An interval observer-based anti-disturbance control strategy for rigid satellite

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Outline

Model and Target

Main Work
- Disturbance Reconstruction
- PPC Controller

Simulation

System Model

Based on the MRPs, the rigid satellite attitude system is given as,

$$\dot{\sigma} = \frac{1}{4}\left[\left(1 - \sigma^T\sigma\right) I_3 + 2\sigma^{\times} + 2\sigma\sigma^T\right] \omega = G(\sigma)\omega \tag{1.1}$$

$$J\dot{\omega} = -\omega^{\times} J\omega + D\tau + d \tag{1.2}$$

where
σ is the system's attitude,
ω is the angular velocity,
τ is the control input,
d is a bounded external disturbance.

In addition, $d^- \leq d(t) \leq d^+$ where d^- and d^+ are two known vectors.

Control Targets

1. Designing an interval observer-based disturbance reconstruction strategy;
2. Based on command filter method, developing an intergral sliding mode controller.

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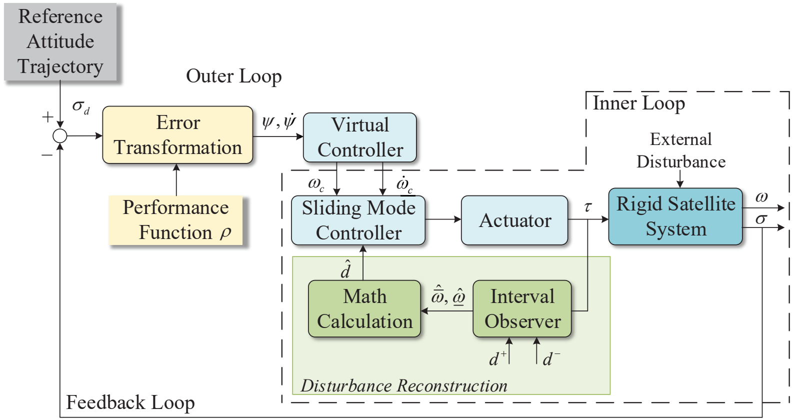

Figure 1: System Block

Disturbance Reconstruction

For (1.2), an interval observer is designed as,

$$\dot{\hat{\bar{\omega}}} = -J^{-1}\omega^{\times}J\omega + J^{-1}D\tau + (J^{-1})^{+}d^{+} - (J^{-1})^{-}d^{-} - L(\omega - \hat{\bar{\omega}}) \quad (2.1)$$

$$\dot{\underline{\hat{\omega}}} = -J^{-1}\omega^{\times}J\omega + J^{-1}D\tau + (J^{-1})^{+}d^{-} - (J^{-1})^{-}d^{+} - L(\omega - \underline{\hat{\omega}}) \quad (2.2)$$

Furthermore, we have,

$$\omega(t) = \Omega(t)\hat{\bar{\omega}}(t) + (I_3 - \Omega(t))\underline{\hat{\omega}}(t) \quad (2.3)$$

where

$\Omega(t) = \mathrm{diag}\{a_v(t)\}$, $a_v(t) = [a_1(t); a_2(t); a_3(t)]$

and $0 \leq a_i(t) \leq 1, \ i = 1, 2, 3.$

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Disturbance Reconstruction

By referring (1.2), (2.1)-(2.3), d is formulated as,

$$d = J \cdot \left[Q_1 \left(\hat{\hat{\omega}} - \underline{\hat{\omega}} \right) + Q_2 d^+ + Q_3 d^- \right]$$

where

$N_1 = -L(\Omega - I_3)$ and $N_2 = -L\Omega$,

$Q_1 = \Omega N_1 + (I_3 - \Omega) N_2 + \dot{\Omega}$,

$Q_2 = \Omega (J^{-1})^+ - (I_3 - \Omega)(J^{-1})^-$,

$Q_3 = -\Omega (J^{-1})^- + (I_3 - \Omega)(J^{-1})^+$.

To calculate $\dot{\Omega}$, the Levant differentiator[1] is used here. Then we have,

$$\hat{Q}_1 = \Omega N_1 + (I_3 - \Omega) N_2 + \hat{\dot{\Omega}}$$

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[1] Levant A. Robust exact differentiation via sliding mode technique. Automatica 1998;34(3):379–84.

Disturbance Reconstruction

Algorithm

1. Design an interval observer for ω;
2. Constructing $\omega(t) = \Omega(t)\hat{\bar{\omega}}(t) + (I_3 - \Omega(t))\underline{\hat{\omega}}(t)$;
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Merits

1. Different selections of d^+ and d^- will produce almost the same disturbance reconstruction;
2. The control input is decoupled from the expression of d;
3. The reconstruction can be obtained within a finite time.

