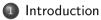
Observability of Nonlinear Systems and Injectivity

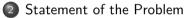
Bernd Tibken

Bergische Universität Wuppertal Chair of Automation and Control Theory

International Online Seminar on Interval Methods in Control Engineering 12.02.2021

Introduction						
0						
Table of Contents						





3 Lie Series

4 Reformulation

5 Injectivity Condition

Introduction		Injectivity Condition
•		0000
Introduction		

Systems

$$\dot{x}(t) = f(x(t)), \quad x(0) = x^{0}$$

 $y(t) = h(x(t))$

Definitions

 y^1 corresponds to $x^0=z^1$ y^2 corresponds to $x^0=z^2$

Observability

For all
$$z^1 \neq z^2$$
 we have $y^1 \neq y^2$.

S
•

Lie Series

Statement of the Problem

- How to check observability?
- Find computational condition!
- Simpler problem: Check For all $z^1 \neq z^2 \in x^I$ we have $y^1 \neq y^2$.
- $\bullet \ x^I$ interval vector

	Lie Series	
	•	
Lie Series		

Taylor series expansion of y

$$y(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left(L_f^k h \right) \left(x^0 \right)$$

Lie derivative

$$\begin{array}{rcl} \left(L_{f}h\right)\left(x\right) &=& \left(\frac{\partial h}{\partial x}\right)f(x) \\ \left(L_{f}^{0}h\right)\left(x\right) &=& h(x) \\ \left(L_{f}^{l+1}h\right)\left(x\right) &=& \left(L_{f}\ L_{f}^{l}h\right), l \geq 1 \\ f,h \text{ real analytic } \Rightarrow y(t) \text{ real analytic} \end{array}$$

Introduction O		Reformulation ●	
Reformulation			

$$y^{i}\left(t\right) = \sum_{k=0}^{\infty} \frac{t^{k}}{k!} \left(L_{f}^{k}h\right) \left(z^{i}\right)$$

When is $y^1(t) = y^2(t)$?

Observability mapping

$$F(x) := \begin{pmatrix} h(x) \\ (L_f h)(x) \\ (L_f^2 h)(x) \\ \vdots \end{pmatrix}$$

			Injectivity Condition ●000
Injectivity Co	ndition		

$$F(z^1) = F(z^2) \Rightarrow z^1 = z^2$$
 uniquely
 \Rightarrow System is observable.

- This is injectivity of *F*!
- First step: finite number of columns

			Injectivity Condition ○●○○
1	11.1		

Injectivity Condition

$$\ \, \bullet \ \, F(z^1)=F(z^2) \\ {\rm for} \ \, z^1,z^2\in x^I\Rightarrow z^1=z^2 \ {\rm uniquely} \\ \ \ \, \label{eq:final}$$

•
$$F(z^1) - F(z^2) = 0$$

• Idea: Apply mean value theorem

$$g\left(z^{1}\right) - g\left(z^{2}\right) = \frac{\partial g}{\partial x}\left(\xi\right)\left(z^{1} - z^{2}\right),$$

$$g \text{ scalar, } \xi \text{ between } z^{1} \text{ and } z^{2}$$

$$g = \begin{pmatrix} g_{1} \\ \vdots \\ g_{p} \end{pmatrix}, g_{i}\left(z^{1}\right) - g_{i}\left(z^{2}\right) = \frac{\partial g_{i}}{\partial x}\left(z^{1} - z^{2}\right)$$

$$g(z^{1}) - g(z^{2}) = \begin{pmatrix} \frac{\partial g_{1}}{\partial x}(\xi_{1}) \\ \vdots \\ \frac{\partial g_{p}}{\partial x}(\xi_{p}) \end{pmatrix} (z^{1} - z^{2})$$
$$M = \begin{pmatrix} \frac{\partial g_{1}}{\partial x}(\xi_{1}) \\ \vdots \\ \frac{\partial g_{p}}{\partial x}(\xi_{p}) \end{pmatrix}, M \neq \frac{\partial g}{\partial x}(\xi)$$
$$M \in \frac{\partial g}{\partial x}(x^{I}) = M^{I} \text{ Interval matrix}$$
$$M \text{ full rank} \Rightarrow z^{1} - z^{2} = 0 \Rightarrow g \text{ injective}$$

			Injectivity Condition 00●0
Injectivity Cor	ndition		

- Interval matrix M^I full rank $\Rightarrow M$ full rank $\Rightarrow F$ injective \Rightarrow system observable on x^I
- $\bullet \ M^I \ {\rm full \ rank?}$
- Apply result of Jiri Rohn (sufficient condition)
- In general NP hard
- Application in the talk by Thomas Paradowski

Thank you for your attention