Interval estimation for linear discrete-time delay systems

International Online Seminar: Interval Methods in Control Engineering

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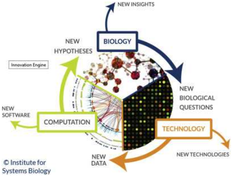
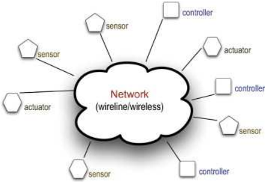
Time-delay systems





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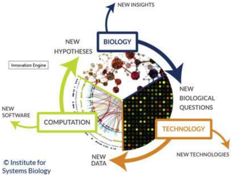
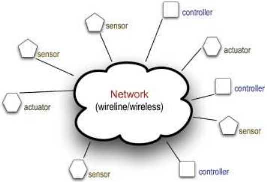




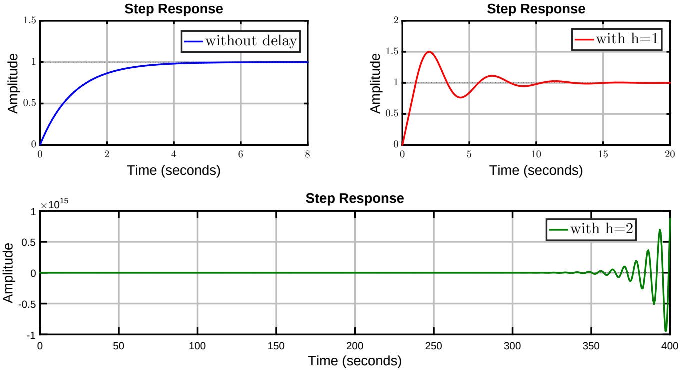
Why TDS are of interest ?

Delays may induce :

- Poor performances
- Instability
- Oscillations

Time-delay effects

$$\dot{x}(t) = -x(t-h)$$



About complexity

- Consider system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t-h) \\ x(t) = \phi(t) \quad t \in [-h, 0] \end{cases}$$

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Infinite dimensional system

- Time-delay systems belong to the class of functional differential equations.

State estimation

State estimation of TDS

- Luenberger-type observers [Hewer et al., 1973] [Gressang, 1974] [Bhat et al., 1976]
- Unknown input observers (UIO) [Sename, 1997] [Fattouh et al., 1999] [fu et al., 2004] [Darouach, 2004] [Hassan et al., 2013] [Zheng et al., 2015] [Warrad et a., 2016]
- H_∞ observers [Choi et al., 1996] [Fattouh et al.,1999] [Fridman et al, 2001] [Briat, 2008]
- Sliding-mode observers [Seuret et al, 2007] [Niu et al., 2004] [Hu et al., 2012]

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When there exist uncertainties

- Point estimation cannot converge to the real states.
- Interval estimation calculates lower and upper bounds enclosing all the feasible states in a guaranteed way.

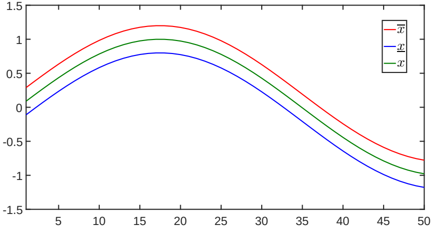
Classification of interval estimation approaches

Interval estimation
- Interval observers
- Set-membership estimation
  - Zonotopes
  - Polytopes
  - Ellipsoids
  - Parallelotopes

Interval observers

Concept

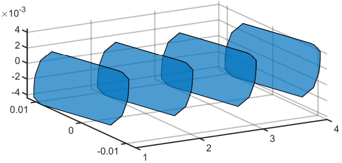
- Provide upper and lower bounds of the state vector using the available information.

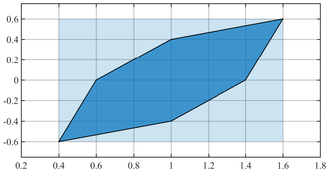


Limitation

- Design two point observers such that the estimation error dynamics are both cooperative and stable.
- Relaxation via coordinate transformation [Mazenc and Bernard, 2011] [Raïsi et al, 2012] $\Longrightarrow$ additional conservatism [Chambon et al., 2016]

Zonotope-based interval estimation





Zonotope-based interval estimation

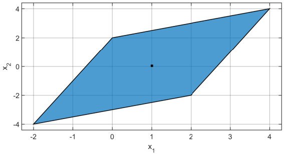
- Good trade-off between estimation accuracy and computational complexity [Tang et al., 2019].
- Intuitive and independent of cooperativity constraint and coordinate transformation.

