Universität Rostock
Traditio et Innovatio

# Control-Oriented Applications of Simulation Techniques for Non-Smooth Dynamic Systems

## Pre-Conference Workshop at ECC 2015
## 14th European Control Conference

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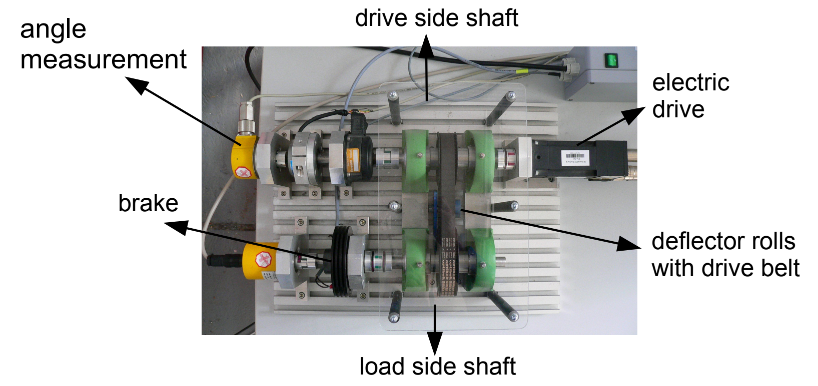
- Motivation: Verified parameter identification for a drive train test rig
- System significantly influenced by static/ sliding friction
- Necessity for handling transitions between different discrete model states (non-smooth right-hand side of an ODE)
- Verified integration approach for corresponding initial value problems
- Identification procedures
- Experimental results and comparison with an interval-based sliding mode observer
- Conclusions and outlook on future work

# Verified Parameter Identification for a Drive Train Test Rig

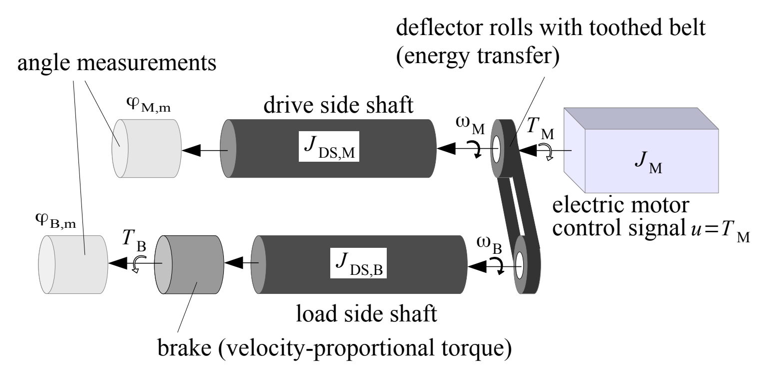
## Experimental setup




- Identification of static friction, sliding friction
- Identification of mass moment of inertia

# Verified Parameter Identification for a Drive Train Test Rig

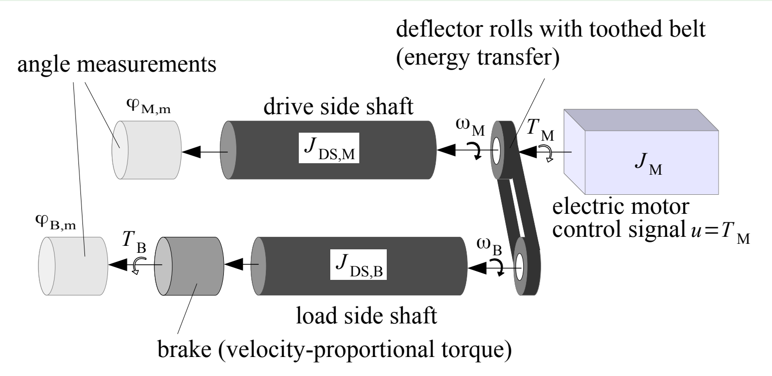
## Experimental setup




- Measurements: Angle(s) and angular velocities
- Actuation of brake: Non-modeled disturbance

# Verified Parameter Identification for a Drive Train Test Rig

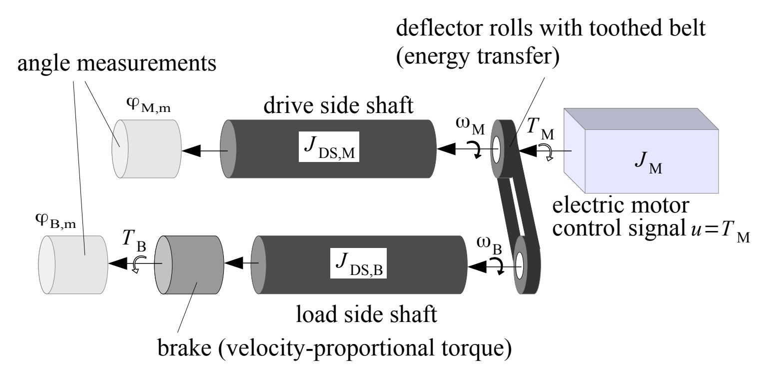
## System model: Sliding friction for $\dot{\varphi}_{\mathrm{M}} = \omega_{\mathrm{M}} = x_2(t) \neq 0$




$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ \alpha \cdot x_2(t) + \beta \cdot (u(t) - T_{\mathrm{F}}(t)) \end{bmatrix} \; , \quad T_{\mathrm{F}}(t) = T_{\mathrm{F,s}} \cdot \operatorname{sign}(x_2(t))$$

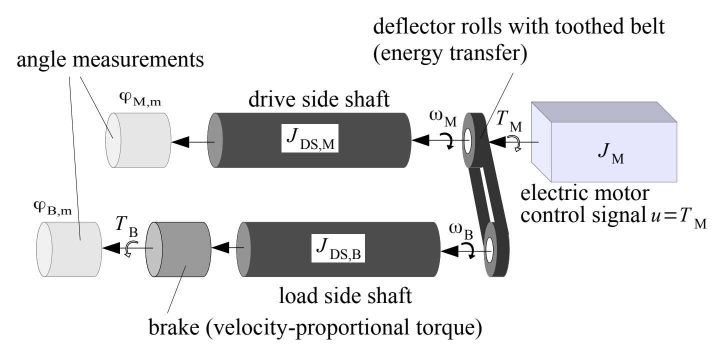
# Verified Parameter Identification for a Drive Train Test Rig

