International Online Seminar on Interval Methods in Control Engineering

Robust state estimation for switched systems - application to fault detection

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Switched systems

Definition

Switched systems represent a class of complex systems, composed of :

- a finite number of continuous subsystems (modes)
- a logical rule operates switching between subsystems

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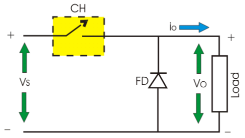
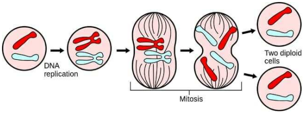
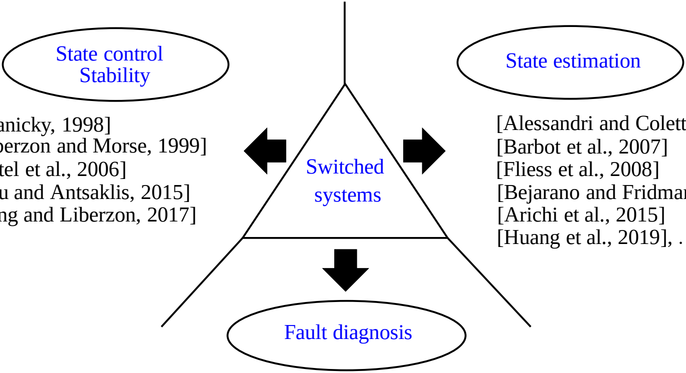
Industrial systems

Switched systems represent an effective tool for describing industrial systems :





Switched systems

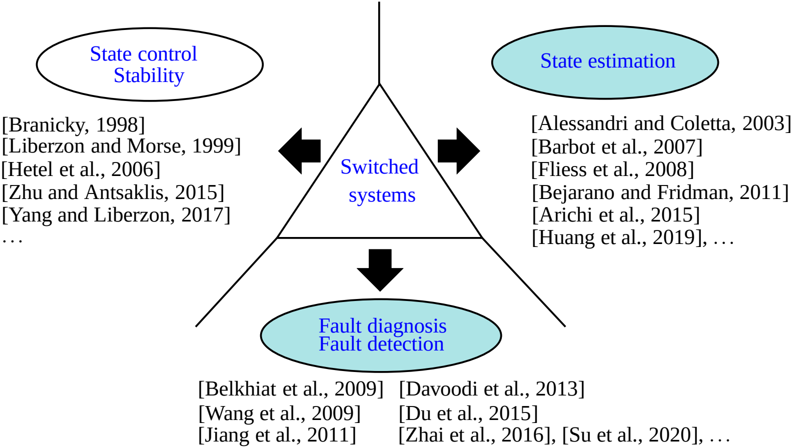


[Branicky, 1998]
[Liberzon and Morse, 1999]
[Hetel et al., 2006]
[Zhu and Antsaklis, 2015]
[Yang and Liberzon, 2017]
...

[Alessandri and Coletta, 2003]
[Barbot et al., 2007]
[Fliess et al., 2008]
[Bejarano and Fridman, 2011]
[Arichi et al., 2015]
[Huang et al., 2019], ...

[Belkhiat et al., 2009] [Davoodi et al., 2013]
[Wang et al., 2009] [Du et al., 2015]
[Jiang et al., 2011] [Zhai et al., 2016], [Su et al., 2020], ...

Switched systems



State estimation

A number of results are available for the estimation problem :

Luenberger Observers	Kalman Filter	Sliding mode observers	Other observers
[Luenberger, 1964]	[Kalman, 1960]	[Levant, 2014]	[Raïssi et al., 2012]
[Jouili et al., 2012]	[Welch et al., 1995]	[Fridman et al., 2008]	[Ríos et al., 2015]
[Rego et al., 2017]	[Yu et al., 2005]	[Van Gorp et al., 2014]	[Chen et al., 2016]
[Lu and Yang, 2017]	[Beccuti et al., 2009]	[Teong Ooi et al., 2015]	[Alhelou et al., 2019]
⋮	⋮	⋮	⋮

Interval estimation

In the presence of uncertainties

- In some cases, point estimation (classical observer) cannot converge to the real states.

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- Interval observers
  - compute the set of admissible values,
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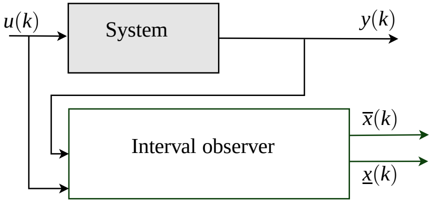

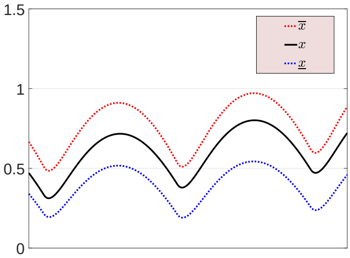

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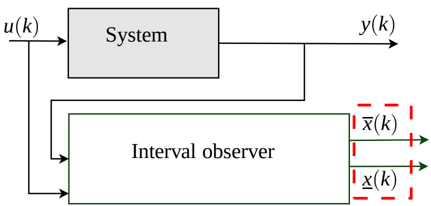

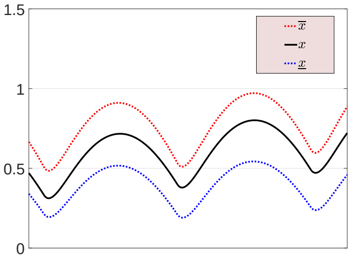

Interval estimation

Bibliography

- Interval estimation, introduced in [Gouzé et al., 2000] and applied for :
  - Linear time-invariant systems [Mazenc and Bernard, 2011]
  - Linear time-varying systems [Efimov et al., 2013]
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  - Nonlinear systems [Meslem et al., 2008]

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  - Nonlinear systems [Meslem et al., 2008]
- Interval estimation for switched systems
  - Continuous time switched systems
    [He and Xie, 2015, Ethabet et al., 2017, Ifqir et al., 2018]
  - Discrete time switched systems
    [Guo and Zhu, 2017, Rabehi et al., 2017, Dinh et al., 2019]

Interval estimation

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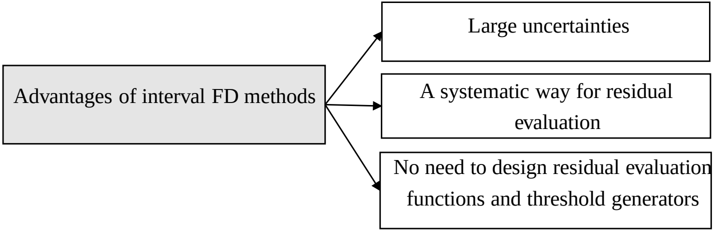
Contribution

The design of a new interval observer for Linear Parameter Varying switched systems subject to measurement noise and state disturbances using a polytopic formulation.

Interval Fault Detection



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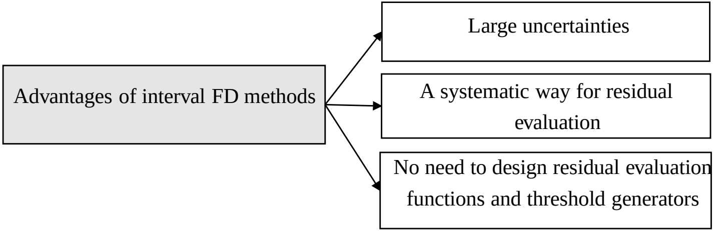

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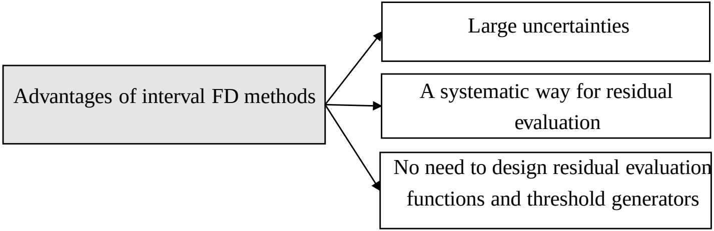

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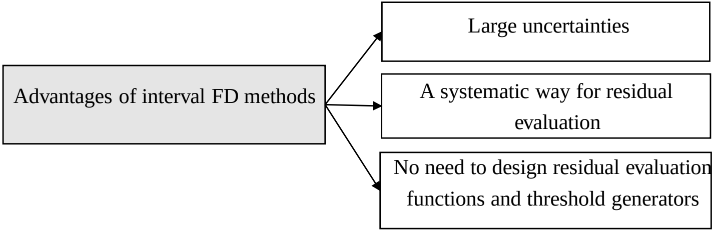
$\Rightarrow$ Some existing results cannot provide accurate FD results.

Interval Fault Detection



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⇒ Some existing results cannot provide accurate FD results.

⇒ Robust fault detection design is needed.

Robustness

H_∞ performances : energy-to-energy

- Simultaneous FD and control for switched linear systems [Zhai et al., 2016]
- Robust FD filter for time-varying delays switched systems [Wang et al., 2016]
- Robust FD observer design for nonlinear systems [Zhou et al., 2017b]

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L_∞ performances : peak-to-peak

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- FD observer for Takagi-Sugeno fuzzy systems [Zhou et al., 2017a]
- L_∞ observer for uncertain linear systems [Han et al., 2019]

Robustness

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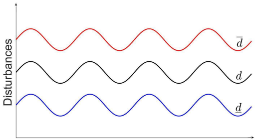
Contribution

An interval method to detect sensor faults for discrete-time switched systems subject to unknown but bounded disturbances is addressed based on the L_∞ formalism.