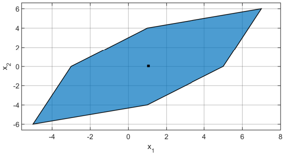
Zonotopes

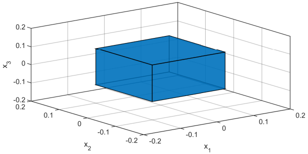
Definition

- An s-zonotope $\mathcal{Z} \subseteq \mathbb{R}^n$ is the affine image of a hypercube $\mathbb{B}^s = [-1, 1]^s$ in $\mathbb{R}^n$ and can be expressed as :

$$\mathcal{Z} = \langle p, H \rangle = p + H\mathbb{B}^s = \{z \in \mathbb{R}^n : z = p + Hb\}$$







Properties of zonotopes

Minkowski Sum

Property 1 : [Combastel, 2003] The Minkowski sum of two zonotopes $\mathcal{Z}_1 = \langle p_1, H_1 \rangle$ and $\mathcal{Z}_2 = \langle p_2, H_2 \rangle$ is also a zonotope

$$\mathcal{Z} = \mathcal{Z}_1 \oplus \mathcal{Z}_2 = \langle p_1 + p_2, [H_1 H_2] \rangle$$

Linear transformation

Property 2 : [Combastel, 2003] The linear transformation of a given zonotope $\mathcal{Z} = \langle p, H \rangle$ is

$$K \odot \mathcal{Z} = \langle Kp, KH \rangle$$

Reduction order

Property 3 : [Combastel, 2005] A high dimensional zonotope can be bounded by a lower one via a reduction operator

$$\mathcal{Z} = \langle p, H \rangle \subseteq \langle p, \downarrow_q (H) \rangle$$

State of art

Interval state estimation methods applied for :

- Biological systems [Gouzé., 2000]
- Bioreactors [Moisan et al., 2007]
- Nonlinear systems control [Raïssi et al., 2012]
- LPV systems [Efimov et al., 2012]
- Switched systems [Ethabet et al., 2018] [Zammali et al., 2020]
- Descriptor systems [Wang et al., 2018] [Tang et al, 2020]
- Fault diagnosis [Wang et al., 2018]

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Interval state estimation for time-delay systems

- Interval observer design for descriptor linear and nonlinear systems with time-delay [Efimov et al., 2015] [Zheng et al., 2016]
- Reduced-order interval observer design for linear and nonlinear systems with time-delay [Efimov et al., 2013] [Huong et al., 2020]
- Interval observers for time-invariant exponentially stable linear systems [Mazenc et al., 2012]

Contributions

Motivation

- Existing solutions have been developed only for continuous-time delay systems
- Zonotope-based interval estimation for time-delay systems has not been fully considered

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Contribution

- Interval observer design and zonotope-based methods for linear discrete-time systems with constant time-delay with external disturbances and measurement noises.

Outline

1. Interval observer design for linear discrete-time delay systems
2. Zonotope-based interval estimation for linear discrete-time delay systems
3. Simulations
   - Example1 : Case of Cooperative Error System
   - Example2 : Case of Non-cooperative Error System
4. Conclusions & Perspectives

Outline

Linear discrete-time delayed system

System description

Consider the following linear discrete-time delay system :

$$\begin{cases} x(k+1) = A_0 x(k) + A_1 x(k-h) + Bu(k) + Dw(k), \\ y(k) = Cx(k) + Fv(k), \\ x(s) = \phi(s), \quad s = -h, \ldots, 0 \end{cases}$$

- $x \in \mathbb{R}^{n_x}$ is the state vector
- $u \in \mathbb{R}^{n_u}$ is the input vector
- $y \in \mathbb{R}^{n_y}$ is the output vector
- $w \in \mathbb{R}^{n_w}$ is the state disturbance
- $v \in \mathbb{R}^{n_v}$ is the measurement noise
- h is a constant time-delay
- ϕ is the initial function vector

