nu \sigma}\right)$ if $\frac{|m|}{\lambda_\nu \sigma} \geq 10$ otherwise 0

Operations: Follow from the properties of the Gaussian distribution

Software: CADNA, `www-pequan.lip6.fr/cadna/`, estimates round-off errors

Example: $P(x, y) = 9x^4 - y^4 + 2y^2$, $P(10864, 18817) = 2.0$ (wrong, exact=1.0), $P(1/3, 2/3) = 0.802...$ (correct). Is there any way to distinguish the quality?
In CADNA, the number of significant digits is computed to be zero in the first case, normal (15) in the second!

Fuzzy: Short Overview

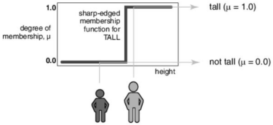
Goal: Model impreciseness/vaugness (*a little*, *very*, etc.)

Definition: Pair (U, μ) is a fuzzy set, where U is a set and $\mu : U \mapsto [0, 1]$ a membership function, $\mu(x)$ the grade of membership of x in U

Set vs. fuzzy set

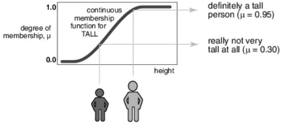
Traditional

$x \in U$ or not



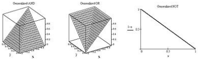
Fuzzy

μ decides if $x \in U$



Fuzzy number Convex, normalized fuzzy set $A \subseteq \mathbb{R}$ with continuous $\mu_A(x) = 1$ at precisely one element

Operations: xAND$y = \min(x, y)$, xOR$y = \max(x, y)$, NOT$x = 1 - x$



Fuzzy decisions: Rules for credibility, aggregation etc.

www.calvin.edu/~pribeiro/othrlnks/Fuzzy/fuzzysets.htm

Mixed Methods: Some References

A set-membership method to characterize a probabilistic set:
L. Jaulin et al., *Inner and outer approximations of probabilistic sets*, 2014
`www.ensta-bretagne.fr/jaulin/teaching.html`

Fuzzy probability theory: M. Beer, *Fuzzy Probability Theory*, In: Meyers, R. (ed.), Encyclopedia of Complexity and Systems Science, 2009

Comparisons: M Beer, V. Kreinovich, *Interval or Moments: Which Carry More Information?*, 2012

Imprecise probabilities: `http://www.sipta.org/`

Intervals and probabilities:
`http://ualr.edu/jdberleant/intprob/`

Approaches to Uncertainty Visualization

Goal: Present data with auxiliary uncertainty information

L. Gosink et al., *Characterizing and Visualizing Predictive Uncertainty*, 2013, graphics.cs.ucdavis.edu/~joy/NSF-IIS-1018097/

Approaches to Uncertainty Visualization

Goal: Present data with auxiliary uncertainty information

⚠ Most works deal with random aleatory uncertainty

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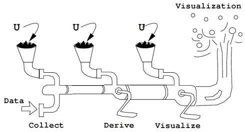

→ Depends on uncertainty *sources*,

→ figure does not consider *model* uncertainty

Figure from A. Pang et al., *Approaches to uncertainty visualization*, 1997

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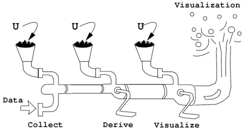

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Taxonomies: Dimensionality of data vs that of uncertainty

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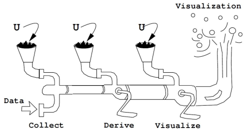

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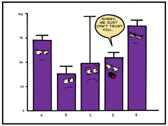
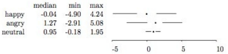
Methods: For example, for scalar/multivariate/vector discrete data

Error bars

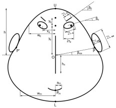


by Velica

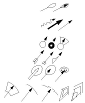
Tufte plots



Chernoff



Glyphs



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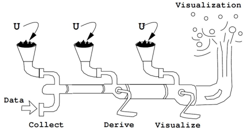

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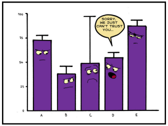

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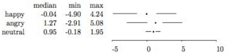

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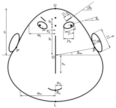

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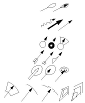

Evaluation: e.g. K. Potter et al., *From Quantification to Visualization...*, 2011

L. Gosink et al., *Characterizing and Visualizing Predictive Uncertainty*, 2013, graphics.cs.ucdavis.edu/~joy/NSF-IIS-1018097/

Summary

Learned in this workshop:

Uncertainty: Definition, types, sources, models

Model uncertainty: Mostly not considered here

Parametric uncertainty: At modeling and simulation level

- → Classical uncertainty: Monte-Carlo, polynomial chaos expansions
- → *Bounded uncertainty:* intervals, Taylor models etc.
- → Imprecise probabilities: *p-boxes, DS-structures*
- → Impreciseness: fuzzy sets

Interesting research topics:

- → Uncertainty visualization
- → Interoperability/implementation issues
- → Parallelization/templatization (e.g. on the GPU)