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PPC-Error Transformation

The PPC stands for prescribed performance control.

Objectives

1. The transient response is bounded by a preset boundary pair;
2. The steady-state response will not violate the boundary set.

The output constraint for $\tilde{\sigma}$ is set as,

$$-\underline{\delta}(t)\rho(t) < \tilde{\sigma} < \overline{\delta}(t)\rho(t)$$

Importing $S(\cdot)$ as $S(\psi_i) = \frac{\overline{\delta}_i(t)e^{\psi_i} - \underline{\delta}_i(t)e^{-\psi_i}}{e^{\psi_i} + e^{-\psi_i}}$, it has,

$$\dot{\psi} = \varrho\dot{\tilde{\sigma}} + \nu$$

Then the constrained output problem is transformed into the stability problem of ψ.

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Outer Loop

For the outer loop, the sliding mode surface is selected as,

$$s_\psi = \psi(t) + \Gamma_\psi \psi_I(t)$$
$$= \psi(t) + \Gamma_\psi \int_0^t k_{\psi 1}\psi(\tau_\psi) + k_{\psi 2}\psi^{q_\psi/p_\psi}(\tau_\psi)d\tau_\psi$$

$$\dot{\psi}_I = k_{\psi 1}\psi(t) + k_{\psi 2}\psi^{q_\psi/p_\psi}(t), \quad \psi_I(0) = -\frac{\psi(0)}{\Gamma_\psi}$$

Due to the implement of command filter, the compensated tracking error Υ is given like,

$$\Upsilon = s_\psi - \zeta$$

Then the virtual controller ω_d is chosen as,

$$\omega_d = G^{-1}\varrho^{-1}\left(-k_1 s_\psi - \nu - k_2 \text{sig}^{l_1}(\Upsilon) - \Gamma_\psi k_{\psi 1}\psi(t) - \Gamma_\psi k_{\psi 2}\psi^{q_\psi/p_\psi}(t)\right)$$

and ζ is governed by,

$$\dot{\zeta} = -k_1\zeta + \varrho G(\omega_c - \omega_d)$$

Inner Loop

For the inner loop, the sliding mode surface is selected as,

$$s_b = e_\omega + \Lambda_1 \Upsilon + \Lambda_2 \text{sig}^{l_1}(\Upsilon)$$

Then the finite time controller is chosen as,

$$\begin{aligned}
\tau =& \tau_{a1} + \tau_{a2} \\
\tau_{a1} =& D^\dagger J \Big(-k_5 s_b - k_6 \text{sig}^{l_1}(s_b) + J^{-1}\omega^\times J\omega - J^{-1}\hat{d} \\
& + \dot{\omega}_c - F - \frac{s_b(\|\varrho G\| + \chi)\|\Upsilon^T e_\omega\|}{\|s_b\|^2 + k_3} \Big) \\
\tau_{a2} =& D^\dagger J(-\Lambda_3 s_b - \Lambda_4 \text{sign}(s_b))
\end{aligned}$$

and χ which is used to address coupling between outer and inner loop satisfies,

$$\dot{\chi} = \begin{cases} \frac{\eta}{\chi} \cdot \frac{\chi\|s_b\|^2 - \|\varrho G\| k_4}{\|s_b\|^2 + k_3} \|\Upsilon^T e_\omega\| - l_2\chi - l_3 \text{sig}^{l_1}(\chi), & \chi \neq 0 \\ -k_a e^{-k_a t}, & \chi = 0 \end{cases}$$

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Some key parameters are given as,[2]
$d = 0.01 \times [-2\sin(0.05t); 2\sin(0.07t); \cos(0.05t)] Nm$

Table 1: Parameters of the interval observer

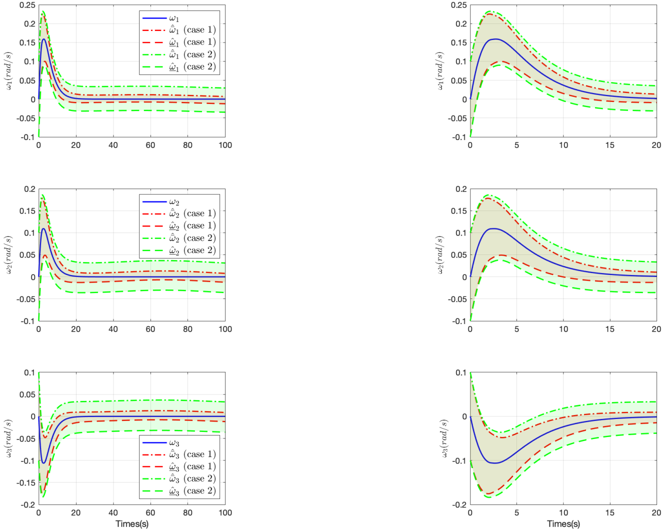
Module	Parameters
Interval observer	$\hat{\bar{\omega}}(0) = [0.1; 0.1; 0.1]$, $\underline{\bar{\omega}}(0) = [-0.1; -0.1; -0.1]$ $L = [-0.6, 0.2, 0.2; 0.2, -0.6, 0.2; 0.2, 0.2, -0.6]$

