INTERVAL METHODS FOR PROCESSING NOISED DATA FROM SEVERAL INFORMATION SOURCES

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The aim of this Presentation is to demonstrate

a survey of tried application of Interval Analysis methods

to one practical problem in Air Traffic Control:

processing the noised data with two-dimensional

uncertainty from several information sources.

Topics of presentation

– Motivations.

– Noised data with uncertainty from several sources.

– Problem formulation.

– Processing procedures.

– References.

Motivation of the research

Nowadays, there are wide known standard approaches to processing the noised information based on methods of mathematical statistics, for instance, "GOST 8.207-76. Methods for Processing the Observation Results..." or "R 40.2.028–2003. Recommendations on Building the Calibration Characteristics..." [1,2]. The most important conditions of their application are:

— a representative sample of measurements (at least 15 ones);

— known probabilistic characteristics of the errors in measurements.

But in practice of Air Traffic Control (ATC), the typical situations are when:

— the sample of measurements of aircraft position is very short (*i.e.*, is not representative), and there can be only $3 \div 4$ measurements at the current instant from a small number of measuring systems;

— the probabilistic characteristics of the errors in measurements are unknown;

— the standard statistical methods do not take into account the so-called "uncertainty set" of each measurement stipulated by specifics of operation of each measuring system.

Motivation of the research (continue)

Under such conditions, it is difficult to validate using the statistical methods and estimate authenticity of output results since absence of any "authentic probabilities" and "authentic intervals".

Remark now another engineering approach to processing the noised information coming from several sources. Often, engineers apply the empirical method of so-called "*averaging over group with weight numbers*" (coefficients). But as a rule, these coefficients are prescribed by experts on the basis of their *subjective reasoning* about the accuracy and reliability of data from each of measuring system without strict validation.

So, in the problem under consideration, the appropriate alternative is application of the Interval Analysis methods to estimation of aircraft motion parameters.

Used methods and procedures for finding the output estimates of aircraft positions

1. For constructing two-dimensional *uncertainty sets* of noised measurements of the aircraft positions, we keep at the general interval ideolody of processing data with uncertainties, for example, "Shary, S.P. Finite–Dimensional Interval Analysis..." or "Jaulin L., Kieffer M., Didrit O., and E. Walter. Applied Interval Analysis." [3,4].

2. For constructing the set of *predicted* possible aircraft positions, we apply usual forecasting the reachable set of aircraft at the current instant on the basis of the information set of the aircraft positions at the previous instant, for instance, "Kostousova E.K. External polyhedral estimates for reachable sets ...", "Patsko V.S., Fedotov A.A., Kumkov S.I., Pyatko S.G. Informational Sets in Model Problems of Aircraft Tracking", or "Kumkov S.I., Patsko V.S. Informational Sets in a Model Problem of Homing." [5,6,7].