Assumptions

- The pair (A_0, C) is observable.
- The initial state vector $x(k)$ satisfies $x(k) \in [\underline{x}(k), \bar{x}(k)]$ for all $k \in [-h, 0]$
- The state disturbance and the measurement noise are unknown but bounded with well known bounds :

$$\underline{w} \leq w(k) \leq \overline{w}; \quad -VE_{n_v} \leq v(k) \leq VE_{n_v}$$

Interval Observer

Goal

- Develop an interval observer which consists of two dynamical systems to estimate respectively the upper and lower bounds of the state.

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Interval Observer Structure

$$\begin{cases} \underline{x}(k+1) = A_0\underline{x}(k) + A_1\underline{x}(k-h) + Bu(k) + L_0(y(k) - C\underline{x}(k)) \\ \qquad + L_1(y(k-h) - C\underline{x}(k-h)) + D^+\underline{w} - D^-\overline{w} \\ \qquad - (|L_0F| + |L_1F|)KE_{n_v}, \\ \overline{x}(k+1) = A_0\overline{x}(k) + A_1\overline{x}(k-h) + Bu(k) + L_0(y(k) - C\overline{x}(k)) \\ \qquad + L_1(y(k-h) - C\overline{x}(k-h)) + D^+\overline{w} - D^-\underline{w} \\ \qquad + (|L_0F| + |L_1F|)KE_{n_v}, \end{cases}$$

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$\Rightarrow$ Finding gain matrices L_0 and L_1 for ensuring :

- Cooperativity condition : $\underline{x}(k) \leq x(k) \leq \overline{x}(k), \quad \forall k \in \mathbb{Z}_+$
- Stability of $\underline{e} = x - \underline{x}$ and $\overline{e} = \overline{x} - x$

Cooperativity condition

Cooperative system [Haddad et al, 2004]

Consider a linear discrete-time dynamical system with a constant time-delay :

$$x(k+1) = A_0 x(k) + A_1 x(k-h) + w(k), \quad w : \mathbb{R}_+ \to \mathbb{R}^n{}_+$$
$$x(k) = \phi(k) \geq 0, \quad k \in [-h, 0]$$

where $x(k) \in \mathbb{R}^n$ is the state vector, h is a constant time-delay and the matrices A_0 and A_1 have appropriate dimensions.
This system is called cooperative or nonnegative for all $h \in \mathbb{Z}_+$ if the matrices A_0 and A_1 are nonnegative.

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$$\begin{cases} \underline{e}(k+1) = (A_0 - L_0 C)\underline{e}(k) + (A_1 - L_1 C)\underline{e}(k-h) + B_d \underline{d}(k), \\ \overline{e}(k+1) = (A_0 - L_0 C)\overline{e}(k) + (A_1 - L_1 C)\overline{e}(k-h) + B_d \overline{d}(k), \end{cases}$$

State Estimation Error Convergence

Stability Condition

- Lyapunov-Krasovskii functional for the upper estimation error (similarly for the lower estimation error)

$$V(\overline{e}(k)) = \overline{e}(k)^T P \overline{e}(k) + \sum_{j=1}^{h} \overline{e}(k-j)^T Q_d \overline{e}(k-j),$$

$$P^T = P \succ 0, Q^T = Q \succ 0$$

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Uncertainties Attenuation Condition

- The robustness interval observer design against the unknown disturbances is definded :

$$||\overline{e}||_2 < \gamma^2 ||\overline{d}||_2 \;\; ||\underline{e}||_2 < \gamma^2 ||\underline{d}||_2$$