Set-membership techniques

Intervals



$$\underline{d} \leq d \leq \overline{d}$$

Set-membership techniques

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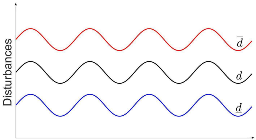

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- Positive system theory
- Cooperativity constraint

Set-membership techniques

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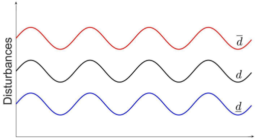

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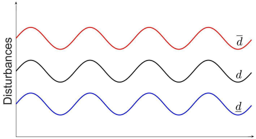

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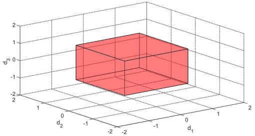
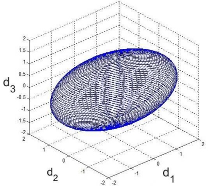


Cooperativity

Accurate FD

$$d \in \langle p, D \rangle,\ D = \text{diag}\left(\frac{\overline{d} - \underline{d}}{2}\right)$$

Ellipsoids



Cooperativity

Accurate FD

$$d \in \mathscr{E}(0, D),\ D = \|d\|_\infty^2 I_{n_d}$$

[Combastel, 2003], [Polyak et al., 2004], [Combastel, 2015], [Tang et al., 2019], ...

Set-membership techniques

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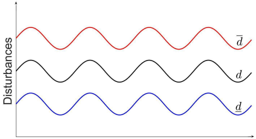

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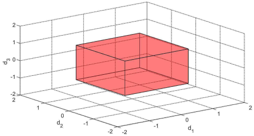

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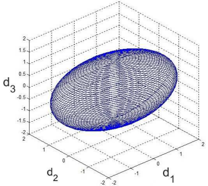

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Contributions

Zonotopic appraoch

- A fault detection method for a class of discrete-time switched systems with actuator faults is proposed.
  - Pole assignment technique to improve fault sensitivity.
  - H_∞ design for the disturbance attenuation.
  - A residual evaluation based on a zonotopic method.

Ellipsoidal analysis

- A robust fault detection method for discrete-time switched systems with sensor faults is achieved.
  - An L_∞ criterion to attenuate the effects of uncertainties.
  - Ellipsoidal analysis for residual evaluation.

Outline

1. Interval estimation for synchronous switched systems
2. Set-membership fault detection frameworks for switched systems
   - Interval based-fault detection method
   - Zonotope based-fault detection method
   - Ellipsoid based-fault detection method
3. Conclusion

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3. Conclusion

LTI systems

System description

Consider the following LTI system :

$$\begin{cases} x_{k+1} & = & Ax_k + \phi_k \\ y_k & = & Cx_k \end{cases}$$

$A \in \mathbb{R}^{n_x \times n_x}$, $\phi \in \mathbb{R}^{n_x}$ and $C \in \mathbb{R}^{n_y \times n_x}$.

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Assumptions

1. There exist two known functions $\overline{\phi}, \underline{\phi} : \mathbb{R} \to \mathbb{R}^{n_x}$ such that $\underline{\phi}_k \leq \phi_k \leq \overline{\phi}_k$.
2. The initial state x_0 satisfies $\underline{x}_0 \leq x_0 \leq \overline{x}_0$ with $\underline{x}_0, \overline{x}_0 \in \mathbb{R}^{n_x}$.
3. There exists a gain L such that $A - LC$ is Schur Stable and Nonnegative.
4. The pair (A, C) is supposed to be detectable.

LTI systems

Goal

Estimate two states : an upper state $\overline{x}_k$ and a lower one $\underline{x}_k$ satisfying :

$$\underline{x}_k \leq x_k \leq \overline{x}_k, \quad k \geq 0$$

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Interval observer structure

$$\begin{cases} \overline{x}_{k+1} = A\overline{x}_k + L(y_k - C\overline{x}_k) + \overline{\phi}_k \\ \underline{x}_{k+1} = A\underline{x}_k + L(y_k - C\underline{x}_k) + \underline{\phi}_k \end{cases}$$

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The interval observer should verify two conditions :

1. Cooperativity : $\underline{x}_k \leq x_k \leq \overline{x}_k, \forall k \geq 0$
2. Stability of $\overline{e}_k = \overline{x}_k - x_k$ and $\underline{e}_k = x_k - \underline{x}_k$

Interval observer design

Cooperative system

Consider a system described by :

$$x_{k+1} = Ax_k + u_k \quad , \quad u : \mathbb{Z}_+ \to \mathbb{R}_+^{n_x},\ k \in \mathbb{Z}_+$$

with $x \in \mathbb{R}^{n_x}$. This system is cooperative or nonnegative if and only if $u_k \geq 0$ for all $k \geq 0$, $x_0 \geq 0$ and A is a nonnegative matrix.

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$$\overline{e}_k = \overline{x}_k - x_k$$
$$\underline{e}_k = x_k - \underline{x}_k$$

$$\overline{e}_{k+1} = (A - LC)\,\overline{e}_k + \overline{\phi}_k - \phi_k$$
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Nonnegative

$$\overline{e}_{k+1} = (A - LC)\overline{e}_k + \overline{\phi}_k - \phi_k$$
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$$\Rightarrow \quad \begin{aligned} \overline{e}_{k+1} &= (A - LC)\overline{e}_k + \overline{\phi}_k - \phi_k \\ \underline{e}_{k+1} &= (A - LC)\underline{e}_k - \underline{\phi}_k + \phi_k \end{aligned}$$

Interval observer design

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Schur Stable

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$$\overline{e}_k = \overline{x}_k - x_k$$
$$\underline{e}_k = x_k - \underline{x}_k$$

Bounded

$$\overline{e}_{k+1} = (A - LC)\,\overline{e}_k + \overline{\phi}_k - \phi_k$$
$$\underline{e}_{k+1} = (A - LC)\,\underline{e}_k - \underline{\phi}_k + \phi_k$$

LPV switched system

System description

Consider the following discrete-time LPV switched system :

$$\begin{cases} x_{k+1} = A_{\sigma(k)}(\eta_{\sigma(k)})x_k + B_{\sigma(k)}(\eta_{\sigma(k)})u_k + W_{\sigma(k)}w_k \\ y_k = Cx_k + v_k \end{cases}, \quad (1)$$

- $W_{\sigma(k)}w_k = w_{\sigma(k)} \in \mathbb{R}^{n_x}$ is the state disturbance.
- $v \in \mathbb{R}^{n_y}$ is the measurement noise.
- $\eta_{\sigma(k)} = [\eta_{q_1}, \ldots, \eta_{q_r}]^T$ the collection of measured time varying parameters.
- $\sigma(k) : \mathbb{Z}_+ \rightarrow \mathscr{I}$ is the index of the active subsystem and assumed to be known.

  $\mathscr{I} = \overline{1,N}, N \in \mathbb{Z}_+, N$ is the number of subsystems.

Assumptions

$q = \sigma(k) \in \mathscr{I}$

1. $A_q(\eta_q)$, $B_q(\eta_q)$ depend affinely on η_q :

$$\begin{array}{l} A_q(\eta_q) = A_{q0} + \eta_{q1} A_{q1} + \ldots + \eta_{qr} A_{qr} \\ B_q(\eta_q) = B_{q0} + \eta_{q1} B_{q1} + \ldots + \eta_{qr} B_{qr} \end{array} , q \in \mathscr{I}$$

2. The initial state x_0 satisfies $\underline{x}_0 \leq x_0 \leq \overline{x}_0$ with $\underline{x}_0, \overline{x}_0 \in \mathbb{R}^{n_x}$.
3. The measurement noise and the state disturbance are unknown but bounded :

$$\underline{w}_q \leq w_q \leq \overline{w}_q, \quad |v| \leq \overline{v} J_{n_y} \quad , q \in \mathscr{I}$$

4. $\eta_q = [\eta_{q_1}, \ldots, \eta_{q_r}]^T$ are constrained in polytopes E_q. We denote by $\eta_q^{(i)}$, $i = 1, \ldots, g$ the vertices of each E_q.
5. For all vertices of E_q and for all $q \in \mathscr{I}$, the pairs $(A_q(\eta_q^{(i)}), C)$ are detectable.

Interval observer

Goal

The aim is to design an interval observer for discrete-time LPV switched systems defined by (1) using a polytopic representation.

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Interval observer structure

$$\begin{cases} \overline{x}_{k+1} = \big(A_q(\eta_q) - L_q(\eta_q)C\big)\,\overline{x}_k + B_q(\eta_q)u_k + \overline{w}_q + L_q(\eta_q)y_k + |L_q(\eta_q)|\overline{v}J_{n_y} \\ \underline{x}_{k+1} = \big(A_q(\eta_q) - L_q(\eta_q)C\big)\,\underline{x}_k + B_q(\eta_q)u_k + \underline{w}_q + L_q(\eta_q)y_k - |L_q(\eta_q)|\overline{v}J_{n_y} \end{cases}, q \in \mathscr{I}$$

Interval observer

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The aim is to design an interval observer for discrete-time LPV switched systems defined by (1) using a polytopic representation.

