Vector set inversion interval filtering based fault observer design

International Online Seminar on Interval Methods in Control Engineering

Ziyun Wang

February 26, 2021

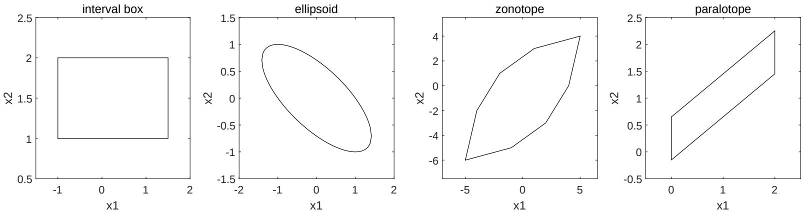
SET-MEMBERSHIP FILTERING

Feasible set

state, measurement output, disturbance, noise, . . .

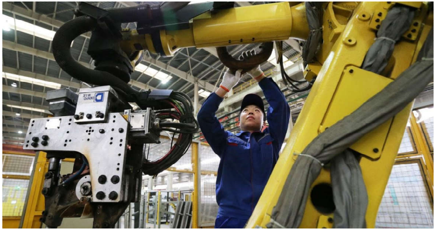
Geometry

interval, ellipsoid, zonotope, paralotope, . . .



Set-membership filtering

- Assume that the disturbances and noises of system are unknown but bounded
- Express feasible set with simple geometry
- Eliminate errors caused by inaccurate system models
- The computational complexity increasing with the increase of the system dimensions and high conservative
- It is widely used in signal processing, fault diagnosis, robot and other fields



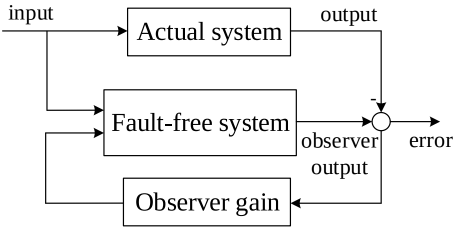
INTERVAL OBSERVER

Design criterion

The error system between the estimated state value and the true value of system is stable.

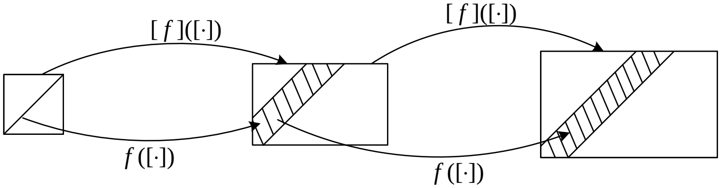
Common design methods:

- Coordinate transformation
- Pole placement
- LMI



Wrapping effect

- The axis-aligned box used in interval recursive calculation process brings errors in feasible solution set
- As the amount of calculation increases, the accumulation of errors will cause too large enclosures



Given a linear time-invariant system described by:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Ew_k + Ff_k \\ y_k = Cx_k + Dv_k \end{cases}$$

- $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^p$, $y_k \in \mathbb{R}^q$ are the state, input, and output vectors, respectively
- $f_k \in \mathbb{R}^m$ is actuator fault vector
- $w_k \in \mathbb{R}^s$ and $v_k \in \mathbb{R}^l$ denote system disturbances and the measurement noises
- A, B, C, D, E and F are constant matrices with appropriate dimensions

Augmented state vector

By combining the actuator fault f_k and the state vector x_k:

$$\bar{x}_k = \begin{bmatrix} x_k \\ f_k \end{bmatrix}.$$

Augmented system

Construct the augmented system:

$$\begin{cases} \bar{x}_{k+1} = \overline{A}\bar{x}_k + \overline{B}u_k + \overline{E}w_k + \overline{G}\Delta f_k, \\ y_k = \overline{C}\bar{x}_k + \overline{D}v_k, \end{cases}$$

where $\overline{A} = \begin{bmatrix} A & F \\ 0 & I_m \end{bmatrix}$, $\overline{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $\overline{E} = \begin{bmatrix} E \\ 0 \end{bmatrix}$, $\overline{G} = \begin{bmatrix} 0 \\ I_m \end{bmatrix}$,
$\overline{C} = \begin{bmatrix} C & 0 \end{bmatrix}$, $\overline{D} = D$, $\Delta f_k = f_{k+1} - f_k$.