State Estimation Error Convergence

LMI-based Conditions

In order to satisfy the above conditions, the following inequalities hold

$$\begin{bmatrix} -P+Q+I_{n_x} & * & * & * & * & * \\ 0 & -Q & * & * & * & * \\ 0 & 0 & -\gamma^2 I_{n_w} & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I_{n_v} & * & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I_{n_v} & * \\ PA_0 - K_0 C & PA_1 - K_1 C & P & K_0 F & K_1 F & -P \end{bmatrix} \prec 0,$$

$$P \succ 0,$$

$$Q \succ 0,$$

$$PA_0 - K_0 C \geq 0,$$

$$PA_1 - K_1 C \geq 0,$$

- Positive scalar γ
- $P \in \mathbb{R}^{n_x \times n_x}$ diagonal matrix, $Q \in \mathbb{R}^{n_x \times n_x}$ symmetric matrix
- $L_0 = P^{-1} K_0$ and $L_1 = P^{-1} K_1$ are the observer gain matrices

Outline

Problem formulation

- Consider the following linear discrete-time delay system :

$$\begin{cases} x(k+1) = A_0 x(k) + A_1 x(k-h) + Bu(k) + Dw(k), \\ y(k) = Cx(k) + Fv(k), \\ x(s) = \phi(s), \quad s = -h, \ldots, 0 \end{cases}$$

- The initial system state vector $x(k)$, disturbances vector $w(k)$ and measurement noises vector $v(k)$ are assumed to be unknown but bounded

$$x(k) \in \langle p_k, H_k \rangle, \quad k = -h, \ldots, 0$$

$$w(k) \in \mathcal{W} = \langle 0, H_w \rangle, \, v(k) \in \mathcal{V} = \langle 0, H_v \rangle$$

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Goal

- Estimate the intervals of state vector as accurate as possible

Main idea

Proposed method

- Zonotope-based interval estimation method for linear discrete-time delay systems

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  1. Pointwise observer design via H_∞ formalism

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  2. Zonotope-based state reachable set estimation

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Proposed method

- Zonotope-based interval estimation method for linear discrete-time delay systems
  1. Pointwise observer design via H_∞ formalism
  2. Zonotope-based state reachable set estimation
  3. Interval estimation via the smallest outer box

Step 1 : H_∞ observer design

Luenberger-type observer structure

$$\begin{aligned}\hat{x}(k+1) = {} & A_0\hat{x}(k) + A_1\hat{x}(k-h) + Bu(k) + L_0(y(k) - C\hat{x}(k)) \\ & + L_1(y(k-h) - C\hat{x}(k-1))\end{aligned}$$

Step 1 : H_∞ observer design

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Estimation error dynamics $e(k) = x(k) - \hat{x}(k)$

$$e(k+1) = (A_0 - L_0C)e(k) + (A_1 - L_1C)e(k-h) + Ed(k)$$

Delay-Independent Stability Condition

$$V(e(k)) = e(k)^T Pe(k) + \sum_{j=1}^{h} e(k-j)^T Qe(k-j), \quad P^T = P \succ 0, Q^T = Q \succ 0$$

Uncertainties attenuation condition

$$||e|_2| < \gamma^2 ||d||_2$$

Step 1 : H_∞ observer design

LMI-based Optimization Problem

In order to satisfy the above conditions, the following LMI holds

$$\begin{bmatrix} -P+Q+I_{n_x} & * & * & * & * & * \\ 0 & -Q & * & * & * & * \\ 0 & 0 & -\gamma^2 I_{n_w} & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I_{n_v} & * & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I_{n_v} & * \\ (PA_0 - R_0 C) & (PA_1 - R_1 C) & PD & -R_0 F & -R_1 F & -P \end{bmatrix} \prec 0$$

- $\gamma > 0$ is a scalar
- $P, Q \in \mathbb{R}^{n_x \times n_x}$ symmetric and positive definite matrices
- $L_0 = P^{-1} R_0$ and $L_1 = P^{-1} R_1$ observer gain matrices

Step 2 : Zonotope-based state reachable set estimation

- The reachable set estimation for the error dynamic system :

$$
\begin{aligned}
e(k+1) = (A_0 - L_0 C)e(k) + (A_1 - L_1 C)e(k-h) \\
+ Dw(k) - L_0 F v(k) - L_1 F v(k-h)
\end{aligned}
$$

where

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e(k) \in \langle 0, H_k \rangle, \quad k = -h, ..., 0 \\
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- Set propagation equation :

$$e(k+1) \in \Omega_{k+1} = (A_0 - L_0C) \odot \langle 0, \hat{H}_k \rangle \oplus (A_1 - L_1C) \odot \langle 0, \hat{H}_{k-h} \rangle \\ \oplus D \odot \langle 0, H_w \rangle \oplus (-L_0F) \odot \langle 0, H_v \rangle \oplus (-L_1F) \odot \langle 0, H_v \rangle \\ = \langle 0, \hat{H}_{k+1} \rangle$$