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The interval observer should verify two conditions :

1. Cooperativity : $\underline{x}_k \leq x_k \leq \overline{x}_k, \forall k \geq 0$
2. Stability of $\overline{e}_k = \overline{x}_k - x_k$ and $\underline{e}_k = x_k - \underline{x}_k$

Cooperativity

- Define the estimation errors $\overline{e}_k = \overline{x}_k - x_k$ and $\underline{e}_k = x_k - \underline{x}_k$.
- The dynamics of the estimation errors are given by :

$$\begin{cases} \overline{e}_{k+1} &= (A_q(\eta_q) - L_q(\eta_q)C)\overline{e}_k + \overline{\chi}_q \\ \underline{e}_{k+1} &= (A_q(\eta_q) - L_q(\eta_q)C)\underline{e}_k + \underline{\chi}_q \end{cases} \qquad \begin{cases} \overline{\chi}_q &= \overline{w}_q - w_q + L_q(\eta_q)v + |L_q(\eta_q)|\overline{v}J_{n_y} \\ \underline{\chi}_q &= w_q - \underline{w}_q - L_q(\eta_q)v + |L_q(\eta_q)|\overline{v}J_{n_y} \end{cases}$$

Convex Form [1]

- $A_q(\eta_q)$ and $L_q(\eta_q)$ depend affinely of η_q. They can be written as a convex combination

$\Rightarrow$ $A_q(\eta_q) = \sum_{i=1}^{g} \lambda_i A_q(\eta_q^{(i)})$ $\quad A_q(\eta_q{}^{(i)})$ the vertices of the state matrices of each polytope E_q.

$\Rightarrow$ $L_q(\eta_q) = \sum_{i=1}^{g} \lambda_i L_q(\eta_q^{(i)})$ $\quad L_q(\eta_q{}^{(i)})$ the vertices of the observer gain of each polytope E_q.

1. [Hetel et al., 2006]

Cooperativity

The error dynamics

$$\begin{cases} \overline{e}_{k+1} = (A_q(\eta_q) - L_q(\eta_q)C)\overline{e}_k + \overline{\chi}_q = \sum_{i=1}^{g} \lambda_i \Big(A_q(\eta_q{}^{(i)}) - L_q(\eta_q{}^{(i)})C \Big) \overline{e}_k + \overline{\chi}_q \\ \underline{e}_{k+1} = (A_q(\eta_q) - L_q(\eta_q)C)\underline{e}_k + \underline{\chi}_q = \sum_{i=1}^{g} \lambda_i \Big(A_q(\eta_q{}^{(i)}) - L_q(\eta_q{}^{(i)})C \Big) \underline{e}_k + \underline{\chi}_q \end{cases}$$

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$$\begin{cases} \overline{e}_{k+1} = (A_q(\eta_q) - L_q(\eta_q)C)\overline{e}_k + \overline{\chi}_q = \sum_{i=1}^{g} \lambda_i \Big(A_q(\eta_q{}^{(i)}) - L_q(\eta_q{}^{(i)})C \Big) \overline{e}_k + \overline{\chi}_q \\ \underline{e}_{k+1} = (A_q(\eta_q) - L_q(\eta_q)C)\underline{e}_k + \underline{\chi}_q = \sum_{i=1}^{g} \lambda_i \Big(A_q(\eta_q{}^{(i)}) - L_q(\eta_q{}^{(i)})C \Big) \underline{e}_k + \underline{\chi}_q \end{cases}$$

Proof

- $A_q(\eta_q) - L_q(\eta_q)C \geq 0 \Longleftrightarrow A_q(\eta_q^{(i)}) - L_q(\eta_q^{(i)})C \geq 0$.
- $\overline{\chi}_q = \overline{w}_q - w_q + L_q(\eta_q)v + |L_q(\eta_q)|\overline{v}J_{n_y} \geq 0.$
- $\overline{e}_0 = \overline{x}_0 - x_0 \Rightarrow \overline{e}_k \geq 0$, for all $k \geq 0 \Rightarrow \overline{x}_k \geq x_k$, for all $k \geq 0$.
- Same arguments to prove that $\underline{x}_k \leq x_k$.
- Then, the following inclusion is satisfied :

$$\underline{x}_k \leq x_k \leq \overline{x}_k$$

Stability

Definition

The stability analysis is the property of a system to return to its equilibrium point after it has been deviated from its initial position.

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Input to State Stability (ISS)

When systems are subject to disturbances and noises, the ISS can be introduced.

- The ISS property provides a natural framework about stability with respect to input perturbations.
- A system should be bounded if bounded inputs are injected and should converge to equilibrium when inputs tend to zero.

Stability

Lemma : Input to State Stability (ISS) [2]

Consider the switched system (1), and let $0 < \alpha < 1$, $\mu > 1$. Suppose that there exist $V_{\sigma_k} : \mathbb{R}^{n_x} \rightarrow \mathbb{R}$ and two $\mathscr{K}_\infty$ functions α_1 and α_2 such that for each $\sigma_k = q, q \neq l$, if the following conditions hold :

$$\alpha_1(\|e_k\|_2) \leq V_q(e_k) \leq \alpha_2(\|e_k\|_2)$$

$$\Delta V_q(e_k) \leq -\alpha V_q(e_k)$$

$$V_q(e_k) \leq \mu V_l(e_k)$$

then, the error system is ISS for any switching signal with an ADT $\tau_a \geq \tau_a^* = -\frac{ln(\mu)}{ln(1-\alpha)}$.
τ_a^* is the lower bound of τ_a determined by both parameters α and μ.

2. [Zhu et al., 2018]

Stability

Stability of each subsystem

The stability of each subsystem is satisfied based on :

$$\Delta V_q(\overline{e}_k) \leq -\alpha V_q(\overline{e}_k) \Rightarrow \Delta V_q(\overline{e}_k) < -\alpha \overline{e}_k^T P_q \overline{e}_k + \gamma^2 \|\overline{\chi}_q(k)\|_2^2$$

LMI-based problem

$$\begin{bmatrix} -(1-\alpha)P_q & 0 & A_q(\eta_q^{(i)})^T P_q - C^T Q_q(\eta_q^{(i)})^T \\ (*) & -\gamma^2 I_n & P_q \\ (*) & (*) & -P_q \end{bmatrix} \preceq 0$$

- A scalar $0 < \alpha < 1$, a positive definite matrix $P_q \in \mathbb{R}^{n_x \times n_x}$.

Stability

Stability at the switching instants

The stability at the switching instants is satisfied based on :

$$V_q(\overline{e}_k) \leq \mu V_l(\overline{e}_k)$$

LMI-based problem

$$\begin{bmatrix} W_l & P_q \\ P_q & P_q \end{bmatrix} \succeq 0$$

- where $W_l = \mu P_l$
- A scalar $\mu > 1$.

Stability

Bounded interval error width

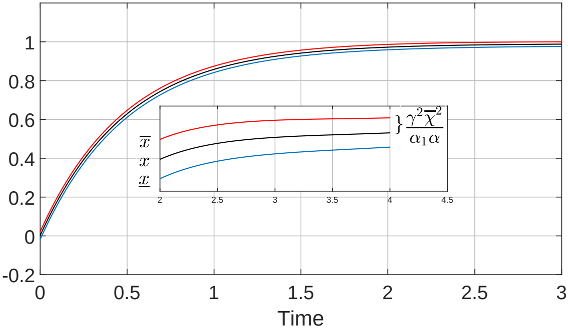

FIGURE – $\lim_{k\to\infty} \|\overline{e}_k\|_2 < \frac{\gamma^2}{\alpha_1 \alpha} \overline{\chi}^2$

Stability

Remark

The interval error width is upper bounded by $\frac{\gamma^2}{\alpha_1\alpha}\overline{\chi}^2$ which should be made as small as possible to enhance the performance of the proposed observer.

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- The minimization of γ allows reducing the upper estimation error.
- The minimization of μ allows looking for an optimum dwell time.

The optimum solution can be obtained by minimizing the objective function :

$$\beta\mu + (1-\beta)\gamma$$

The weight β is in the range $[0,1]$

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Remark

The same arguments show the boundedness of the lower estimation error $\underline{e_k}$

Simulation results

A discrete-time LPV switched system defined with three subsystems, $N = 3$ is considered.

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A discrete-time LPV switched system defined with three subsystems, $N = 3$ is considered.