## System model: Static friction for $x_2(t) = 0$ and $|u(t)| \leq T_{\mathrm{F,s}}$




$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

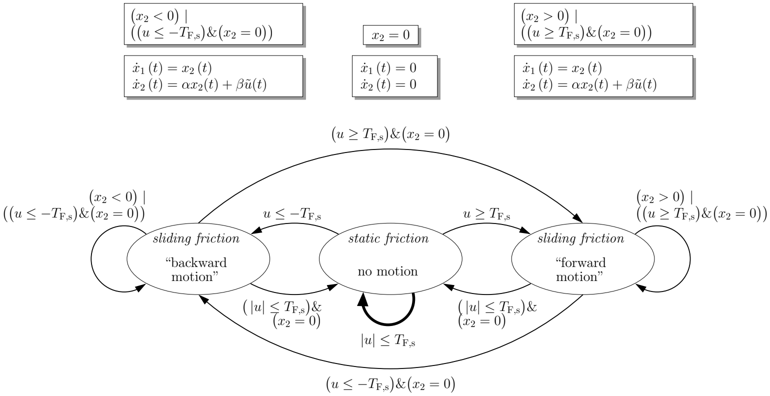
# Verified Parameter Identification for a Drive Train Test Rig




- $\alpha$ – (velocity-proportional friction)/(mass moment of inertia)
- $\beta$ – 1/(mass moment of inertia)
- $T_{\mathrm{F,s}}$ – static friction, possibly varying after standstill

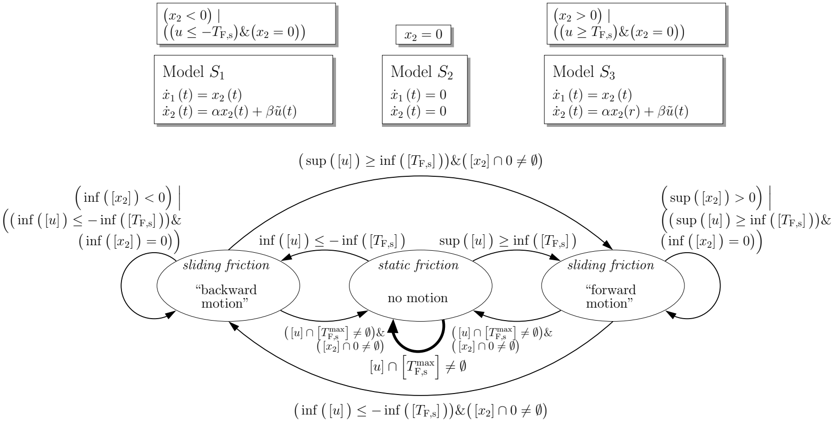
# System Model with Non-Smooth Right-Hand Side (1)

## Nominal system model, $\tilde{u}(t) := u(t) - T_{\mathrm{F}}(t)$



# System Model with Non-Smooth Right-Hand Side (2)

## Uncertain model, $\tilde{u}(t) := u(t) - T_{\mathrm{F}}(t)$, $[T_{\mathrm{F}}^{\max}] := \left[-\overline{T}_{\mathrm{F,s}}\ ;\ \overline{T}_{\mathrm{F,s}}\right]$



# Taylor Series-Based Enclosure Method (1)

- Discretization of considered time horizon
- Taylor series expansion of solution of the IVP with respect to time according to

$$\mathbf{x}\left(t_{k+1}\right) = \mathbf{x}\left(t_k\right) + \sum_{i=1}^{\nu} \frac{h^i}{i!} \mathbf{f}^{(i-1)}\left(\mathbf{x}\left(t_k\right), \mathbf{p}, \mathbf{u}\left(t_k\right), t_k\right) + \mathbf{e}\left(\mathbf{x}\left(\xi\right), \mathbf{p}, \mathbf{u}\left(\xi\right), \xi\right) \quad , \quad \dot{\mathbf{x}}(t_k) := \mathbf{f}\left(\mathbf{x}\left(t_k\right), \mathbf{p}, \mathbf{u}\left(t_k\right), t_k\right)$$

  with the integration step-size $h$, i.e., $t_k = kh$, $t_{k+1} = (k+1)h$, and $t_k \leq \xi \leq t_{k+1}$

## Note

- System parameters $\mathbf{p} \in [\mathbf{p}]$ are piecewise constant
- Changes of control signals $\mathbf{u}(t_k)$ only occur at the points $t = t_k$

# Taylor Series-Based Enclosure Method (2)

- Recursive computation of the total derivatives $\mathbf{f}^{(i-1)}$ (resp. Taylor series coefficients) in terms of the smooth right-hand side of the ODE with $\dot{\mathbf{p}} = \mathbf{0}$ and $\dot{\mathbf{u}}(t) = \mathbf{0}$, $t \in (t_k \,;\, t_{k+1})$
- Calculation of guaranteed bounds of the discretization error

$$\mathbf{e}\left(\mathbf{x}\left(\xi\right), \mathbf{p}, \mathbf{u}\left(\xi\right), \xi\right) \subseteq [\mathbf{e}_k] := \frac{h^{\nu+1}}{(\nu+1)!}\mathbf{f}^{(\nu)}\left([\mathbf{B}_{x,k}], [\mathbf{p}], \mathbf{u}\left([\tau_k]\right), [\tau_k]\right)$$