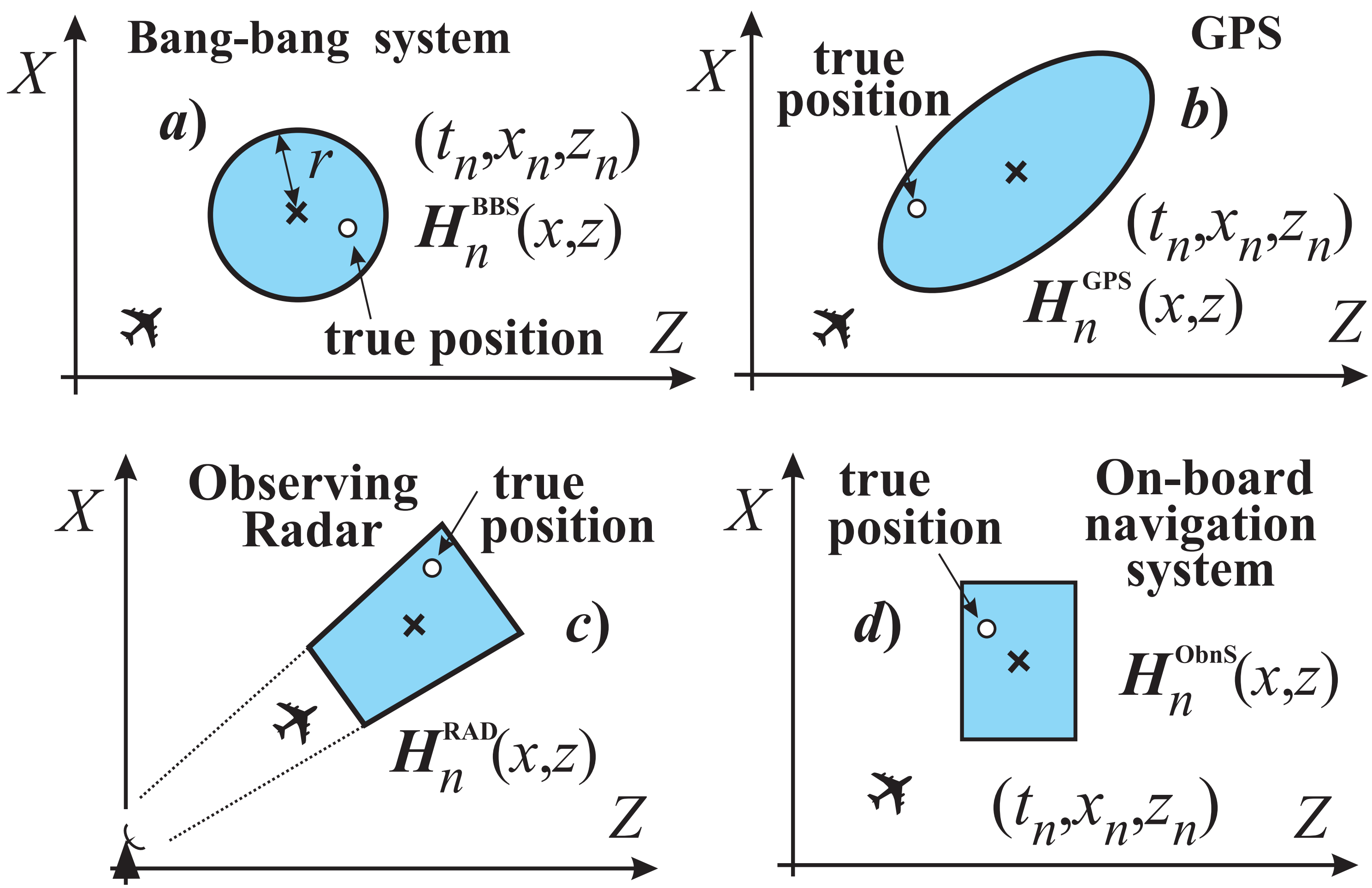
Used methods and procedures for finding the output estimates of aircraft positions (continue)

3. For constructing the *current information set* of aircraft positions, we use the interval procedures of intersection of the two-dimensional measurements' uncertainty sets – polytopes – and their convex envelopes (the outer minimal ones by inclusion) [6,7].

4. To calculate the *current output estimate* of the aircraft position, we chose the special internal point as the estimate.

Noised data with uncertainty. Several sources (Fig. 1)

Let four measurements (the black crosses) of the current aircraft position come from four measuring systems [8,9]. The cases are shown when the measurements at the current instant t_n are **closing**, *i.e.*, their uncertainty sets contain of the true aircraft position (white circles).



Problem formulation

Having the measurements with their uncertainty sets, to improve the set of current aircraft positions compatible with the given input data in situations:

— when all the measurements *are closing*;
— when measurements *are not closing* (partially or completely).