Table 2: Parameters of disturbance bounds

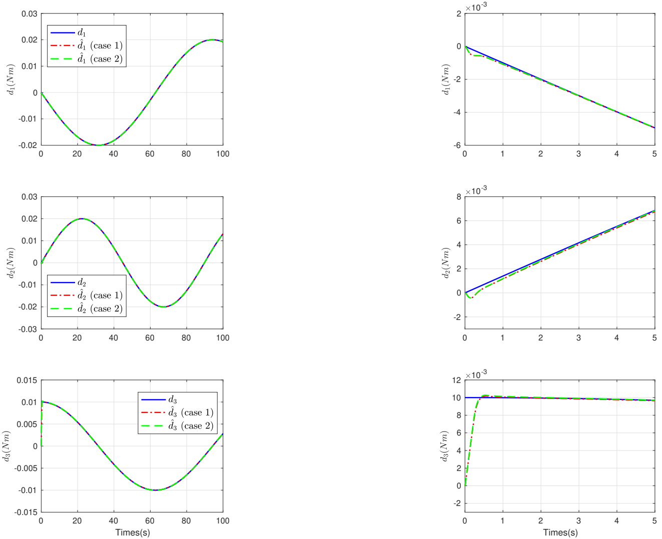
Observer	Parameters
Case 1	$d^{+} = [0.03; 0.03; 0.03]$, $d^{-} = -[0.03; 0.03; 0.03]$
Case 2	$d^{+} = [0.1; 0.1; 0.1]$, $d^{-} = -[0.1; 0.1; 0.1]$

[2]The detailed parameters are in https://doi.org/10.1016/j.isatra.2020.05.048

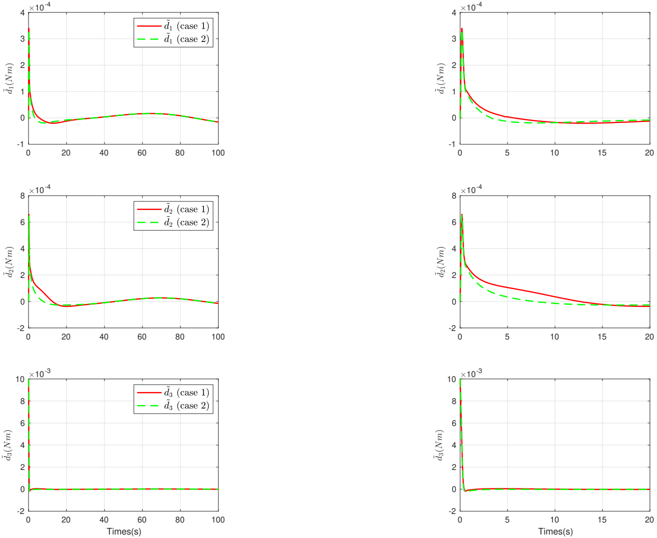
Interval Estiamtion



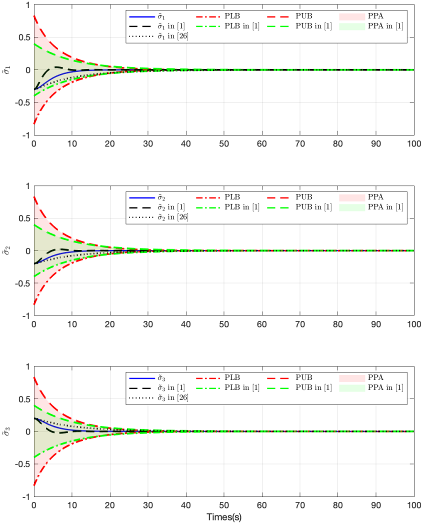
Disturbance Reconstruction



Reconstruction Error



Tracking Error Trajectory



The references are given as,
[1]DOI: 10.1109/TIE.2018.2838065
[26]DOI: 10.1016/j.ast.2012.11.007

Thank for your attention.