- Prerequisites: Differentiability of $\mathbf{f} \in C^{\nu}$
- Bounding box $[\mathbf{B}_{x,k}]$, parameter and control enclosures $[\mathbf{p}]$ and $\mathbf{u}\left([\tau_k]\right)$ for the time interval $[\tau_k] := [t_k \,;\, t_{k+1}]$ have to be available

$\Longrightarrow$ Use of the Picard iteration to determine $[\mathbf{B}_{x,k}]$

# Extension to Systems with Non-Smooth Right-Hand Sides

Step 1 Calculation of bounding box $[\mathbf{B}_{a,k}]$ for the time interval $[\tau_k]$ for the union of all system models which are *active* at $t = t_k$ with a continuously differentiable function $\mathbf{f}_a$ enclosing the right-hand sides of all active models

Step 2 Check for additionally activated models

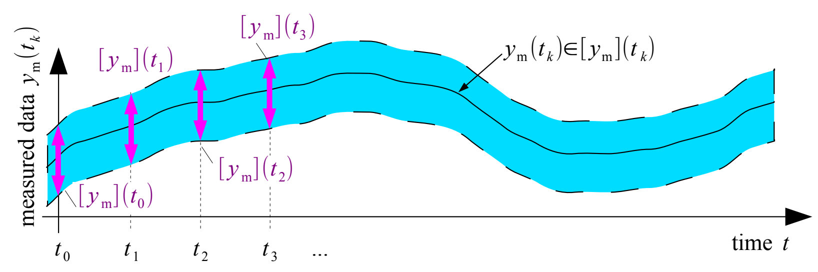
- Repeat **Step 1** if additional models are activated $\Longrightarrow$ modification of $\mathbf{f}_a$ by consideration of additionally activated models
- Otherwise, continue with **Step 3**

Step 3 Interval evaluation of series expansion for $\mathbf{f}(\cdot) = \mathbf{f}_a(\cdot)$
Subsequently: $\nu \equiv 1$

Step 4 Deactivation of system models which can no longer be active at $t = t_{k+1}$

# Verified Methods for Parameter Identification

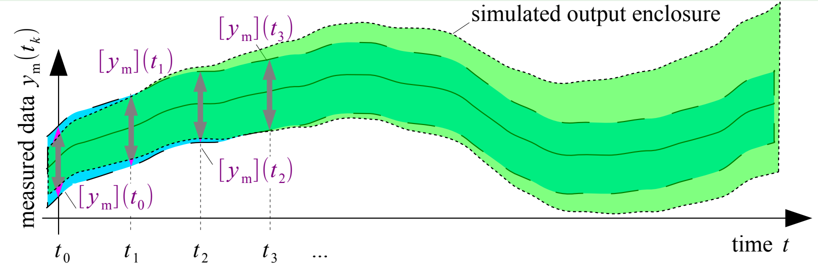
## Offline procedure: Multiple simulations over complete horizon of gathered measured data




- Measured data are available at discrete points of time
- Worst-case bounds for measurement tolerances
- Information about uncertain initial states and bounds on uncertain parameters

# Verified Methods for Parameter Identification

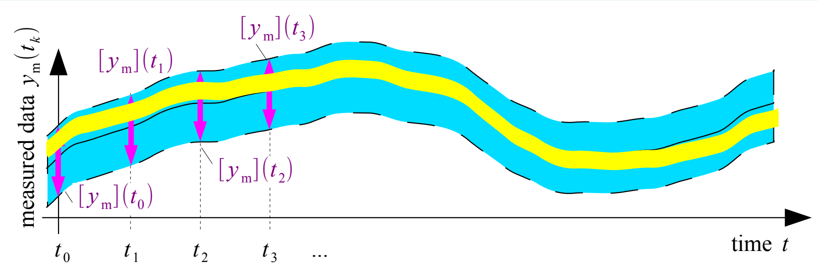
## Offline procedure: Multiple simulations over complete horizon of gathered measured data




- Prerequisite: Correctness of model structure
- Initial state/ parameter intervals are subdivided for candidates, for which no decision about admissibility can be made
- Intersection of directly measured and simulated state intervals possible

# Verified Methods for Parameter Identification

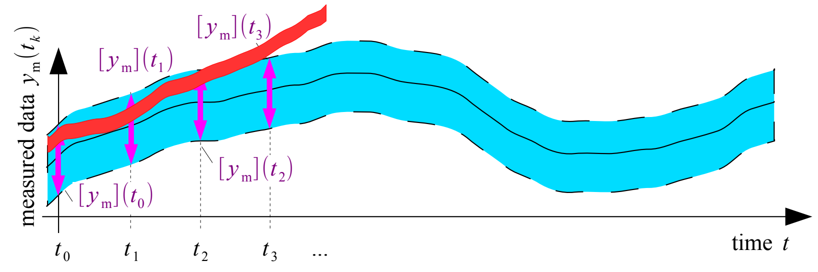
## Offline procedure: Multiple simulations over complete horizon of gathered measured data




- Search for guaranteed admissible initial state/ parameter intervals
- Subdivision until undecided region is sufficiently small
- Needs to be fulfilled for each available sensor if $\dim(\mathbf{y}_\mathrm{m}) > 1$

# Verified Methods for Parameter Identification

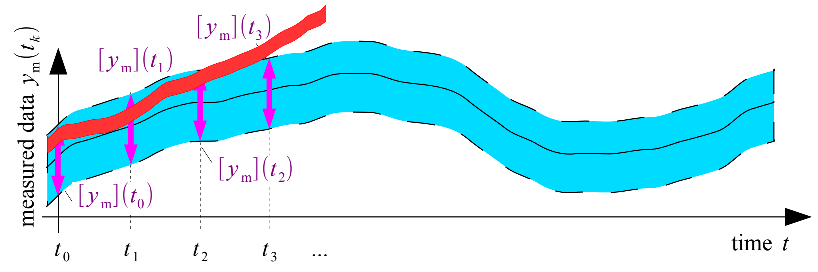
## Offline procedure: Multiple simulations over complete horizon of gathered measured data