- $x = [x_1, x_2]^T \in \mathbb{R}^2$ is the state vector.
- The state initial conditions are set as $x_0 = [1,1]^T$, $\underline{x}_0 = [-2,-2]^T$ and $\overline{x}_0 = [2,2]^T$ such that :

$$\underline{x}_0 \leq x_0 \leq \overline{x}_0$$

- $y \in \mathbb{R}$ is the output.
- $u = [1,1]^T \in \mathbb{R}^2$ is the known input.
- The measured time varying parameters $\eta_q = [\eta_{q1}, \eta_{q2}]^T$ for $q = 1,2,3$ are defined by :

$$\eta_1 = \begin{bmatrix} 0.5(|sin(0.1k)|+1) \\ 0.5(|cos(0.1k)|+1) \end{bmatrix}, \eta_2 = \begin{bmatrix} 1.5|sin(0.1k)|+0.5 \\ 1.5|cos(0.1k)|+0.5 \end{bmatrix}, \eta_3 = \begin{bmatrix} 2.5|cos(0.1k)|+0.5 \\ 2.5|sin(0.1k)|+0.5 \end{bmatrix}$$

- v is a uniformly distributed signal bounded by $\overline{v} = 0.5$.
- $w_q \in \mathbb{R}^2$ for $q = 1,2,3$ is the vector of disturbance with :

$$w_1 = [0.9 \quad 0.8]^T sin(0.1k) \quad w_2 = [0.6 \quad 0.7]^T sin(0.2k) \quad w_3 = [0.9 \quad 0.8]^T sin(0.3k)$$

Simulation results

Using the Matlab LMI toolbox Yalmip/Sedumi

- The parameters $\alpha = 0.9$, $\alpha_1 = 2$ are chosen to solve the optimization problem.
- The following Lyapunov matrices are obtained :

$$P_1 = \begin{bmatrix} 2.71 & 0 \\ 0 & 2.43 \end{bmatrix}, P_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, P_3 = \begin{bmatrix} 2.48 & 0 \\ 0 & 2.58 \end{bmatrix}$$

- The observer gains L_q are given by :

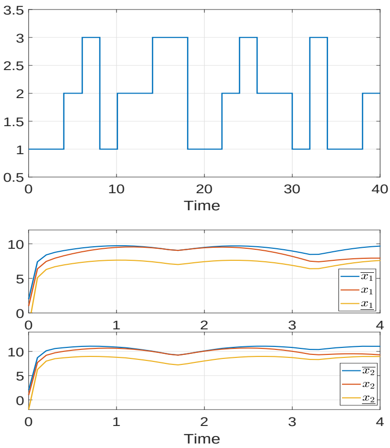
$$L_{10} = \begin{bmatrix} -0.0035 & 0 \end{bmatrix}^T, L_{11} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, L_{12} = \begin{bmatrix} 0 & -0.045 \end{bmatrix}^T$$

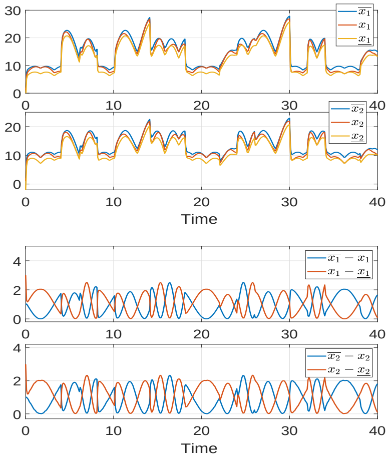
$$L_{20} = \begin{bmatrix} -0.0263 & 0 \end{bmatrix}^T, L_{21} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, L_{22} = \begin{bmatrix} 0 & 0.0130 \end{bmatrix}^T$$

$$L_{30} = \begin{bmatrix} 0.0118 & 0 \end{bmatrix}^T, L_{31} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, L_{32} = \begin{bmatrix} 0 & -0.0100 \end{bmatrix}^T$$

- $\mu = 2 \Rightarrow \tau_a > 0.301$.
- $\gamma = 2.12$.

Simulation results





Outline

1 Interval estimation for synchronous switched systems

2 Set-membership fault detection frameworks for switched systems
- Interval based-fault detection method
- Zonotope based-fault detection method
- Ellipsoid based-fault detection method

3 Conclusion

Switched system

System description

Consider the following discrete-time switched system :

$$\begin{cases} x_{k+1} = A_q x_k + B_q u_k + D_q w_k \\ y_k = C x_k + D_v v_k + F f_k, \end{cases} \quad (2)$$

- The known matrices A_q, B_q, C, D_q, D_v and F are given with appropriate dimensions.

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- The known matrices A_q, B_q, C, D_q, D_v and F are given with appropriate dimensions.

Assumptions

1. The measurement noise and the state disturbance are unknown but bounded :

$$\underline{w} \leq w \leq \overline{w}, \quad \underline{v} \leq v \leq \overline{v}$$

2. The initial state x_0 satisfies $\underline{x}_0 \leq x_0 \leq \overline{x}_0$ with $\underline{x}_0, \overline{x}_0 \in \mathbb{R}^{n_x}$.
3. The pairs (A_q, C) are detectable, $\forall q = 1, \ldots, N$.

Interval observer structure

The FD interval observer is proposed as follows :

$$\begin{cases} \overline{\xi}_{k+1} &= \overline{T}_q A_q \overline{x}_k + \overline{T}_q B_q u_k + \overline{L}_q (y_k - C\overline{x}_k) + \overline{\Delta} \\ \overline{x}_k &= \overline{\xi}_k + \overline{N}_q y_k \\ \underline{\xi}_{k+1} &= \underline{T}_q A_q \underline{x}_k + \underline{T}_q B_q u_k + \underline{L}_q (y_k - C\underline{x}_k) + \underline{\Delta} \\ \underline{x}_k &= \underline{\xi}_k + \underline{N}_q y_k \\ \overline{y}_k &= C^+ \overline{x}_k - C^- \underline{x}_k + D_v{}^+ \overline{v} - D_v{}^- \underline{v} \\ \underline{y}_k &= C^+ \underline{x}_k - C^- \overline{x}_k + D_v{}^+ \underline{v} - D_v{}^- \overline{v} \\ \overline{r}_k &= \overline{y}_k - y_k \\ \underline{r}_k &= \underline{y}_k - y_k \end{cases}$$

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\end{cases}
$$

- $\overline{\xi}_k,\ \underline{\xi}_k \in \mathbb{R}^{n_x}$ are intermediate variables.
- $\overline{x}_k,\ \underline{x}_k \in \mathbb{R}^{n_x}$ are the estimated upper and lower bounds of x_k.
- $\underline{\Delta}$ and $\overline{\Delta}$ are bounded and given by :

$$
\begin{cases}
\overline{\Delta} = (\overline{T}_q D_q)^+ \overline{w} - (\overline{T}_q D_q)^- \underline{w} + (\overline{L}_q D_v)^+ \overline{v} - (\overline{L}_q D_v)^- \underline{v} + (\overline{N}_q D_v)^+ \overline{v} - (\overline{N}_q D_v)^- \underline{v} \\
\underline{\Delta} = (\underline{T}_q D_q)^+ \underline{w} - (\underline{T}_q D_q)^- \overline{w} + (\underline{L}_q D_v)^+ \underline{v} - (\underline{L}_q D_v)^- \overline{v} + (\underline{N}_q D_v)^+ \underline{v} - (\underline{N}_q D_v)^- \overline{v}.
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- $\overline{L}_q \in \mathbb{R}^{n_x \times n_y}$ and $\underline{L}_q \in \mathbb{R}^{n_x \times n_y}$ are the observer gains.
- $\overline{T}_q \in \mathbb{R}^{n_x \times n_x}$, $\underline{T}_q \in \mathbb{R}^{n_x \times n_x}$, $\overline{N}_q \in \mathbb{R}^{n_x \times n_y}$ and $\underline{N}_q \in \mathbb{R}^{n_x \times n_y}$ are constant matrices that should be designed to satisfy

$$\begin{aligned} \overline{T}_q + \overline{N}_q C &= I_{n_x} \\ \underline{T}_q + \underline{N}_q C &= I_{n_x} \end{aligned}$$

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- When $\overline{T}_q = I_{n_x}$, $\underline{T}_q = I_{n_x}$, $\overline{N}_q = O_{n_x}$ and $\underline{N}_q = O_{n_x}$, the proposed interval observer is equivalent to the classical interval observer.

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- In the case of classical interval observers, the gain matrices $\overline{L}_q$ and $\underline{L}_q$ should be designed such that $A_q - \overline{L}_q C$ and $A_q - \underline{L}_q C$ are nonnegative $\forall q = 1, \ldots, N$.

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- This Assumption is restrictive and conservative.

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- This Assumption is restrictive and conservative.
- By introducing weighted matrices $\overline{T}_q$, $\underline{T}_q$, $\overline{N}_q$ and $\underline{N}_q$, the proposed FD observer can reduce the conservatism of gain matrices and provide more degree of freedom.

Error dynamics

The error dynamics

Let $\overline{e}_k = \overline{x}_k - x_k$ and $\underline{e}_k = x_k - \underline{x}_k$.

$$\begin{cases} \overline{e}_{k+1} &= (\overline{T}_q A_q - \overline{L}_q C)\overline{e}_k + \overline{H}_q \overline{d}_k + \overline{F}_q \tilde{f}_k \\ \underline{e}_{k+1} &= (\underline{T}_q A_q - \underline{L}_q C)\underline{e}_k + \underline{H}_q \underline{d}_k + \underline{F}_q \tilde{f}_k, \end{cases}$$

$$\overline{H}_q = \begin{bmatrix} I_n \\ \overline{L}_q{}^T \\ \overline{N}_q{}^T \end{bmatrix}^T, \underline{H}_q = \begin{bmatrix} I_n \\ \underline{L}_q{}^T \\ \underline{N}_q{}^T \end{bmatrix}^T, \overline{F}_q = \begin{bmatrix} (\overline{L}_q F)^T \\ (\overline{N}_q F)^T \end{bmatrix}^T, \underline{F}_q = \begin{bmatrix} -(\underline{L}_q F)^T \\ -(\underline{N}_q F)^T \end{bmatrix}^T,$$

$$\overline{d}_k = \begin{bmatrix} \overline{\Delta} - \overline{T}_q D_q w_k \\ D_v v_k \\ D_v v_{k+1} \end{bmatrix}, \underline{d}_k = \begin{bmatrix} -\underline{\Delta} + \underline{T}_q D_q w_k \\ -D_v v_k \\ -D_v v_{k+1} \end{bmatrix}, \tilde{f}_k = \begin{bmatrix} f_k \\ f_{k+1} \end{bmatrix}.$$

Nonnegativity

Theorem

For the system (2), let Assumptions 1-3 be satisfied. Then, the relation $\underline{x}_k \leq x_k \leq \overline{x}_k$ holds in fault free case ($f = 0$) for all $k \geq 0$ if $\overline{T}_q A_q - \overline{L}_q C$ and $\underline{T}_q A_q - \underline{L}_q C$ are nonnegative.