- Estimate the set of the state vector :

$$\begin{cases} x(k+1) = \hat{x}(k+1) + e(k+1) \\ e(k+1) \in \langle 0, \hat{H}_{k+1} \rangle \end{cases}$$

$$\Rightarrow x(k+1) \in \langle \hat{x}(k+1), \hat{H}_{k+1} \rangle$$

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$\Rightarrow x(k+1) \in \langle \hat{x}(k+1), \hat{H}_{k+1} \rangle$ How to obtain $\hat{H}_{k+1}$?

Step 2 : Zonotope-based state reachable set estimation

- Applying Proporties 1 and 2 of zonotopes to the set propagation equation :

$$\begin{cases} \hat{H}_{k+1} = [(A_0 - L_0 C)\hat{H}_k \;\; (A_1 - L_1 C)\hat{H}_{k-h} \;\; DH_w \;\; -L_0 FH_v \;\; -L_1 FH_v] \\ \hat{H}_k = H_k, \quad k \in [-h, 0] \end{cases}$$

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- Applying Property 3 to the matrix $\hat{H}_{k+1}$:

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- From $x(k) = \hat{x}(k) + e(k)$, the estimated set of the states are given :

$$x(k) \in \langle \hat{x}(k), \hat{H}_k \rangle$$

Step 3 : Interval estimation via the smallest outer box

Smallest outer box [Combastel, 2003]

Definition. For a zonotope $\mathcal{Z} = \langle p, H \rangle \subset \mathbb{R}^n$, its smallest outer box $Box(\mathcal{Z})$ is the smallest interval vector containing it, which is denoted by :

$$\mathcal{Z} \subset Box(\mathcal{Z}) = \{z \in \mathbb{R}^n : z = p + diag\{\sum_{j=1}^{m} |H_{1,j}| \cdots \sum_{j=1}^{m} |H_{n,j}|\}\mathbb{B}^s\}$$

Interval State Estimation

- Smallest interval vector $[\underline{x}(k), \overline{x}(k)]$ that contains the real state :

$$\begin{cases} \underline{x}(i,k) = \hat{x}(i,k) - \sum_{j=1}^{m} |\hat{H}_{i,j}|, \quad i = 1, \ldots, n \\ \overline{x}(i,k) = \hat{x}(i,k) + \sum_{j=1}^{m} |\hat{H}_{i,j}|, \quad i = 1, \ldots, n \end{cases}$$

Outline

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Simulation Results

- A linear discrete-time delay system is considered with following parameters :

$$A_0 = \begin{bmatrix} 0.5 & 0.3 \\ -0.8 & 0.1 \end{bmatrix}, A_1 = \begin{bmatrix} -0.11 & 0.03 \\ 0.17 & 0.11 \end{bmatrix}, F = 0.1$$

$$B = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, D = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, h = 3.$$

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- The initial state vector is bounded by $\mathcal{X} = \langle 0, H_k \rangle$ where

$$H_k : \begin{cases} 0, & \text{k=-h,...,-1} \\ 0.5 \times I, & \text{k=0} \end{cases}$$

Simulation Results

- In this case, the cooperativity condition is satisfied and the interval observer is designed with $\gamma = 3.8$ and

$$L_0 = \begin{bmatrix} 0.36 \\ -0.91 \end{bmatrix}, L_1 = \begin{bmatrix} -0.25 \\ 0.04 \end{bmatrix},$$

$$A_0 - L_0 C = \begin{bmatrix} 0.13 & 0.3 \\ 0.11 & 0.10 \end{bmatrix}, A_1 - L_1 C = \begin{bmatrix} 0.14 & 0.03 \\ 0.12 & 0.11 \end{bmatrix}.$$