Interval observer

Construct state interval observer of augmented system*:

$$\hat{\bar{x}}_k = T\bar{A}\hat{\bar{x}}_{k-1} + T\bar{B}u_{k-1} + L(y_{k-1} - \bar{C}\hat{\bar{x}}_{k-1}) + Ny_k$$

- $\hat{\bar{x}}_k \in \mathbb{R}^{n+m}$ is estimated vector of augmented state vector $\bar{x}_k$
- $L \in \mathbb{R}^{(n+m)\times q}$ is observer gain
- $T \in \mathbb{R}^{n+m}$ and $N \in \mathbb{R}^{(n+m)\times q}$ are constant matrices
  *see in *Wang Z H, et al., Systems &Control Letters 2018*

Error system

Define error system:

$$\begin{aligned} e_k &= \bar{x}_k - \hat{\bar{x}}_k \\ &= (T\bar{A} - L\bar{C})e_{k-1} + T\bar{E}w_{k-1} + T\bar{G}\Delta f_{k-1} - L\bar{D}v_{k-1} - N\bar{D}v_k. \end{aligned}$$

Theorem

Given a scalar $\gamma > 0$, *if there are positive definite matrices* $P \in \mathbb{R}^{n+m}$, $Y \in \mathbb{R}^{(n+m)\times n+m+q}$, $Z \in \mathbb{R}^{(n+m)\times q}$ *satisfy*

$$\begin{bmatrix} I_{n+m} - P & * & * & * & * & * \\ 0 & -\gamma^2 I_s & * & * & * & * \\ 0 & 0 & -\gamma^2 I_m & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I_l & * & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I_l & * \\ P(T\overline{A} - L\overline{C}) & PT\overline{E} & PT\overline{G} & -PL\overline{D} & -PN\overline{D} & -P \end{bmatrix} < 0,$$

the error system is stable.

Using the proposed theorem:

- e_k is robust to disturbance and noise
- Design interval observer by H_∞ technique
- Calculate error interval $[e_k]$ and observer state estimation interval $[\overline{x}_k]^o = \hat{\overline{x}}_k + [e_k]$ by interval operation

Question: How to reduce the impact of the wrapping effect?

Vector set inversion problem

$$X = \{[\overline{x}_k]^v \in \mathbb{IR}^n | O_{(k:k+s)}[\overline{x}_k]^v \subset [Y_k]\}$$
$$= O^{-1}_{(k:k+s)}[Y_k]$$

where, X is solution set, $[\overline{x}_k]^v$ is vector set inversion interval

$$[Y_k] = y_{(k:k+s)} - O_{u(k:k+s)} u_{(k:k+s)} - O_{f(k:k+s)}[\Delta f_{(k:k+s)}]$$
$$- O_{w(k:k+s)}[w_{(k:k+s)}] - O_{v(k:k+s)}[v_{(k:k+s)}],$$

$y_{(k:k+s)}$, $u_{(k:k+s)}$, $[\Delta f_{(k:k+s)}]$, $[w_{(k:k+s)}]$ and $[v_{(k:k+s)}]$ are the output, input, fault difference interval, disturbance interval and noise interval of the system from time instant k to $k+s$, respectively

Vector set inversion problem

$$X = O^{-1}_{(k:k+s)}(y_{(k:k+s)} - O_{u(k:k+s)}u_{(k:k+s)} - O_{f(k:k+s)}[\Delta f_{(k:k+s)}] - O_{w(k:k+s)}[w_{(k:k+s)}] - O_{v(k:k+s)}[v_{(k:k+s)}])$$

where,

$$O_{(k:k+s)} = \begin{bmatrix} \overline{C} \\ \overline{C}\,\overline{A} \\ \vdots \\ \overline{C}\,\overline{A}^{s} \end{bmatrix}, O_{v(k:k+s)} = \begin{bmatrix} \overline{D} & 0 & \dots & 0 \\ 0 & \overline{D} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \overline{D} \end{bmatrix}$$

$$O_{u(k:k+s)} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \overline{C}\,\overline{B} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \overline{C}\,\overline{A}^{s-1}\,\overline{B} & \overline{C}\,\overline{A}^{s-2}\,\overline{B} & \dots & \overline{C}\,\overline{B} \end{bmatrix}$$

Vector set inversion problem

$$X = O^{-1}_{(k:k+s)}\big(y_{(k:k+s)} - O_{u(k:k+s)}u_{(k:k+s)} - O_{f(k:k+s)}[\Delta f_{(k:k+s)}] - O_{w(k:k+s)}[w_{(k:k+s)}] - O_{v(k:k+s)}[v_{(k:k+s)}]\big)$$

where,

$$O_{f(k:k+s)} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \overline{C}\,\overline{G} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \overline{C}\,\overline{A}^{s-1}\,\overline{G} & \overline{C}\,\overline{A}^{s-2}\,\overline{G} & \dots & \overline{C}\,\overline{G} \end{bmatrix}$$