- Exclusion of inadmissible intervals (for at least one of the sensors)
- Drawback: Conservativeness for systems with non-smooth right-hand sides (large number of subintervals)

# Verified Methods for Parameter Identification

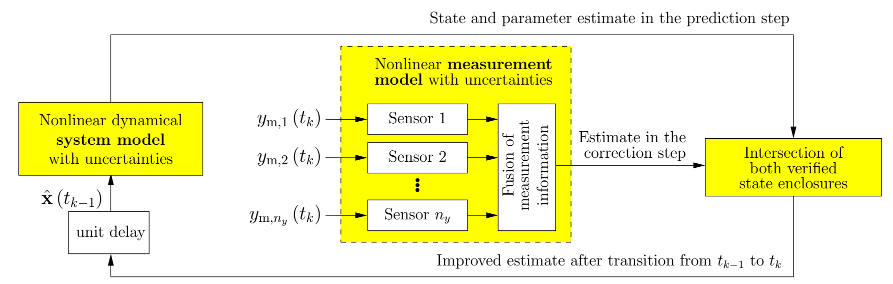
## Offline procedure: Multiple simulations over complete horizon of gathered measured data




- Exclusion of inadmissible intervals (for at least one of the sensors)
- Drawback: Conservativeness for systems with non-smooth right-hand sides (large number of subintervals)
- **Parameter reset (after standstill) cannot be handled efficiently**

# Verified Methods for Parameter Identification

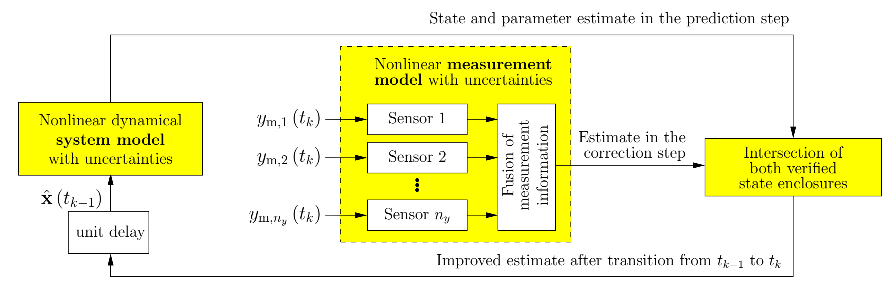
## Online procedure: Observer-based approach




- Verified integration of state equations between two subsequent measurement points $\Longrightarrow$ Structure close to Luenberger observer/ (Extended) Kalman Filter
- Exclusion of inadmissible intervals

# Verified Methods for Parameter Identification

## Online procedure: Observer-based approach




- Verified integration of state equations between two subsequent measurement points $\Longrightarrow$ Structure close to Luenberger observer/ (Extended) Kalman Filter
- Exclusion of inadmissible intervals
- **Parameter reset easily possible at specific points of time**

# Observer-Based Verified Parameter Identification (1)

1. Description of the state enclosure by a list of $L$ interval boxes

$$\left[\mathbf{z}^{<l>}\right](t_k) := \begin{bmatrix} \left[\mathbf{x}^{<l>}\right](t_k) \\ \left[\mathbf{p}^{<l>}\right](t_k) \end{bmatrix} \;, \quad l = 1, \ldots, L$$

2. Perform $M$ subdivisions of intervals, if at least one interval $l$ is characterized by

$$\prod_{j=1}^{n+n_{\mathrm{p}}} \mathrm{diam}\left\{\left[z_j^{<l>}\right](t_k)\right\} \neq 0$$

$\Longrightarrow$ New interval list of length $L + M - 1$

3. Verified integration of the IVP until the next measurement point $t_{k+1}$ $\Longrightarrow \left[\mathbf{z}^{<l>}\right](t_{k+1})$

# Observer-Based Verified Parameter Identification (2)

4. Intersection of all interval boxes with measured data $z_1(t_{k+1}) \in [y_\mathrm{m}]\,(t_{k+1})$ (assumption first state is directly measurable)

$$\left[\tilde{z}_1^{<l>}\right](t_{k+1}) := \left[z_1^{<l>}\right](t_{k+1}) \cap [y_\mathrm{m}]\,(t_{k+1})$$

5. Replace $\left[z_1^{<l>}\right](t_{k+1})$ by $\left[\tilde{z}_1^{<l>}\right](t_{k+1})$ for all $l = 1, \ldots, L + M - 1$
6. Delete all subintervals with $\left[\tilde{z}_1^{<l>}\right](t_{k+1}) = \emptyset$ from the interval list
7. Replace static friction subintervals with initial range if standstill is detected for a minimum time span (by binary signal from velocity sensor)
8. Reduce the number of subintervals by a convex hull with sufficiently small overestimation: New list length $L := L^*$

Note: Interval replacement (step 7) and reduction of interval number (step 8) can be employed interchangeably

# Bisectioning Strategy in Step 2

## Selection of the candidates to be subdivided

$$l^* = \arg\max_{l=1,\ldots,L'} \prod_{j=1}^{n+n_\mathrm{p}} \operatorname{diam}\left\{\left[z_j^{<l>}\right](t_k)\right\} , \quad L' \geq L$$

## Reduce ambiguities between static and sliding friction

- Split static friction interval if

$$[u](t_k) \cap \operatorname{hull}\left\{-\left[T_{\mathrm{F,s}}^{<l^*>}\right], \left[T_{\mathrm{F,s}}^{<l^*>}\right]\right\} \neq \emptyset$$

# Bisectioning Strategy in Step 2

## Selection of the candidates to be subdivided

$$l^* = \arg\max_{l=1,\dots,L'} \prod_{j=1}^{n+n_\mathrm{p}} \mathrm{diam}\left\{\left[z_j^{<l>}\right](t_k)\right\} \ , \quad L' \geq L$$