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- Zonotope-based interval estimation method is achieved with H_∞ index $\gamma = 1.94$ and the following matrices :

$$L_0 = \begin{bmatrix} 0.5 \\ -0.79 \end{bmatrix}, L_1 = \begin{bmatrix} -0.10 \\ 0.16 \end{bmatrix}.$$

Simulation Results

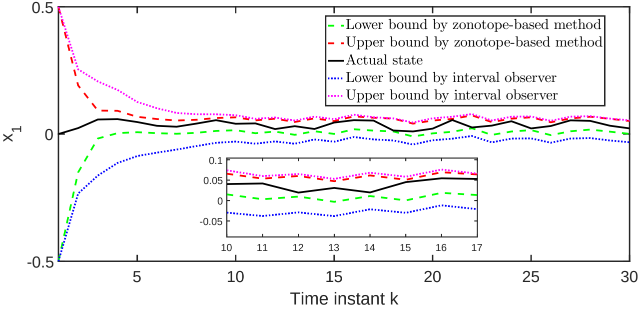

FIGURE: x_1 and its interval estimation

Simulation Results

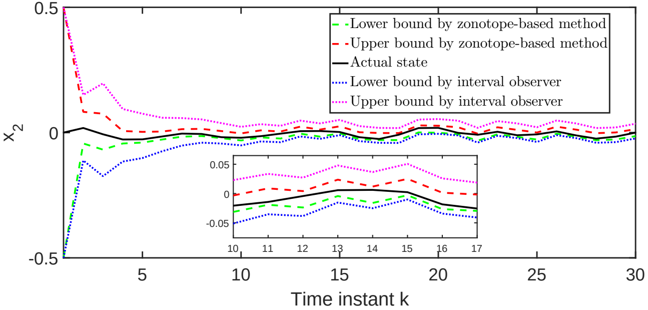

FIGURE: x_2 and its interval estimation

Outline

1. Interval observer design for linear discrete-time delay systems
2. Zonotope-based interval estimation for linear discrete-time delay systems
3. Simulations
   - Example1 : Case of Cooperative Error System
   - Example2 : Case of Non-cooperative Error System
4. Conclusions & Perspectives

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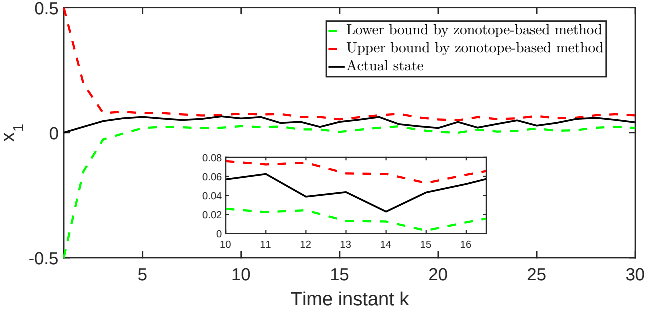

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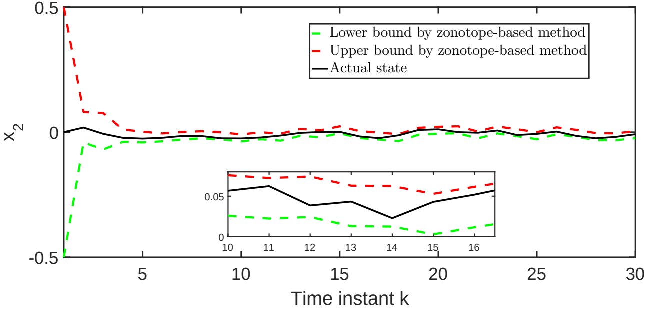

FIGURE: x_2 and its interval estimation

Outline

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Interval Estimation for Linear Discrete-Time Delay Systems

- An interval observer design and zonotope-based interval estimation methods are proposed for linear discrete-time systems with time-delay affected by bounded disturbances and measurement noises.

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- Compared with interval observers, the proposed method gives less conservative estimation results and it is independent of cooperativity constraint and coordinate transformation.
- The interval observer has less computational complexity than the zonotope-based method based on set operations.

Perspectives

Further works

- Extension to delay-dependent stability approach.

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- Extension to delay-dependent stability approach.
- Robust fault diagnosis for discrete-time delay systems with time-varying delay.

Thank you for your attention

Questions ?