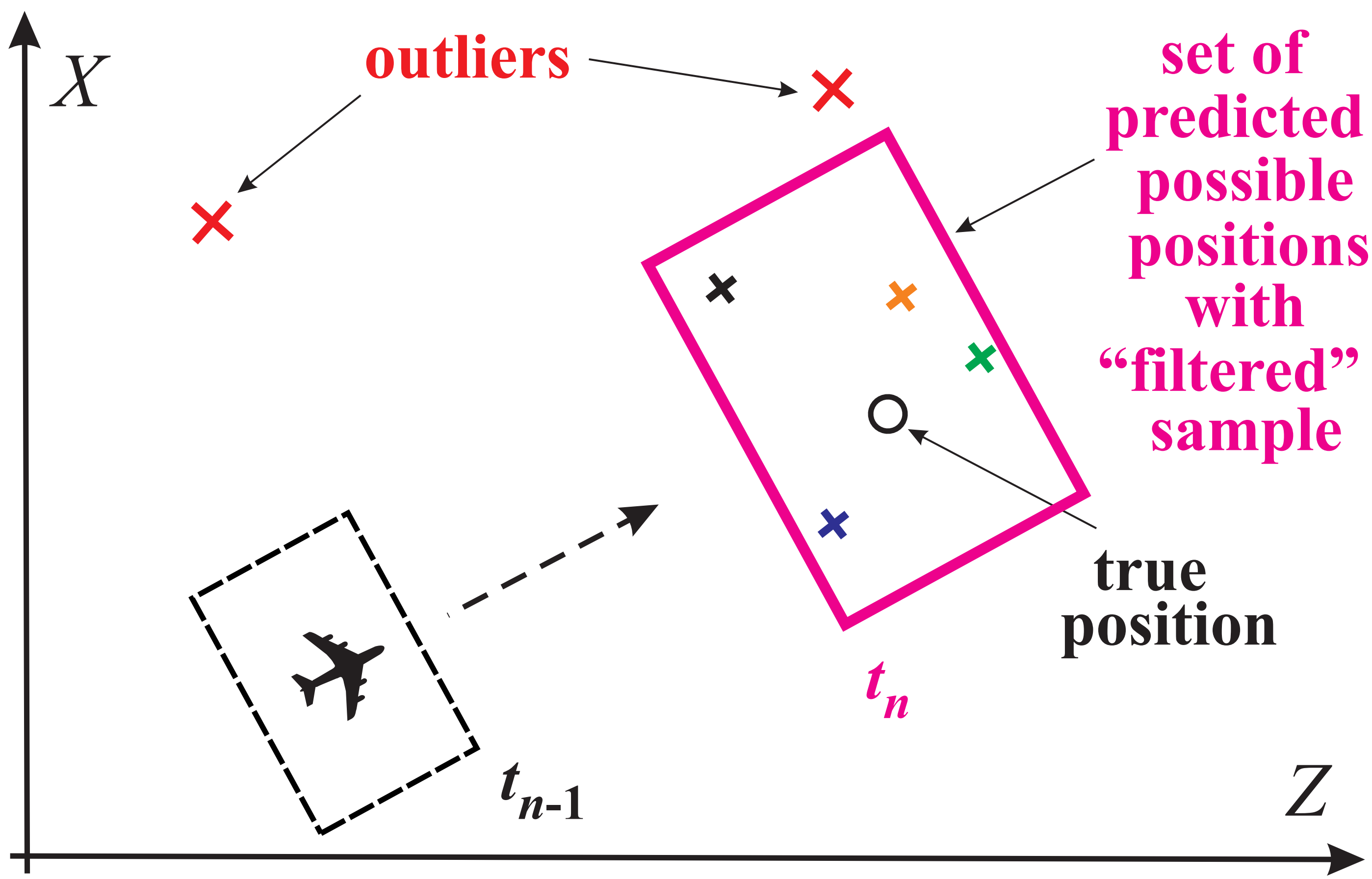
The main processing procedure

Building the information set $\boldsymbol{I}(x,z)$ by intersection of all the uncertainty sets [7,8]

$$\begin{aligned}\boldsymbol{I}(x,z) = \boldsymbol{H}^{\mathsf{BBS}}(x,z) \cap \boldsymbol{H}^{\mathsf{GPS}}(x,z) \\ \cap \boldsymbol{H}^{\mathsf{RAD}}(x,z) \cap \boldsymbol{H}^{\mathsf{ObnS}}(x,z).\end{aligned} \tag{1}$$