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Proof

- In fault free case ($f = 0$),

$$\begin{cases} \overline{e}_{k+1} &= (\overline{T}_q A_q - \overline{L}_q C)\overline{e}_k + \overline{\Delta} + \overline{L}_q D_v v_k + \overline{N}_q D_v v_{k+1} - \overline{T}_q D_q w_k \\ \underline{e}_{k+1} &= (\underline{T}_q A_q - \underline{L}_q C)\underline{e}_k - \underline{\Delta} - \underline{L}_q D_v v_k - \underline{N}_q D_v v_{k+1} + \underline{T}_q D_q w_k. \end{cases}$$

Nonnegativity

Theorem

For the system (2), let Assumptions 1-3 be satisfied. Then, the relation $\underline{x}_k \leq x_k \leq \overline{x}_k$ holds in fault free case ($f = 0$) for all $k \geq 0$ if $\overline{T}_q A_q - \overline{L}_q C$ and $\underline{T}_q A_q - \underline{L}_q C$ are nonnegative.

Proof

- In fault free case ($f = 0$),

$$\begin{cases} \overline{e}_{k+1} = (\overline{T}_q A_q - \overline{L}_q C)\overline{e}_k + \overline{\Delta} + \overline{L}_q D_v v_k + \overline{N}_q D_v v_{k+1} - \overline{T}_q D_q w_k \\ \underline{e}_{k+1} = (\underline{T}_q A_q - \underline{L}_q C)\underline{e}_k - \underline{\Delta} - \underline{L}_q D_v v_k - \underline{N}_q D_v v_{k+1} + \underline{T}_q D_q w_k. \end{cases}$$

- $\overline{T}_q A_q - \overline{L}_q C$ and $\underline{T}_q A_q - \underline{L}_q C$ are nonnegative. According to Assumption 1,

$$\begin{aligned} \overline{\Delta} - \overline{T}_q D_q w_k + \overline{L}_q D_v v_k + \overline{N}_q D_v v_{k+1} &\geq 0 \\ -\underline{\Delta} + \underline{T}_q D_q w_k - \underline{L}_q D_v v_k - \underline{N}_q D_v v_{k+1} &\geq 0. \end{aligned}$$

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- $\overline{e}_0 \geq 0$ and $\underline{e}_0 \geq 0 \Rightarrow \overline{e}_k \geq 0$ and $\underline{e}_k \geq 0 \Rightarrow \underline{x}_k \leq x_k \leq \overline{x}_k$.

Stability

In order to study the stability of the residual framers, a new augmented state is proposed.

Augmented system

Let $\mathscr{E}_k = [\overline{e}_k{}^T \quad \underline{e}_k{}^T]^T$ and $\mathscr{R}_k = [\overline{r}_k{}^T \quad \underline{r}_k{}^T]^T$. The following augmented system can be deduced :

$$\begin{cases} \mathscr{E}_{k+1} = \mathscr{A}_q \mathscr{E}_k + \mathscr{H}_q d_k + \tilde{\mathscr{F}}_q \tilde{f}_k \\ \mathscr{R}_k = \mathscr{C}\mathscr{E}_k + \mathscr{V}\tilde{v}_k + \mathscr{F} f_k, \end{cases}$$

$$\mathscr{A}_q = \begin{bmatrix} \overline{T}_q A_q - \overline{L}_q C & 0 \\ 0 & \underline{T}_q A_q - \underline{L}_q C \end{bmatrix}, \mathscr{H}_q = \begin{bmatrix} \overline{H}_q & 0 \\ 0 & \underline{H}_q \end{bmatrix}, \tilde{\mathscr{F}}_q = \begin{bmatrix} \overline{F}_q \\ \underline{F}_q \end{bmatrix}, d_k = \begin{bmatrix} \overline{d}_k \\ \underline{d}_k \end{bmatrix},$$

$$\mathscr{F} = \begin{bmatrix} -F \\ -F \end{bmatrix}, \mathscr{C} = \begin{bmatrix} C^+ & C^- \\ -C^- & -C^+ \end{bmatrix}, \mathscr{V} = \begin{bmatrix} -D_v & D_v{}^+ & -D_v{}^- \\ -D_v & -D_v{}^- & D_v{}^+ \end{bmatrix}, \tilde{v}_k = \begin{bmatrix} v_k \\ \overline{v} \\ \underline{v} \end{bmatrix}.$$

Error dynamics

The error dynamics

$$\begin{cases} \mathscr{E}_{k+1} & = \mathscr{A}_q \mathscr{E}_k + \mathscr{H}_q d_k + \tilde{\mathscr{F}}_q \tilde{f}_k \\ \mathscr{R}_k & = \mathscr{C} \mathscr{E}_k + \mathscr{V} \tilde{v}_k + \mathscr{F} f_k, \end{cases}$$

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A subsystem decoupled from the effects of the sensor fault

$$\begin{cases} \mathscr{E}^d_{k+1} &= \mathscr{A}_q \mathscr{E}^d_k + \mathscr{H}_q d_k \\ \mathscr{R}^d_k &= \mathscr{C} \mathscr{E}^d_k + \mathscr{V} \tilde{v}_k, \end{cases}$$

Error dynamics

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A subsystem affected by the sensor fault

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$\mathscr{E}_k = \mathscr{E}^f_k + \mathscr{E}^d_k.$

Stability and L_∞ performances

Stability and L_∞ disturbance attenuation condition

The aim is to compute the FD observer gains $\overline{L}_q$ and $\underline{L}_q$ such that the following conditions hold :

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The aim is to compute the FD observer gains $\overline{L}_q$ and $\underline{L}_q$ such that the following conditions hold :

1. The error system is stable.
2. Given scalars $\gamma > 0$, $\gamma_1 > 0$, $\gamma_2 > 0$ and $0 < \lambda < 1$, then residual signal should satisfy the following L_∞ performance

$$||\mathscr{R}^d|| < \sqrt{\gamma_1^2(\gamma(\lambda(1-\lambda)^k V_0 + \gamma\theta_d^2)) + \gamma_2^2\theta_v^2},$$

- $V_0 = {\mathscr{E}_0^d}^T P_q \mathscr{E}_0^d$
- $P_q \in \mathbb{R}^{2n_x \times 2n_x}$
- θ_d and θ_v are known constants and represent the L_∞ of d and $\tilde{v}$ such that $\theta_d = \|d\|_\infty$ and $\theta_v = \|\tilde{v}\|_\infty$.

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Sufficient conditions are given in terms of LMIs.

Residual evaluation

FD decision

The corresponding FD decision scheme is made as follows :

$$\begin{cases} 0 \in [\underline{r}_k \quad \overline{r}_k] & \text{Fault-free} \\ 0 \notin [\underline{r}_k \quad \overline{r}_k] & \text{Faulty} \end{cases}$$

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Fault free case

In the fault free case, the output signal is consistent with the estimation of the proposed interval observer.

$$y_k \in [\underline{y}_k \quad \overline{y}_k] \Rightarrow 0 \in [\underline{y}_k - y_k \quad \overline{y}_k - y_k] \Rightarrow 0 \in [\underline{r}_k \quad \overline{r}_k]$$

Residual evaluation

Faulty case

In contrary case, an inconsistency on the output signal is detected and it indicates the existence of a fault.

$$y_k \notin [\underline{y}_k \quad \overline{y}_k] \Rightarrow 0 \notin [\underline{y}_k - y_k \quad \overline{y}_k - y_k] \Rightarrow 0 \notin [\underline{r}_k \quad \overline{r}_k]$$

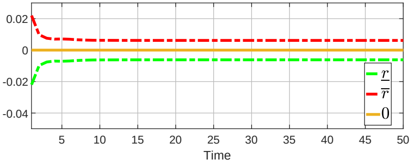

FIGURE – Fault-free case

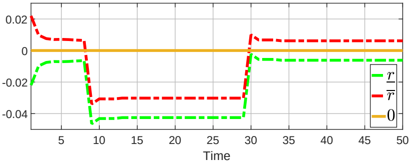

FIGURE – Faulty case

Simulation results

A discrete-time switched system defined with three subsystems, $N = 3$ is considered.

- The state initial conditions are set as
  - $x_0 = [0 \quad 0 \quad 0]^T$
  - $\underline{x}_0 = [-0.1 \quad -0.1 \quad -0.1]^T$
  - $\overline{x}_0 = [0.1 \quad 0.1 \quad 0.1]^T$.
- $w_k \in \mathbb{R}$ and $v_k \in \mathbb{R}^2$ are uniformly distributed signals such that :
  - $|w_k| \leq 1$.
  - $|v_k| \leq [0.1 \quad 0.1]$.
- The numerical simulation was carried out using Matlab optimization tools (Yalmip/Sedumi).
- FD results are given using Multiple Quadratic Lyapunov Functions.