## Reduce ambiguities between static and sliding friction

- Select splitting point $\overline{u}(t_k) + \epsilon$, $\epsilon > 0$ for $[u](t_k) > 0$ with $\underline{T}_{\mathrm{F,s}}^{<l^*>} < \underline{u}(t_k)$ and $\overline{T}_{\mathrm{F,s}}^{<l^*>} > \overline{u}(t_k)$
- Select splitting point $\underline{u}(t_k) - \epsilon$, $\epsilon > 0$ for $[u](t_k) < 0$ with $-\overline{T}_{\mathrm{F,s}}^{<l^*>} < \underline{u}(t_k)$ and $-\underline{T}_{\mathrm{F,s}}^{<l^*>} > \overline{u}(t_k)$
- Else: Splitting of $\left[T_{\mathrm{F,s}}^{<l^*>}\right]$ at its midpoint

# Bisectioning Strategy in Step 2

## Avoid unnecessarily conservative interval bounds

- Split angular velocity interval $\left[x_2^{<l^*>}\right]$ for $\operatorname{diam}\left\{\left[x_2^{<l^*>}\right]\right\} \geq \operatorname{diam}\left\{\left[\beta^{<l^*>}\right]\right\}$
- Split interval $\left[\beta^{<l^*>}\right]$ for

$$\left(\left[\alpha^{<l^*>}\right] \cdot \left[x_2^{<l^*>}\right]\right) \cap \left(\left[\beta^{<l^*>}\right] \cdot \left(\left[u\right](t_k) - \left[T_{\mathrm{F,s}}^{<l^*>}\right]\right)\right) \neq \emptyset$$

- Else: Split interval $\left[\alpha^{<l^*>}\right]$
- Optional: Trisectioning of $\left[x_2^{<l^*>}\right]$ if static and sliding friction are possible simultaneously

# Replacement of Static Friction Interval in Step 7

## Repeat for each subinterval $l = 1, \ldots, L + M - 1$

- Initial range for $T_{\mathrm{F,s}}$: $\left[T_{\mathrm{F,s}}^{\mathrm{ini}}\right]$
- Create subintervals

$$[T_\mathrm{a}] := \left[\underline{T}_{\mathrm{F,s}}^{\mathrm{ini}} \; ; \; \underline{T}_{\mathrm{F,s}}^{<l>}\right]$$

$$[T_\mathrm{b}] := \left[\underline{T}_{\mathrm{F,s}}^{<l>} \; ; \; \overline{T}_{\mathrm{F,s}}^{<l>}\right]$$

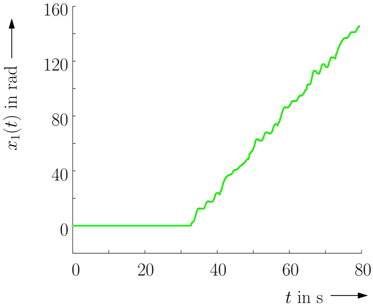
$$[T_\mathrm{c}] := \left[\overline{T}_{\mathrm{F,s}}^{<l>} \; ; \; \overline{T}_{\mathrm{F,s}}^{\mathrm{ini}}\right]$$

- Create a list of up to $3L$ subintervals, where $\left[T_{\mathrm{F,s}}^{<l>}\right]$ is replaced by each of the intervals $[T_\mathrm{a}]$, $[T_\mathrm{b}]$, $[T_\mathrm{c}]$ with non-zero diameter
- Subsequent merging (not necessarily after each time step) avoids combining intervals with different active model states $S_i$, $i = 1, 2, 3$

# Experimental Results for Verified Parameter Identification

- Measurement uncertainty: $[-0.1\ ;\ 0.1]$ rad
- Piecewise const. control: exactly known, interval hull over exp. data
- Sampling time: $10\,\mathrm{ms}$

Estimate of the angle $x_1 = \varphi_\mathrm{M}$

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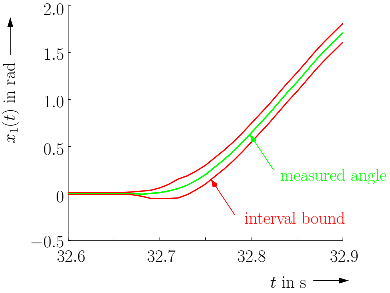

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- Measurement uncertainty: $[-0.1\ ;\ 0.1]\ \mathrm{rad}$
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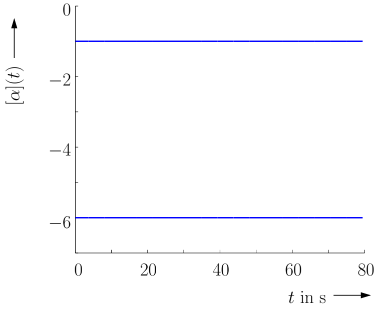
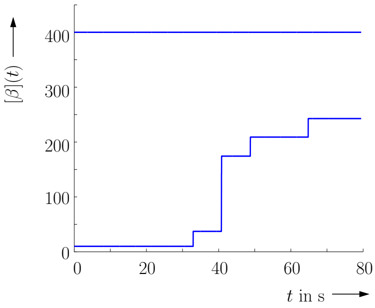
Parameter estimate $\alpha$




Parameter estimate $\beta$



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- Measurement uncertainty: $[-0.1\ ;\ 0.1]$ rad
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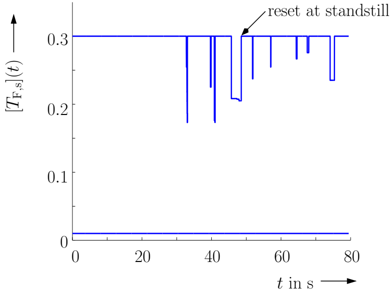
Parameter estimate $T_{\mathrm{F,s}}$