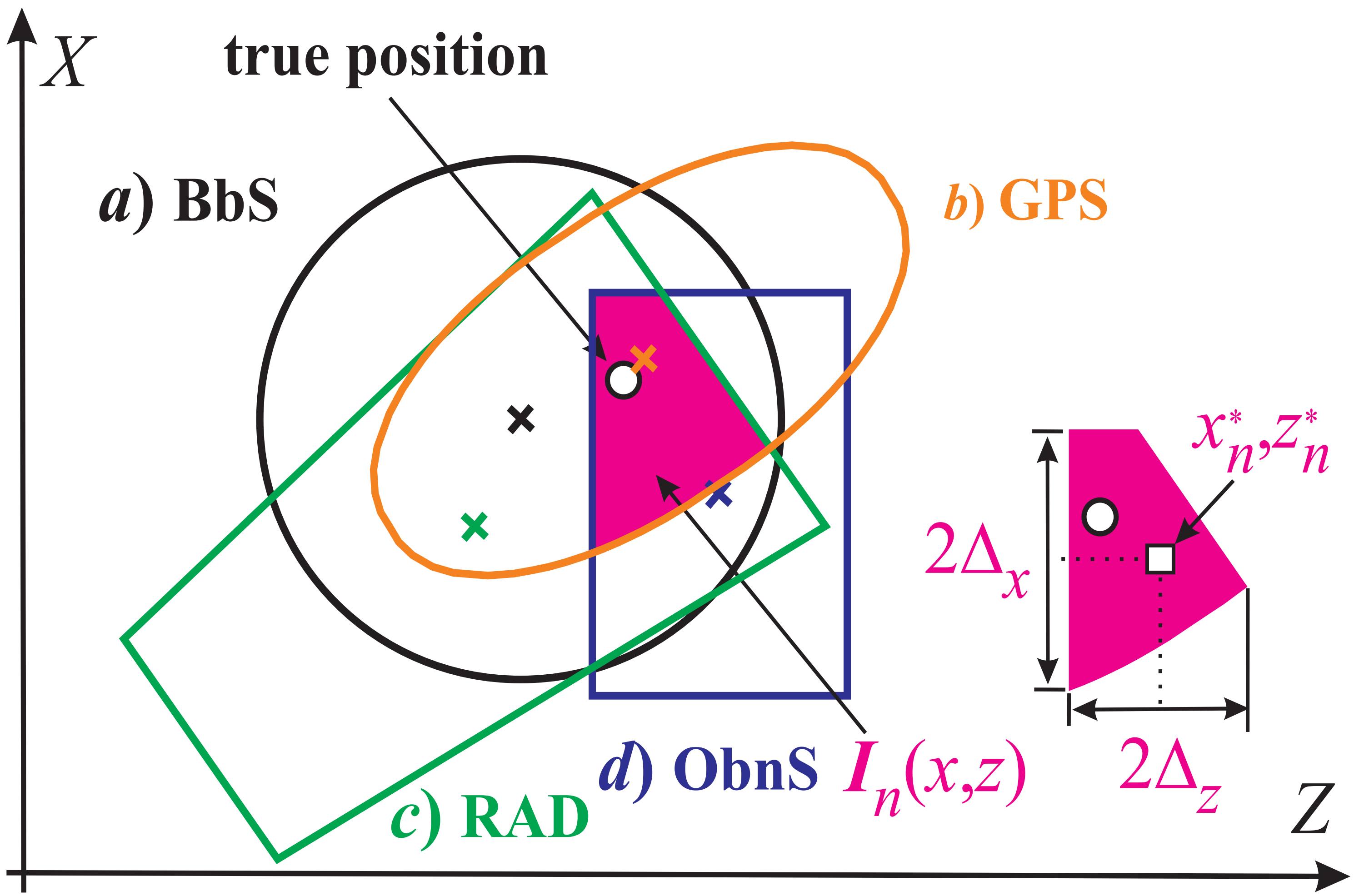
Remark (Fig. 2)

In practice of ATC, preliminary processing the noised data is obligatory implemented. It is performed to clean off "suspicious outliers" in the information coming from the radio-, radar-, GPS- and other systems. Usually, to do this, the simple **"rough filtration"** is implemented by means of approximate sets (so called "strobes") of possible positions of the aircraft, for example, "Korolev E.N. Tech nologies for operation..." or "Pyatko S.G., Krasov A.I., etc. Automated systems..." [8,9]. But the sample is very short, and elimination of "suspicious" measurements has to be seriously validated. Below, we show how it could be done by the Interval Analysis methods.