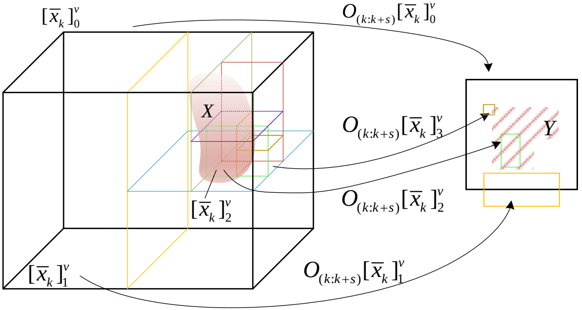
$$O_{w(k:k+s)} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \overline{C}\,\overline{E} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \overline{C}\,\overline{A}^{s-1}\,\overline{E} & \overline{C}\,\overline{A}^{s-2}\,\overline{E} & \dots & \overline{C}\,\overline{E} \end{bmatrix}$$

Use SIVIA algorithm to solve the above problem. The larger the time length s, the higher the accuracy and the greater the amount of calculation.

Solution: Convert the interval boxs to the form of row vectors

$\mathcal{L}$ is a vector group representing all interval boxes in the solution process, There are four different situations:

1. $O_{(k:k+s)}\mathcal{L}_i$ and $[Y_k]$ intersect, but not completely belong to $[Y_k]$
2. $O_{(k:k+s)}\mathcal{L}_i$ and $[Y_k]$ have no intersection
3. $O_{(k:k+s)}\mathcal{L}_i$ belongs entirely to $[Y_k]$
4. $O_{(k:k+s)}\mathcal{L}_i$ have partial intersection with $[Y_k]$ and the width of the corresponding interval box is less than the precision parameter



Test function

$$[t](\cdot) = \begin{cases} in, & O_{(k:k+s)}\mathcal{L}_i \subset [Y_k] \\ out, & O_{(k:k+s)}\mathcal{L}_i \cap [Y_k] = \emptyset \\ eps, & W(\mathcal{L}_i) < \varepsilon \end{cases}$$

- in, out and eps are all column vectors of Boolean variables equal to the dimension of $\mathcal{L}$
- $W(\mathcal{L}_i)$ is the column vector composed of the width of each interval box in $\mathcal{L}_i$
- ε is precision parameter

Test function

$$[t](\cdot) = \begin{cases} in, & O_{(k:k+s)}\mathcal{L}_i \subset [Y_k] \\ out, & O_{(k:k+s)}\mathcal{L}_i \cap [Y_k] = \emptyset \\ eps, & W(\mathcal{L}_i) < \varepsilon \end{cases}$$

1. If $O_{(k:k+s)}\mathcal{L}_i \subset [Y_k]$, $in(i) = 1$. Otherwise, $in(i) = 0$. Push the vector group $\mathcal{L}(in)$ into feasible set $\mathcal{N}$
2. If $O_{(k:k+s)}\mathcal{L}_i \cap [Y_k] = \emptyset$, $out(i) = 1$, $\mathcal{L}(\neg in \wedge \neg out)$ belongs to the uncertain vector group $\mathcal{U}$
3. If $W(\mathcal{L}_i) < \varepsilon$, $eps(i) = 1$, push $\mathcal{U}(eps)$ into uncertain set $\mathcal{E}$
4. Bisect the remaining interval boxes in $\mathcal{U}$
5. Loop until $\mathcal{L}$ is empty

Theorem

The solution set $[\overline{x}_k]^v$ obtained by the vector set inversion interval filtering algorithm satisfies:

$$[\overline{x}_k]^v \subset X \subset [\overline{x}_k]^v \cup \mathcal{E}.$$

Proof. In the process of solving, $O_{(k:k+s)}[\overline{x}_k]_i^v$ completely belongs to $[Y_k]$, $[\overline{x}_k]_i^v$ is a feasible subset satisfying

$$[\overline{x}_k]_i^v \subset \mathcal{N}$$

The union of all feasible subsets in $\mathcal{N}$ is $[\overline{x}_k]^v$, namely

$$\bigcup_{i=1,2,\ldots} [\overline{x}_k]_i^v = [\overline{x}_k]^v \subset O_{(k:k+s)}^{-1}[Y_k] = X$$