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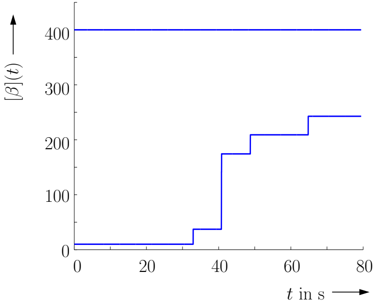

# Experimental Results for Verified Parameter Identification

- Measurement uncertainty: $[-0.1\ ;\ 0.1]\ \mathrm{rad}$
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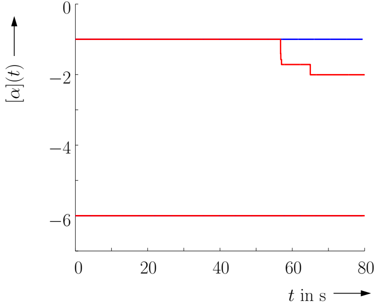
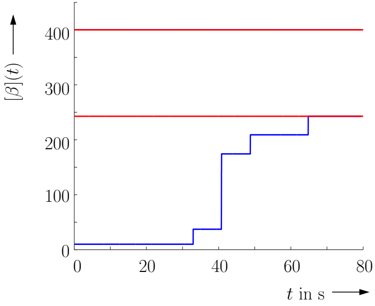
Parameter estimate $\alpha$




Parameter estimate $\beta$



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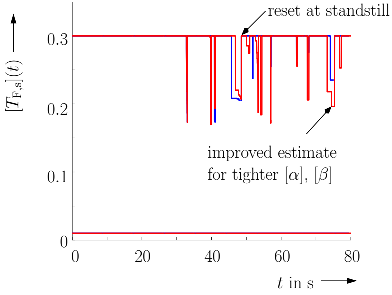


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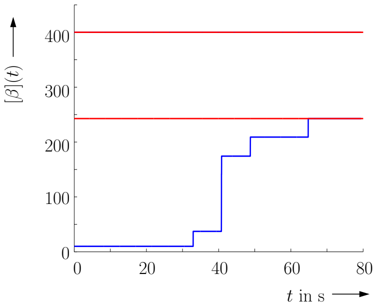
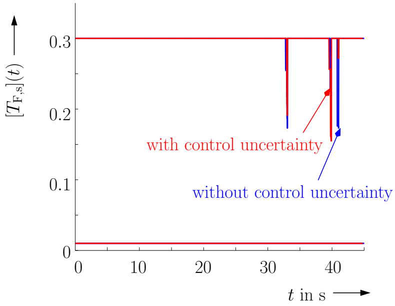


# Experimental Results for Verified Parameter Identification

- Measurement uncertainty: $[-0.1\ ;\ 0.1]\ \mathrm{rad}$
- Piecewise const. control: uncertainty: 10% of the experimental data
- Sampling time: $10\,\mathrm{ms}$

Parameter estimate $T_{\mathrm{F,s}}$

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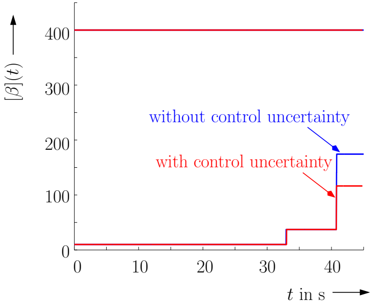

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## Note

- Transformation of state equations is advantageous for large measurement and control sampling times (wrapping effect)
- Minimize the number of unknown parameters to be identified
- Additional splitting of velocity intervals $[x_2]\,(t_k)$ for which a definite distinction of the discrete model states is impossible (A. Rauh et al.: Experimental Comparison of Interval-Based Parameter Identification Procedures for Uncertain ODEs with Non-Smooth Right-Hand Sides, MMAR 2015, Miedzyzdroje, Poland.)
- Dual task: Control optimization

# Optimal Control Tasks in Uncertain Settings (1)

## Continuous-time dynamic optimization problem

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{p}(t), \mathbf{u}(t), t) \; ; \quad 0 \le t \le t_f$$

$$\mathbf{x}(t=0) = \mathbf{x}_0 \longrightarrow \mathbf{x}(t_f) = \mathbf{x}_\mathrm{f}$$

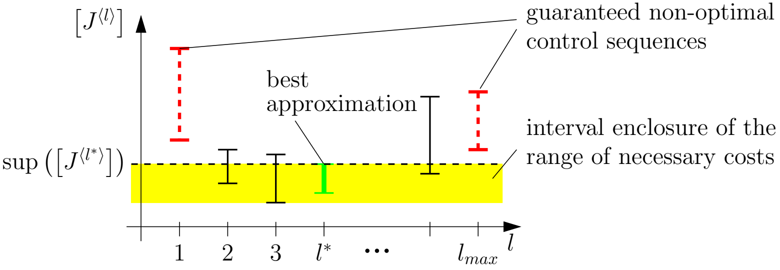
$$J = f_{t_f}(\mathbf{x}(t_f), \mathbf{p}(t_f), t_f) + \int_0^{t_f} f_0(\mathbf{x}(t), \mathbf{p}(t), \mathbf{u}(t), t)\,\mathrm{d}t \overset{!}{=} \min$$

## Continuous-time system model

- State equations $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{p}(t), \mathbf{u}(t), t)$
- State vector $\mathbf{x}(t)$
- Vector of uncertain system parameters $\mathbf{p}(t) \in [\underline{\mathbf{p}}(t) \; ; \; \overline{\mathbf{p}}(t)]$ with additional tolerances for $\dot{\mathbf{p}}(t) = \Delta\mathbf{p}(t)$, $\Delta\mathbf{p}(t) \in [\underline{\Delta\mathbf{p}}(t) \; ; \; \overline{\Delta\mathbf{p}}(t)]$
- Control vector $\mathbf{u}(t) \in [\underline{\mathbf{u}}(t) \; ; \; \overline{\mathbf{u}}(t)]$ with given range constraints

# Optimal Control Tasks in Uncertain Settings (2)

- Specification of robustness in the time domain by definition of admissible and forbidden regions in the state space
- Quantification of the influence of uncertainties on both the system states and the performance index
- How can optimality be defined for uncertain systems?