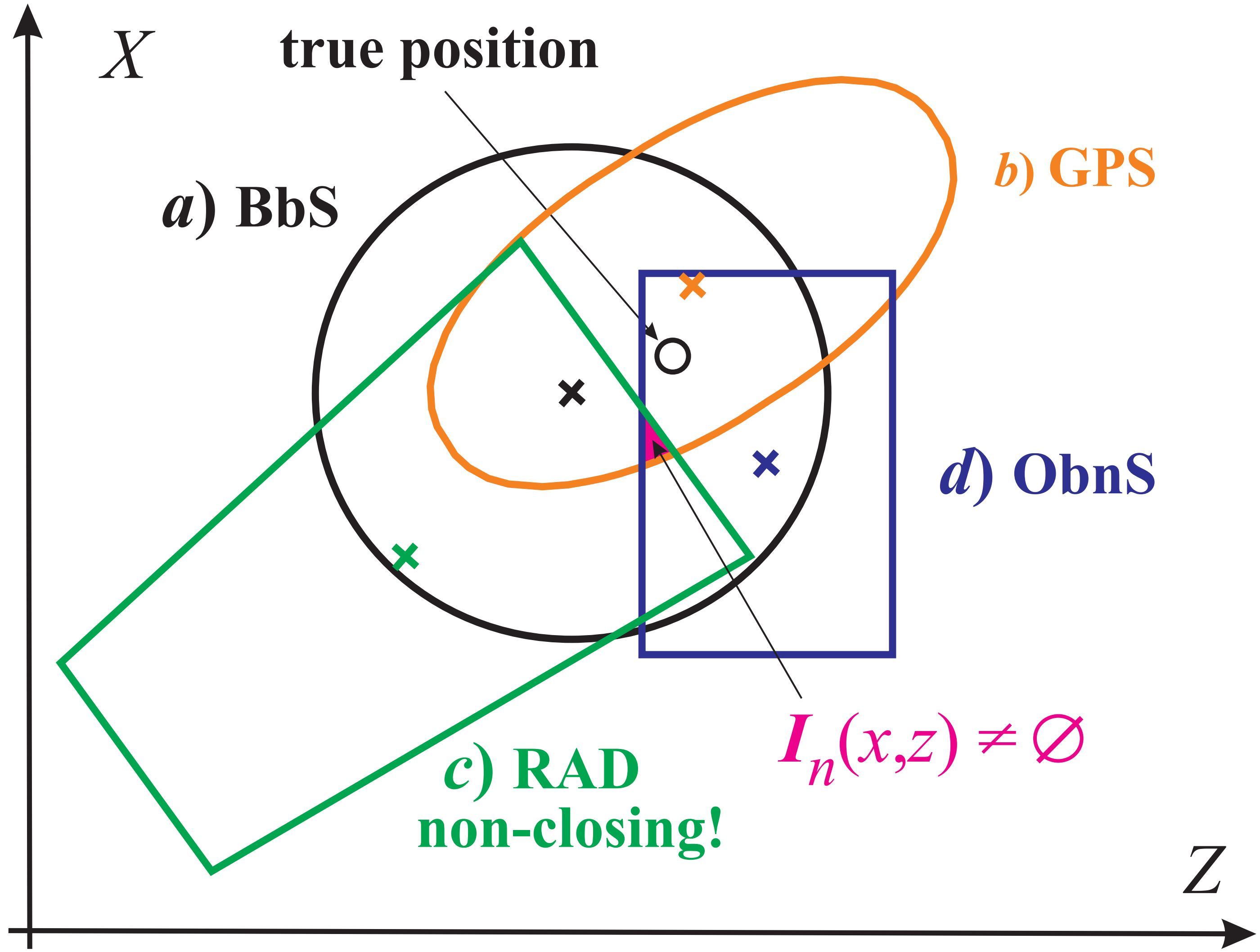
The case when all the measurements are closing (Fig. 3)

Information set exists, *i.e.*, is not empty. The output information: the information set itself, its central point x^*, z^* (square on the insertion), and the values Δ_x and Δ_z as the estimates of its accuracy on the mesuremtn's coordinates.