Theorem

The solution set $[\overline{x}_k]^v$ obtained by the vector set inversion interval filtering algorithm satisfies:

$$[\overline{x}_k]^v \subset X \subset [\overline{x}_k]^v \cup \mathcal{E}.$$

Similarly, when $O_{(k:k+s)}[\overline{x}_k]^v_i$ and $[Y_k]$ have a partial intersection and the width of $[\overline{x}_k]^v_i$ is less than the precision parameter ε, $[\overline{x}_k]^v_i$ is an uncertain subset satisfying

$$[\overline{x}_k]^v_i \subset \mathcal{E}$$

All uncertain subsets form an uncertain layer $\mathcal{E}$ satisfying

$$X \setminus [\overline{x}_k]^v \subset \mathcal{E}$$

Therefore, $[\overline{x}_k]^v \subset X \subset [\overline{x}_k]^v \cup \mathcal{E}$. $\square$

NUMERICAL EXAMPLE

Consider the linear system model with:

- state matrix $A = \begin{bmatrix} 0.9842 & 0.0407 \\ 0 & 0.9590 \end{bmatrix}$
- input matrix $B = \begin{bmatrix} 0.0831 & 0.0007 \\ 0 & 0.0352 \end{bmatrix}$
- output matrix $C = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$
- disturbance matrix $E = \begin{bmatrix} 0.9842 & 0.0407 \\ 0 & 0.9590 \end{bmatrix}$
- noise matrix $D = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.05 \end{bmatrix}$
- actuator fault matrix $F = \begin{bmatrix} 0.8 & 0 \end{bmatrix}^{\mathrm{T}}$

Obtain the interval observer matrix parameters:

$$T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.6550 & 0 \\ -1.2496 & 0.0086 & 1 \end{bmatrix}, L = \begin{bmatrix} 0 & 0 \\ -0.0393 & 0.3780 \\ -1.2241 & -0.0225 \end{bmatrix},$$

$$N = \begin{bmatrix} 2 & 0 \\ 0 & 0.6900 \\ 2.4992 & -0.0173 \end{bmatrix}$$

In simulation,

- initial observation state $\hat{\bar{x}}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}$
- initial error interval $[e_0] = \begin{bmatrix} [e_{0,1}] \\ [e_{0,2}] \end{bmatrix} = \begin{bmatrix} [-0.1 & 0.1] \\ [-0.1 & 0.1] \end{bmatrix}$
- unknown disturbance $|w_k| \leqslant \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}^{\mathrm{T}}$
- unknown noise $|v_k| \leqslant \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}^{\mathrm{T}}$
- input $u = \begin{bmatrix} 3 & 3 \end{bmatrix}^{\mathrm{T}}$

Actuator fault:

$$f_k = \begin{cases} 0, & k < 50, k \geqslant 100 \\ 10, & 50 \leqslant k < 100 \end{cases}$$

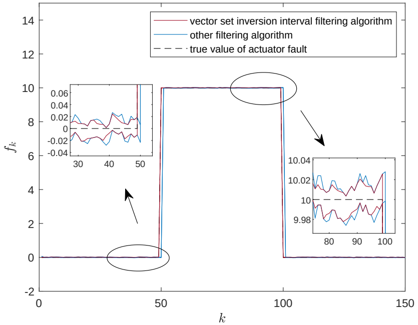


Fig: Fault estimation results

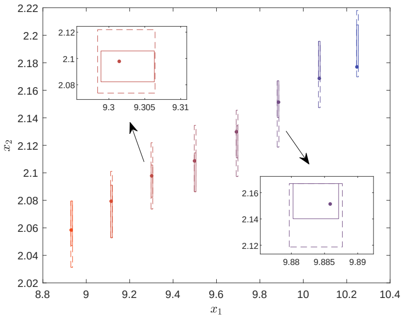


Fig: Comparison of state estimation results at k=38 ∼45 time instant

Conclusions

- The actuator fault observation method of linear system with unknown but bounded disturbance and noise is studied

Conclusions

- The actuator fault observation method of linear system with unknown but bounded disturbance and noise is studied
- Use multi-time output data to reduce the wrapping effect of interval calculation

Conclusions

- The actuator fault observation method of linear system with unknown but bounded disturbance and noise is studied
- Use multi-time output data to reduce the wrapping effect of interval calculation
- Solve the problem that the calculation time of the traditional interval filtering algorithm increases exponentially as the interval dimension increases

Conclusions

- The actuator fault observation method of linear system with unknown but bounded disturbance and noise is studied
- Use multi-time output data to reduce the wrapping effect of interval calculation
- Solve the problem that the calculation time of the traditional interval filtering algorithm increases exponentially as the interval dimension increases
- Vector set inversion interval filtering based fault observer can be extended to deal with fault diagnosis problems in aircraft systems, multi-machine node systems, servo motor systems, diode circuits and other engineering fields

Thank you for your kind attention!