Simulation results

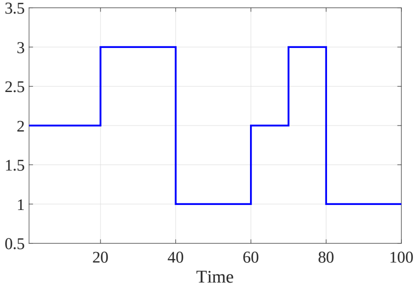

FIGURE – Evolution of the switching signal.

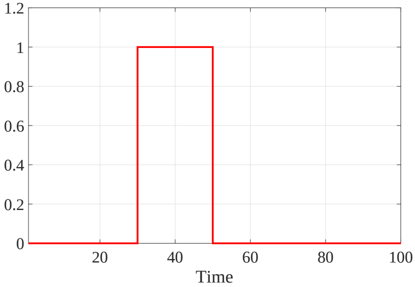

FIGURE – Evolution of the fault.

Simulation results

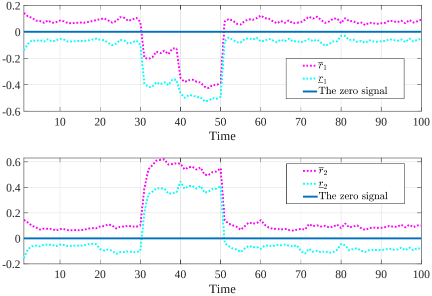

FIGURE – Residual framers using fault detection TNL interval observer.

- In the fault free case, the cooperativity property is ensured, $0 \in [\underline{r}_k \quad \overline{r}_k]$.
- When a fault occurs ($k = 30$), the fault is detected at the time instant $k = 31$ and $0 \notin [\underline{r}_k \quad \overline{r}_k]$.

Simulation results

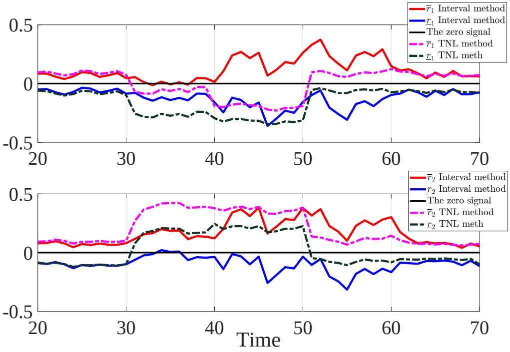

FIGURE – FD performance comparison between the TNL and the interval approaches (Small fault).

- The fault can be detected based on the TNL technique which is not the case when using the classical interval approach.

Interval vs zonotopic techniques

Interval based-fault detection method
(TNL structure)

Interval vs zonotopic techniques

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↓

High computational efficiency

Easy to implement

Interval vs zonotopic techniques

Interval based-fault detection method
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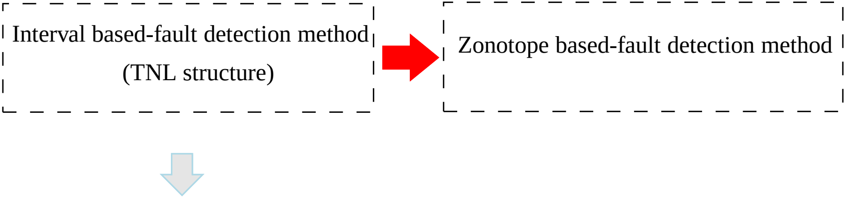
Strict design conditions

Nonnegativity of $\overline{T}_q A_q - \overline{L}_q C$ and $\underline{T}_q A_q - \underline{L}_q C$

Interval vs zonotopic techniques



High computational efficiency

Easy to implement

Strict design conditions

Nonnegativity of $\overline{T}_q A_q - \overline{L}_q C$ and $\underline{T}_q A_q - \underline{L}_q C$

Interval vs zonotopic techniques

Interval based-fault detection method (TNL structure) → Zonotope based-fault detection method

High computational efficiency

Easy to implement

Strict design conditions

Nonnegativity of $\overline{T}_q A_q - \overline{L}_q C$ and $\underline{T}_q A_q - \underline{L}_q C$

Accurate Fault Detection results

Relaxed design conditions

Problem statement

System description

Consider the following discrete-time switched system :

$$\begin{cases} x_{k+1} = A_q x_k + B_q u_k + D w_k + F_q f_k \\ y_k = C x_k + D_v v_k \end{cases}$$

Objective

- The design of a FD approach for the discrete-time switched systems
  - robust against disturbances (H_∞ criterion)
  - sensitive to fault (Pole assignment)
- The residual evaluation is achieved based on
  - zonotopic approaches

Preliminaries

Definition

An s-order zonotope **Z** is the affine image of a hypercube $\mathbb{B}^s = [-1, 1]^s$ as follows :

$$\mathbf{Z} = \langle p, H \rangle = p + H\mathbb{B}^s = \{p + Hz,\, z \in \mathbb{B}^s\}$$

where $p \in \mathbb{R}^n$ is the center of **Z** and $H \in \mathbb{R}^{n \times s}$ denotes the generation matrix of **Z**.

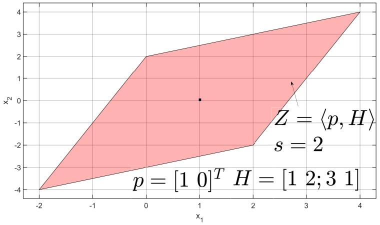
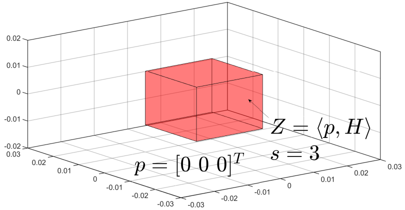


FIGURE – 3-zonotope in a two dimension space



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Assumption

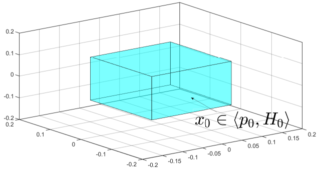

FIGURE – $x_0 \in \langle p_0, H_0 \rangle$, $H_0 = \text{diag}(\bar{x})$

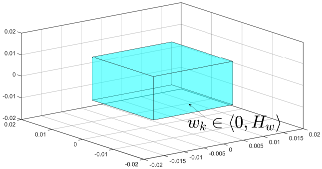

FIGURE – $w_k \in \langle 0, H_w \rangle$, $H_w = \text{diag}(\overline{w})$

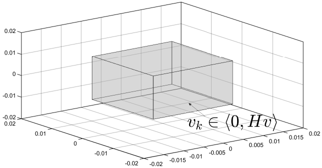

FIGURE – $v_k \in \langle 0, H_v \rangle$, $H_v = \text{diag}(\bar{v})$

Problem statement

Fault detection observer design

FDO structure

$$\begin{cases} \hat{x}_{k+1} = A_q \hat{x}_k + B_q u_k + L_q (y_k - C\hat{x}_k) \\ r_k = y_k - C\hat{x}_k \end{cases}$$

- $\hat{x}_k$ is the estimation of x_k
- $L_q \in \mathbb{R}^{n_x \times n_y}$ are the observer gains.

Objective

Compute the FDO gains L_q :

- sensitive to fault (Pole assignment)
- robust against disturbances (H_∞ criterion)

Error dynamics

The error dynamics

$$\begin{cases} e_{k+1} & = & (A_q - L_q C)e_k + F_q f_k + D w_k - L_q D_v v_k \\ r_k & = & C e_k + D_v v_k \end{cases}$$

Error dynamics

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A subsystem decoupled from the effects of f_k

$$\begin{cases} e^d_{k+1} = (A_q - L_q C)e^d_k + D w_k - L_q D_v v_k \\ r^d_k = C e^d_k + D_v v_k \end{cases}$$

A subsystem affected by the actuator fault

$$\begin{cases} e^f_{k+1} = (A_q - L_q C)e^f_k + F_q f_k \\ r^f_k = C e^f_k \end{cases}$$

- where $e_k = e^f_k + e^d_k$, $e^f_0 = 0$ and $e^d_0 = 0$.

Fault sensitivity condition

FDO gains

The FDO gains L_q are designed to improve fault sensitivity on residual signal such that :

$$(A_q - L_q C)F_q = \lambda F_q \tag{3}$$

where λ is a scalar satisfying $0 < \lambda < 1$.
If the condition (3) holds, it follows that :

$$\begin{aligned} e_k^f &= \lambda^{k-1} F_q f_0 + \ldots + \lambda F_q f_{k-2} + F_q f_{k-1} \\ r_k^f &= \lambda^{k-1} C F_q f_0 + \ldots + \lambda C F_q f_{k-2} + C F_q f_{k-1} \end{aligned}$$

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Remark

- The residual signal r_k depends on a weighting scalar λ.
- $\Rightarrow$ It is required to adjust the value of λ in order to improve fault sensitivity.
- $\Rightarrow$ A pole assignment method is proposed.

Fault sensitivity condition

Lemma 1 [Ben-Israel and Charnes, 1963]

Given matrices $A \in \mathbb{R}^{a\times b}$, $B \in \mathbb{R}^{b\times c}$ and $C \in \mathbb{R}^{a\times c}$, if $\text{rank}(B) = c$, then the general solution of the $AB = C$ is

$$A = CB^{\dagger} + S(I - BB^{\dagger})$$

where $S \in \mathbb{R}^{a\times b}$ is an arbitrary matrix.

FDO gains

The FDO gains L_q can be obtained by solving (3) :

$$L_q \quad = \quad (A_qF_q - \lambda F_q)(CF_q)^{\dagger} + S(I - CF_q(CF_q)^{\dagger})$$

where $S \in \mathbb{R}^{n_x\times n_y}$ is a matrix to be designed.