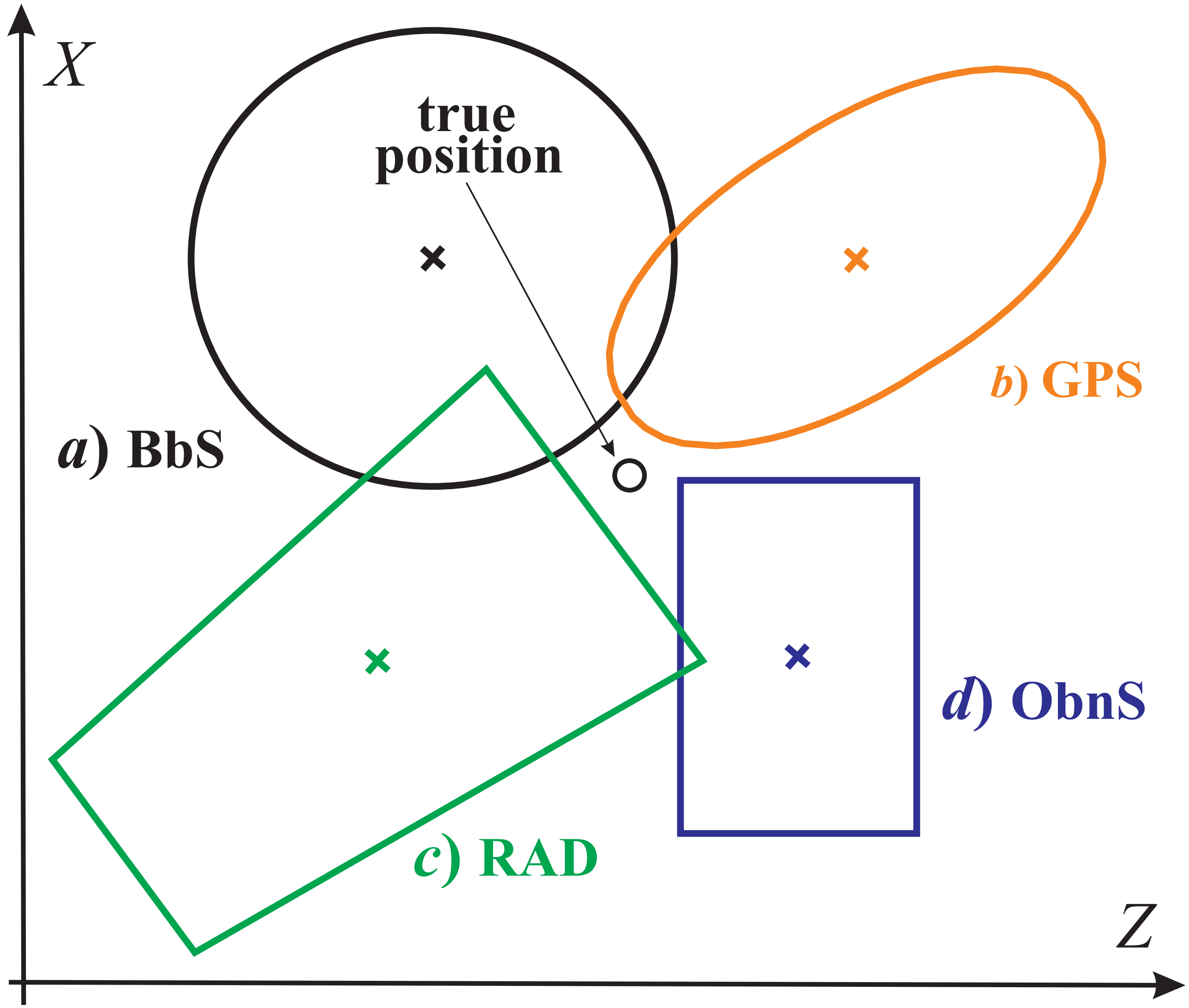


The case: the measurements are partially closing (Fig. 4)

Since the noising influence, the situation is possible when the measurements are only partially closing, but the Information set (1) exists, *i.e.*, is not empty.