# Optimal Control Tasks in Uncertain Settings (3)

- Computation of guaranteed enclosures of trajectories for a given control input
- Evaluation of performance index and robustness requirements

Interval arithmetic techniques are a basis for verified analysis of dynamic systems with uncertainties.
$\Longrightarrow$ Mathematical verification of functionality, robustness, and safety

# Optimal Control Tasks in Uncertain Settings (3)

- Computation of guaranteed enclosures of trajectories for a given control input
- Evaluation of performance index and robustness requirements

Interval arithmetic techniques are a basis for verified analysis of dynamic systems with uncertainties.
$\Longrightarrow$ Mathematical verification of functionality, robustness, and safety

## Dynamic optimization

In order to optimize open-loop or closed-loop controllers for systems with uncertainties, approximations of globally optimal control sequences are determined by piecewise constant (or piecewise linear) solutions.

# Interval Algorithm for Synthesis of Optimal Closed-Loop and Open-Loop Control (1)

1. Direction of evaluation: **GOAL $\longrightarrow$ START**
   - Interval enclosures of region of attraction of final states (controllability problem for complete range of control variables)
   - Interval enclosure of possible range of the performance index
   - Guaranteed minimum and maximum of costs for transfer of the system to the final state
2. Restriction to a specific set of initial states: **START $\longrightarrow$ GOAL**
   - Candidates for admissible control sequences $\{[\mathbf{u}](t_k)\}$
   - Delete control sequences which do not allow to transfer the system from the initial state to the final state
   - Delete control sequences which are certainly not optimal

# Interval Algorithm for Synthesis of Optimal Closed-Loop and Open-Loop Control (2)

3. Reduction of interval diameters of sequence of candidates $\{[\mathbf{x}](t_k)\}$, $\{[\mathbf{u}](t_k)\}$ of the optimal solution
   $\Longrightarrow$ Re-start with **Step 1** as long as improvement is possible
4. Search for **global optimum** $\{\mathbf{u}^*(t_k)\}$
   Stopping criterion depending upon $\{[\tilde{\mathbf{u}}](t_k)\}$, $\{[\tilde{\mathbf{x}}](t_k)\}$, $\left\{\left[\tilde{J}\right](t_k)\right\}$

$$\inf\left(\left[\tilde{J}_0\right]\right) - \min_i\left(\inf\left(\left\{\left[J_0^{\langle i\rangle}\right]\right\}\right)\right) < \epsilon_1$$

$$\text{and} \qquad \operatorname{diam}\left(\left[\tilde{J}_0\right]\right) < \epsilon_2$$

5. Output of $\{[\tilde{\mathbf{x}}](t_k)\}$ as best-known approximation for $\{\mathbf{u}^*(t_k)\}$

# Mechanical Systems with Friction and Hysteresis (1)

## Continuous-time system model

- Equations of motion for a mass $m$

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ \frac{1}{m}\left(F_{\mathrm{a}}(t) - F_{\mathrm{f}}(x_2)\right) \end{bmatrix}$$

with the state vector $\mathbf{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$

- Mathematical model of the resulting friction force

$$F_{\mathrm{f}}(x_2) = \begin{cases} -[F_{\mathrm{s}}] + [\mu] \cdot x_2 & \text{for} \quad S_1 = \mathtt{true} \quad \text{or} \quad S_2 = \mathtt{true} \\ +[F_{\mathrm{s}}] + [\mu] \cdot x_2 & \text{for} \quad S_4 = \mathtt{true} \quad \text{or} \quad S_5 = \mathtt{true} \end{cases}$$

with the static friction

$$F_{\mathrm{f}}(x_2) \in [F_{\mathrm{s}}^{\max}] := \left[-\overline{F}_{\mathrm{s}}\,;\ \overline{F}_{\mathrm{s}}\right] \quad \text{for} \quad S_3 = \mathtt{true}$$

# Mechanical Systems with Friction and Hysteresis (2)

## Interval representation of uncertain parameters

- Uncertainties of static friction coefficient $[F_\mathrm{s}] := \left[\underline{F}_\mathrm{s} \; ; \; \overline{F}_\mathrm{s}\right]$
- Uncertainties of sliding friction coefficient $[\mu] := \left[\underline{\mu} \; ; \; \overline{\mu}\right]$

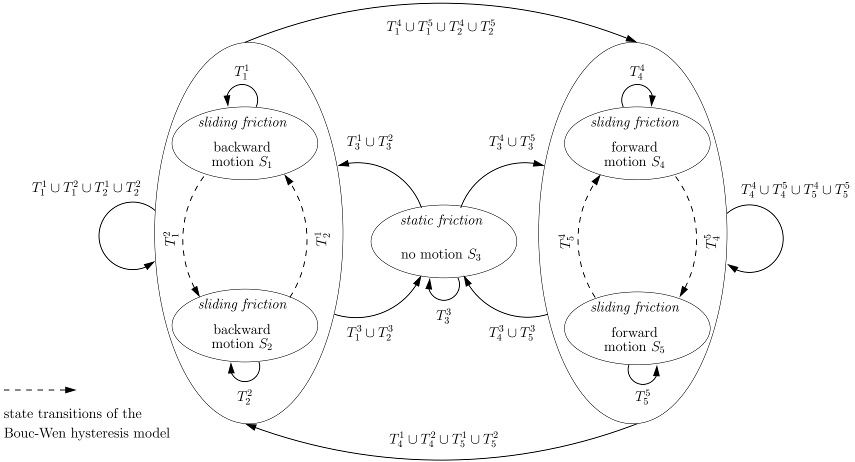
## Hysteresis and friction

- Accelerating force $F_a(t) := u(t) - \phi(x_1(t), \omega(t))$ acting on the mass $m$ as the difference of
  - control variable $u(t)$ provided by an actuator
  - restoring spring force $\phi(x_1(t), \omega(t)) = \kappa_x x_1 + \kappa_\omega \omega$
- Hysteresis of the restoring spring force given by the Bouc-Wen model

$$\dot{\omega}(t) = \rho \cdot \Big( x_2(t) - \sigma \cdot |x_2(t)| \cdot |\omega(t)|^{\nu-1} \cdot \omega(t) + (\sigma - 1) \cdot x_2(t) \cdot |\omega(t)|^{\nu} \Big)$$