Disturbance attenuation condition

H_∞ method

The performance $\|r^d\| < \gamma\sqrt{(\|w\|^2 + \|v\|^2)}$ is considered.

Disturbance attenuation condition

H_∞ method

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LMI-based optimization problem

To prove the stability and the H_∞ performance :

$$\begin{bmatrix} -P + C^T C & * & * & * \\ 0 & -\gamma^2 I_n & 0 & * \\ D_v^T C & 0 & D_v^T D_v - \gamma^2 I_n & * \\ PA_q - Q_q C & PD & -Q_q D_v & -P \end{bmatrix} \prec 0$$

- A scalar $\gamma > 0$, a positive definite matrix $P \in \mathbb{R}^{n_x \times n_x}$, $Q_q \in \mathbb{R}^{n_x \times n_y}$ and $L_q = P^{-1} Q_q$.

Residual evaluation

FD decision

- The corresponding FD decision scheme is made as follows :

$$\begin{cases} r_k \in \mathbf{R_k} & \text{Fault-free} \\ r_k \notin \mathbf{R_k} & \text{Faulty} \end{cases}$$

Theorem

The residual signal r_k is bounded by the zonotope $\mathbf{R_k} = \langle 0, R_k \rangle$ and R_k satisfies the following iteration equation :

$$\begin{cases} R_k & = & [CH_k \quad D_v H_v] \\ H_{k+1} & = & [(A_q - L_q C)\downarrow_l (H_k) \quad DH_w \quad -L_q D_v H_v] \end{cases}$$

Residual evaluation

Lemma 2 [Combastel, 2003]

- A high-dimensional zonotope can be bounded by a lower one via the reduction operation.
- The reduction operator can be described as $\mathbf{Z} = \langle p, H \rangle \subseteq \langle p, \downarrow_l(H) \rangle$.
- $\downarrow_l(H)$ represents the complexity reduction operator
- $n \leq l \leq s$ denotes the maximum number of columns of generator matrix H.

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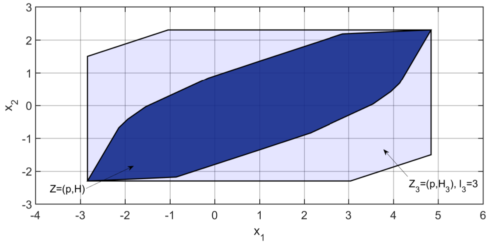

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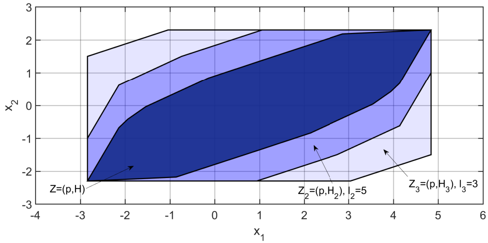

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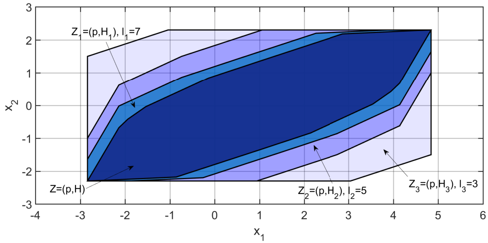


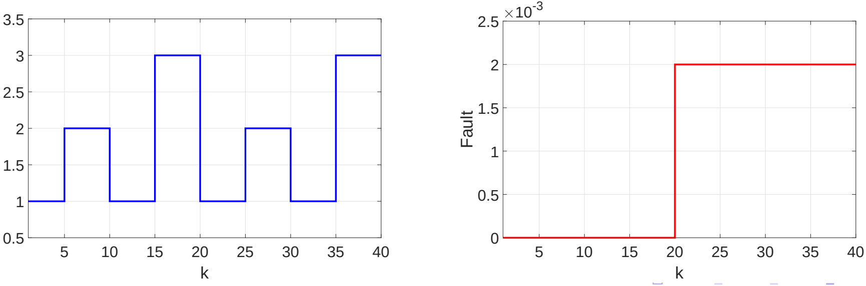
Simulation results

A discrete-time switched system defined with three subsystems, $N=3$ is considered.

- The initial state x_0 is bounded by the zonotope $\mathbf{X}_0 = \langle p_0, H_0 \rangle$ with :

$$p_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \; H_0 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

- $w_k \in \mathbb{R}$ and $v_k \in \mathbb{R}^2$: bounded random signals by $[-0.1, 0.1]$.



Simulation results

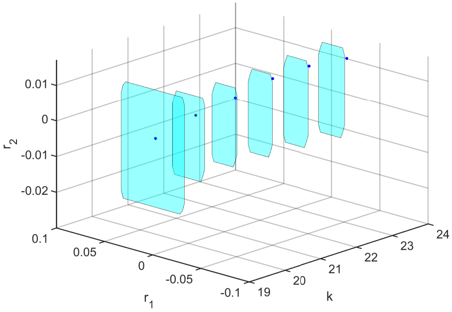


FIGURE – Residual and residual zonotope of the proposed FD observer.

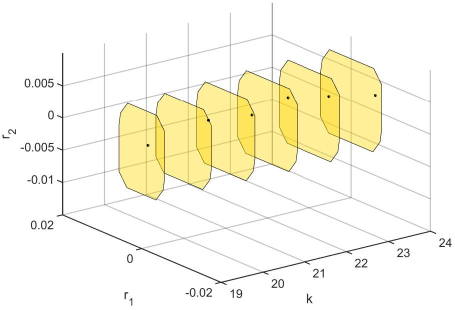


FIGURE – Residual and residual zonotope of method without optimization.

Interval vs zonotopic techniques

Zonotope based-fault detection method

Interval vs zonotopic techniques

Zonotope based-fault detection method

⇓

Accurate Fault Detection results

Relaxed design conditions

Interval vs zonotopic techniques

Zonotope based-fault detection method

↓

Accurate Fault Detection results

Relaxed design conditions

Heavier computational burden

Interval vs zonotopic techniques

Zonotope based-fault detection method → Ellipsoid based-fault detection method

Accurate Fault Detection results

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Interval vs zonotopic techniques

Zonotope based-fault detection method → Ellipsoid based-fault detection method

Zonotope based-fault detection method:

- Accurate Fault Detection results
- Relaxed design conditions
- Heavier computational burden

Ellipsoid based-fault detection method:

- A good trade-off between FD accuracy and computational complexity
- Simple matrix calculations
- No reduction operator

Problem statement

System description

Consider the following discrete-time switched system :

$$\begin{cases} x_{k+1} = A_q x_k + B_q u_k + D_q w_k \\ y_k = C x_k + D_v v(k) + F f_k, \end{cases}$$

- $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$, $f \in \mathbb{R}^{n_f}$, $w \in \mathbb{R}^{n_w}$ and $v \in \mathbb{R}^{n_v}$.
- The known matrices A_q, B_q, C, D_q, D_v and F are given with appropriate dimensions.

Objective

The aim is to develop a FD decision via ellipsoidal techniques for discrete-time switched systems with sensor faults.

Preliminaries

Definition

An ellipsoid set $\mathscr{E}(c,X) \subset \mathbb{R}^n$ is given by :

$$\mathscr{E}(c,X) = \{x \in \mathbb{R}^n : (x-c)^T X^{-1}(x-c) \leq 1\}.$$

The center of $\mathscr{E}(c,X)$ is denoted by $c \in \mathbb{R}^n$. $X \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix and represents the shape and size of the ellipsoid $\mathscr{E}(c,X)$.

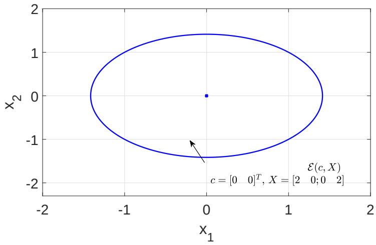

FIGURE – Ellipsoid set

Preliminaries

Assumption

Assume that the initial state x_0, the state disturbances w_k and the measurement noise v_k are unknown but bounded such that

$$x_0 \in \mathscr{E}(c_0, X_0),\ w_k \in \mathscr{E}(0, W) and\ v_k \in \mathscr{E}(0, V).$$

- $c_0 \in \mathbb{R}^{n_x}$ is a known vector
- $X_0 = \tilde{x}_0^2 I_{n_x}$, $W = \|w\|_\infty^2 I_{n_w}$ and $V = \|v\|_\infty^2 I_{n_v}$.
- $\|w\|_\infty$ and $\|v\|_\infty$, assumed to be known, are the L_∞ norm of w and v.
- The known constant $\tilde{x}_0$ is given such that $\|x_0 - c_0\| \leq \tilde{x}_0$.

Problem statement

Fault detection observer design : TNL structure

$$\begin{cases} \hat{x}_{k+1} = T_q A_q \hat{x}_k + T_q B_q u_k + N_q y_{k+1} + L_q (y_k - C\hat{x}_k) \\ r_k = y_k - C\hat{x}_k \end{cases}$$

- $\hat{x}_k$ is the estimation of x_k, r_k is the residual signal and $L_q \in \mathbb{R}^{n_x \times n_y}$ are the observer gains.
- $T_q \in \mathbb{R}^{n_x \times n_x}$ and $N_q \in \mathbb{R}^{n_x \times n_y}$ are constant matrices that should be designed to satisfy

$$T_q + N_q C = I_{n_x}$$

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Remark

- If T_q and N_q are chosen such that $T_q = I_{n_x}$ and $N_q = 0$, the proposed observer is reduced to the commonly used Luenberger form.
- The proposed structure can provide more design degrees of freedom by introducing matrices T_q and N_q.