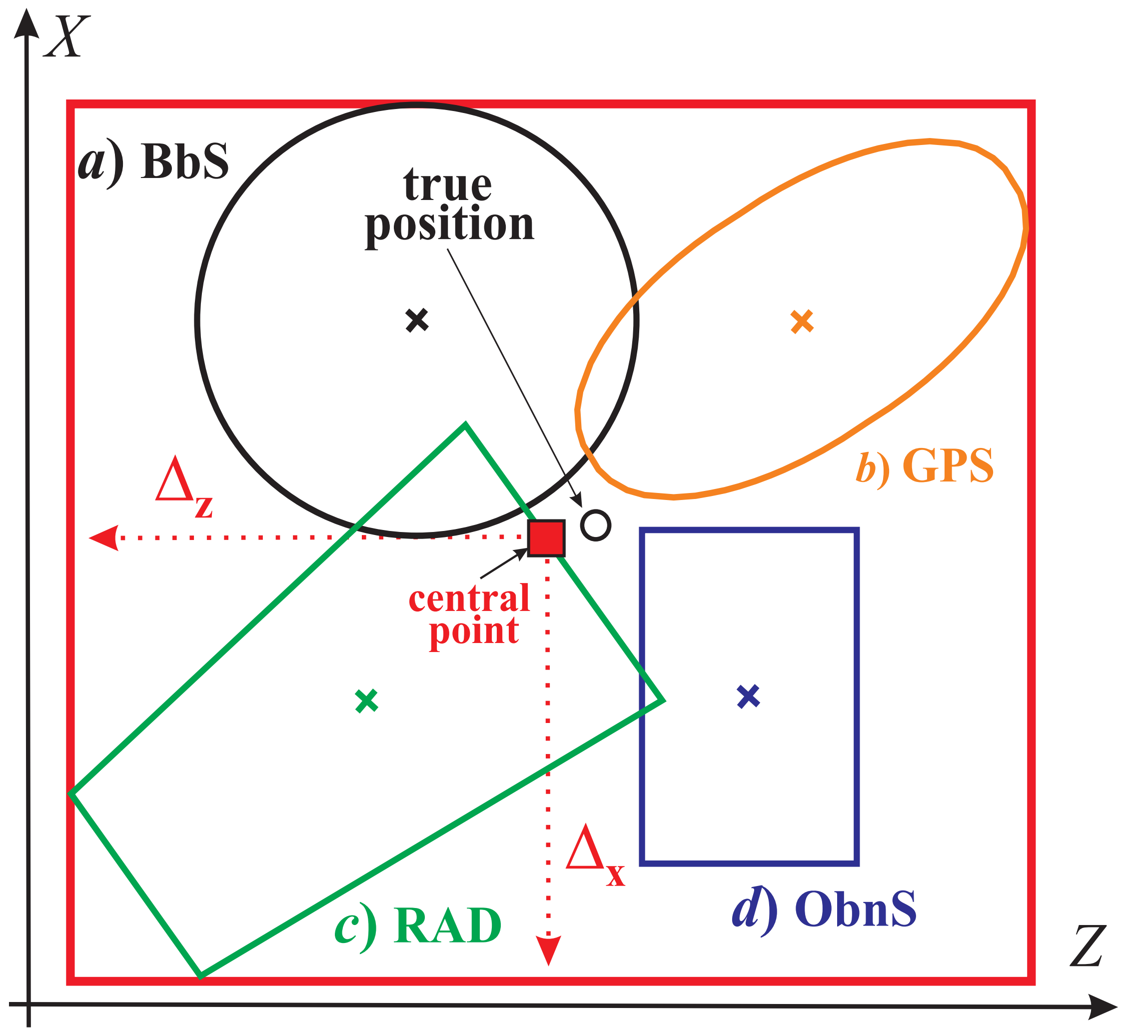


What to do when all the measurements are non-closing (Fig. 5) and the Information set is empty?



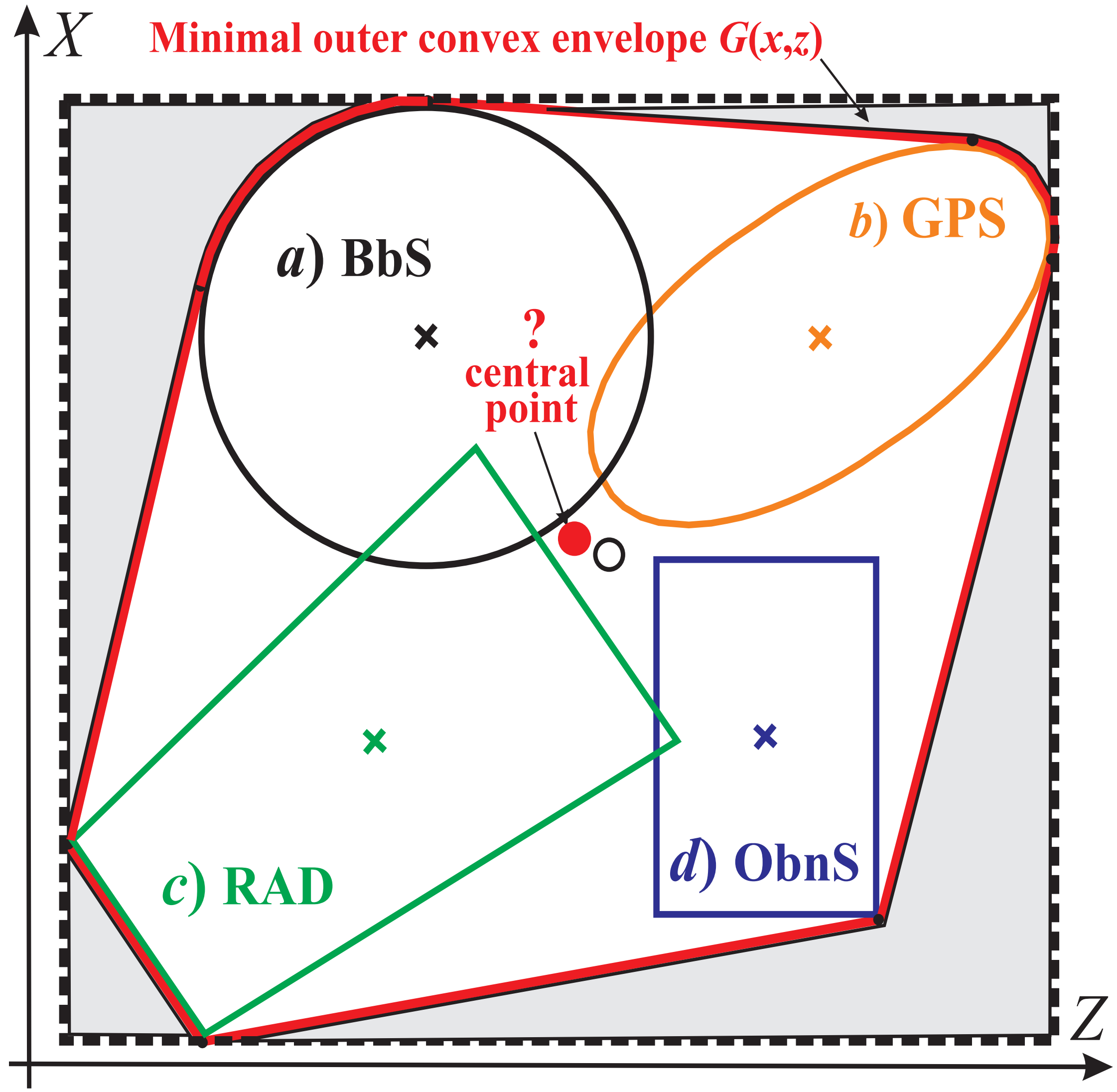
Procedure 1 (Fig. 6)

The most popular and simple procedure: application of the minimal outer parallelotope (the "minimal outer box"). The output information: the central point of the box and the values Δ_x and Δ_z as the estimates of its accuracy on the coordinates.