# Mechanical Systems with Friction and Hysteresis (3)

## State transition diagram



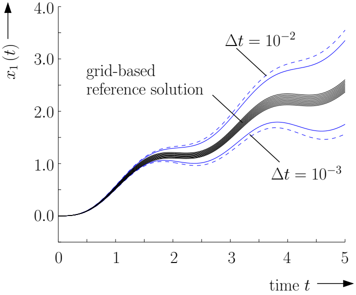
# Simulation Results

## Given control input: $u(t) = 2 \cdot \sin(3t)$

## Uncertain parameters

$m \in [1.1\ ;\ 1.21]$, $F_\mathrm{s} \in [0.015\ ;\ 0.03]$, $\kappa_x = 0.001$, $\kappa_\omega = 0.001$, $\sigma = 0.001$, $\rho = 0.001$, $\nu = 1$, $\mu = 0.001$, $x_1(0) = x_2(0) = 0$, $\omega(0) = -0.001$

Position $x_1$



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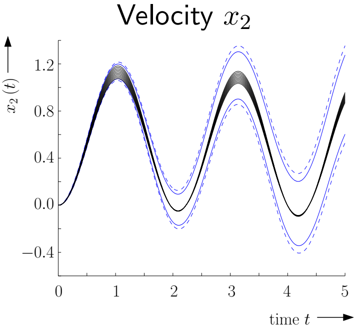

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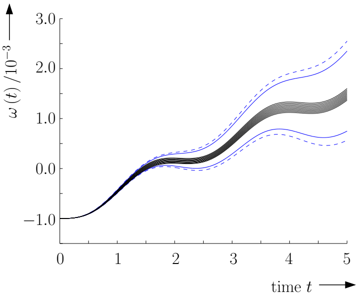
Hysteresis variable $\omega$



# Optimization Results

## Optimality criterion ($\Delta t = 0.1,\ k_{\max} = 50,\ M = 10$)

$$J = \int_0^{t_f} \left( (x_1(t) - 1)^2 + x_2(t)^2 + u(t)^2 \right) \mathrm{d}t + 100\Delta t \sum_{k=1}^{k_{\max}} (u_k - u_{k-1})^2$$

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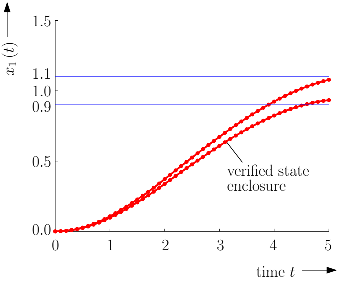
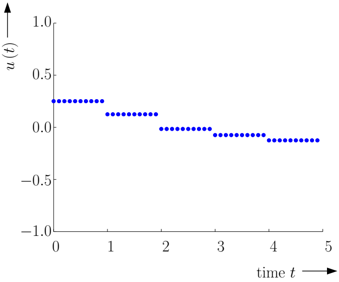



Input force $u$



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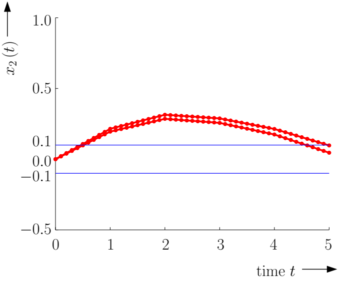
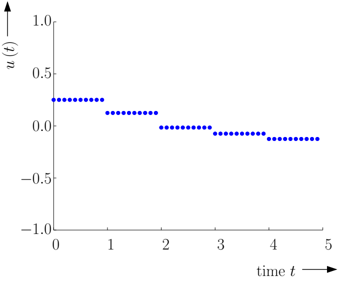
Velocity $x_2$




Input force $u$



# Conclusions and Outlook on Future Work

- Verified integration of ODEs with non-smooth right-hand side
- Observer-based identification of parameter intervals
- Consistency with dynamic system model and (uncertain) measured data
- Robust optimal control synthesis
- Efficiency of subdivision heuristics
- Validation of alternative estimation procedures (e.g. sliding mode) $\Longrightarrow$ Use of interval bounds for system parameters to describe the stability domains in the presentation of L. Senkel: *Interval-Based Design of Sliding Mode Control and State Estimation Procedures*

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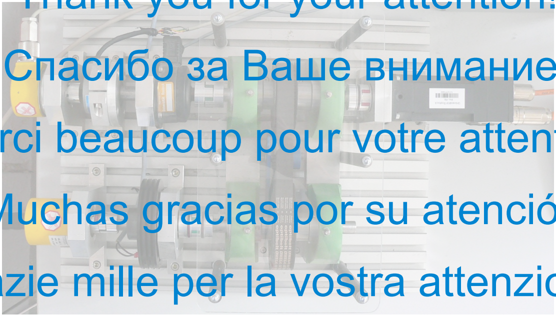
- Implementation of parallelization strategies
- Extension to further (higher-dimensional) nonlinear models
- Derivation of optimal input trajectories wrt. exclusion of infeasible intervals



Dziękuję bardzo za uwagę!

Thank you for your attention!

Спасибо за Ваше внимание!

Merci beaucoup pour votre attention!

¡Muchas gracias por su atención!

Grazie mille per la vostra attenzione!

Vielen Dank für Ihre Aufmerksamkeit!