Error dynamics

The error dynamics

$$\begin{cases} e_{k+1} &= \mathscr{A}_q e_k + \mathscr{D}_{w_q} w_k + \mathscr{D}_{vq} d_{v\,k+1} + \mathscr{F}_q d_{f\,k+1} \\ r_k &= C e_k + D_v v_k + F f_k \end{cases}$$

$$\mathscr{A}_q = T_q A_q - L_q C, \quad \mathscr{D}_{w_q} = T_q D_q, \quad \mathscr{D}_{vq} = [-L_q D_v \quad -N_q D_v],$$
$$\mathscr{F}_q = [-L_q F \quad -N_q F], \quad d_v = [v(k)^T \quad v(k+1)^T]^T, \quad d_f = [f(k)^T \quad f(k+1)^T]^T.$$

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$$\mathscr{F}_q = [-L_q F \quad -N_q F], \qquad d_v = [v(k)^T \quad v(k+1)^T]^T, \quad d_f = [f(k)^T \quad f(k+1)^T]^T.$$

A subsystem decoupled from the effects of f_k

$$\begin{cases} e^d_{k+1} &= \mathscr{A}_q e^d_k + \mathscr{D}_{w_q} w_k + \mathscr{D}_{vq} d_{v\,k+1} \\ r^d_k &= C e^d_k + D_v v_k \end{cases}$$

A subsystem affected by the sensor fault

$$\begin{cases} e^f_{k+1} &= \mathscr{A}_q e^f_k + \mathscr{F}_q d_{f\,k+1} \\ r^f_k &= C e^f_k + F f_k \end{cases}$$

Stability and L_∞ performances

Stability and L_∞ disturbance attenuation condition

The aim is to compute the FD observer gain L_q such that the following conditions hold :

Stability and L_∞ performances

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1. The error system is stable.
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$$\|r_k^d\| < \sqrt{\gamma_1{}^2\ \theta + \gamma_2^2 \|v\|_\infty^2}$$

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$$\|r_k^d\| < \sqrt{{\gamma_1}^2\ \theta + \gamma_2^2 \|v\|_\infty^2}$$

- $\theta = (\gamma_w + \gamma_v)(\lambda(1-\lambda)^k V_{q0} + \gamma_w \|w\|_\infty^2 + \gamma_v \|d_v\|_\infty^2)$.
- $\|d_v\|_\infty$ is the L_∞ norm of d_v.
- $V_{q0} = {e_0^d}^T P_q e_0^d$.
- $P_q \succ 0 \in \mathbb{R}^{n_x \times n_x}$.

Sufficient conditions are given in terms of LMIs.

Residual evaluation

Residual evaluation

- The residual evaluation is based on determining whether the residual signal r_k is excluded from the residual ellipsoid $\mathscr{E}(0, R_k)$ or not.
- The corresponding FD decision scheme is made as follows :

$$\begin{cases} r_k \in \mathscr{E}(0, R_k) & \text{Fault-free} \\ r_k \notin \mathscr{E}(0, R_k) & \text{Faulty} \end{cases}$$

The residual ellipsoid $\mathscr{E}(0, R_k)$ is obtained based on the following theorem.

Residual evaluation

Theorem

Let $x_0 \in \mathscr{E}(c_0, X_0)$ and $\hat{x}_0 = c_0$, then r_k can be bounded by the ellipsoid $\mathscr{E}(0, R_k)$ and R_k satisfies the following iteration equations :

$$p_v^* = \sqrt{\frac{\text{trace}((L_q D_v)V(L_q D_v)^T)}{\text{trace}((N_q D_v)V(N_q D_v)^T)}},\ H_v = (1+\frac{1}{p_v^*})(L_q D_v)V(L_q D_v)^T + (1+p_v^*)(N_q D_v)V(N_q D_v)^T$$

$$p_d^* = \sqrt{\frac{\text{trace}(\mathscr{D}_{w_q} W \mathscr{D}_{w_q}^T)}{\text{trace}(H_v)}},\ H_d = (1+\frac{1}{p_d^*})\mathscr{D}_{w_q} W \mathscr{D}_{w_q}^T + (1+p_d^*)H_v$$

$$p_{x_k}^* = \sqrt{\frac{\text{trace}(\mathscr{A}_q X_k \mathscr{A}_q{}^T)}{\text{trace}(H_d)}},\ X_{k+1} = (1+\frac{1}{p_{x_k}^*})\mathscr{A}_q X_k \mathscr{A}_q{}^T + (1+p_{x_k}^*)H_d$$

$$p_{r_k}^* = \sqrt{\frac{\text{trace}(CX_kC^T)}{\text{trace}(D_v V D_v^T)}},\ R_k = (1+\frac{1}{p_{r_k}^*})CX_kC^T + (1+p_{r_k}^*)D_v V D_v^T$$

Simulation results

A discrete-time switched system defined with three subsystems, $N = 3$ is considered.

- The initial conditions are chosen such that :

$$x_0 = [1 \quad 1 \quad 1]^T, \quad \hat{x}_0 = [1 \quad 1 \quad 1]^T, \quad c_0 = [0 \quad 0 \quad 0]^T \quad and \quad X_0 = I_3.$$

- The numerical simulation was carried out using Matlab optimization tools.
- Two fault scenarios are considered in the sequel.

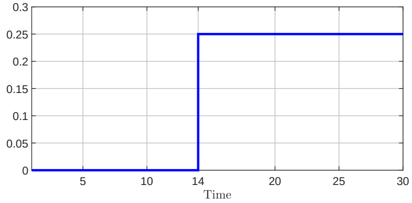

FIGURE – $f = 0.25$

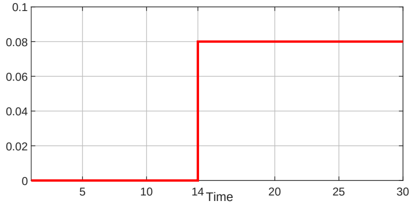

FIGURE – $f = 0.08$

Simulation results

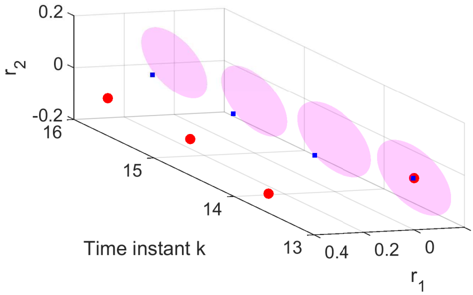

FIGURE – Residual and residual ellipsoid based on the proposed approach.

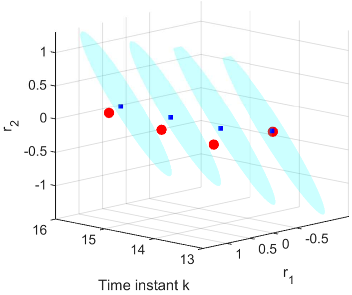

FIGURE – Residual and residual ellipsoid using a Luenberger observer.

Outline

1. Interval estimation for synchronous switched systems
2. Set-membership fault detection frameworks for switched systems
   - Interval based-fault detection method
   - Zonotope based-fault detection method
   - Ellipsoid based-fault detection method
3. Conclusion

Conclusion

- An interval approach is developed for state estimation of discrete-time LPV switched systems [Zammali et al., 2019a].
  - Stability and nonnegativity properties have been relaxed thanks to the polytopic shape of the system parameters.
  - LMIs conditions are expressed on the vertices of each polytope.

  Extensions of these results to the case of unknown switching signal and continuous-time LPV switched systems are investigated and published in [Zammali et al., 2019b], [Zammali et al., 2020a], [Zammali et al., 2020c].

- A new interval observer-based (TNL structure) FD method for discrete-time switched systems is designed using L_∞ performance [Zammali et al., 2020f].
  - The proposed approach allows reducing the conservatism of gain matrices and offers more degrees of design freedom.

  Interval techniques to detect sensor faults using the H_∞ and L_∞ criteria are published in [Zammali et al., 2020b], [Zammali et al., 2020d].

Conclusion

- Set-membership FD frameworks have been developed for switched systems with actuator fault using zonotopic analysis [Zammali et al., 2020g].
  - A novel pole assignment approach is designed to maximize the sensitivity of faults on the residual signal.
  - H_∞ performance is investigated to minimize the effect of disturbances.
- Set-membership FD frameworks have been developed for switched systems with sensor faults using ellipsoidal analysis [Zammali et al., 2020e].
  - A FD observer with a new structure is investigated.
  - The design conditions of the proposed observer are given in terms of LMIs using Multiple Lyapunov Functions, with an Average Dwell Time switching signal.
  - An L_∞ criterion is used to attenuate the effect of unknown but bounded disturbances and measurement noise.

Publications

Three Journal papers

1. European Journal of Control
2. International Journal of Control
3. Acta Cybernetica

Six Conference papers

1. 59th IEEE Conference on Decision and Control
2. 21rst IFAC World Congress
3. 28th Mediterranean Conference on Control and Automation
4. European Control Conference
5. 58th Conference on Decision and Control

An abstract

1. 11th Summer Workshop on Interval Methods

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