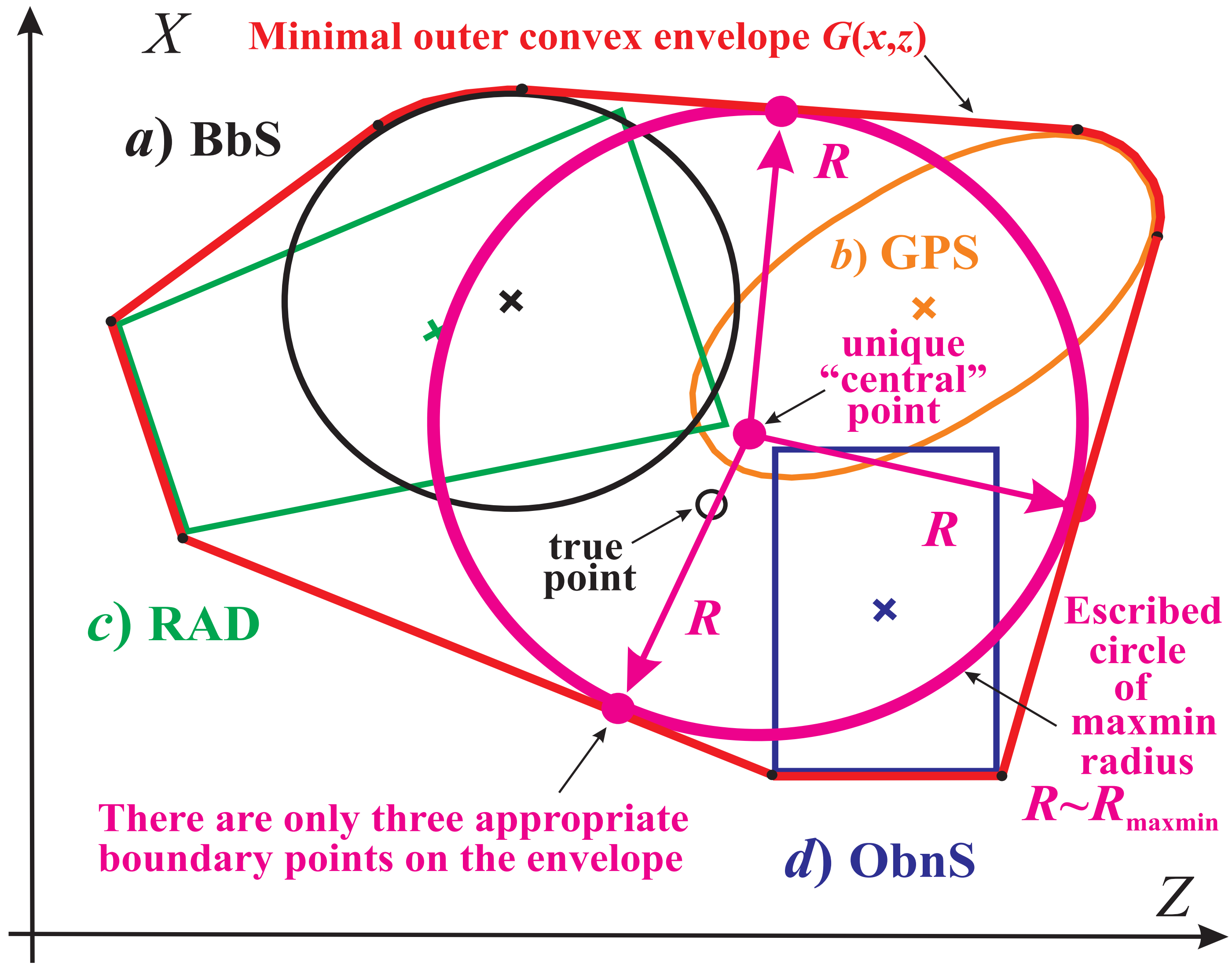
Sophisticated Procedure 2 (Fig. 7)

It is possible to improve the approximate estimates (as in Fig. 6) by building the *minimal outer convex envelope* [7,8] as the *minimal by inclusion* set that contains all the measurements' uncertainty sets without the redundant parts (in grey).



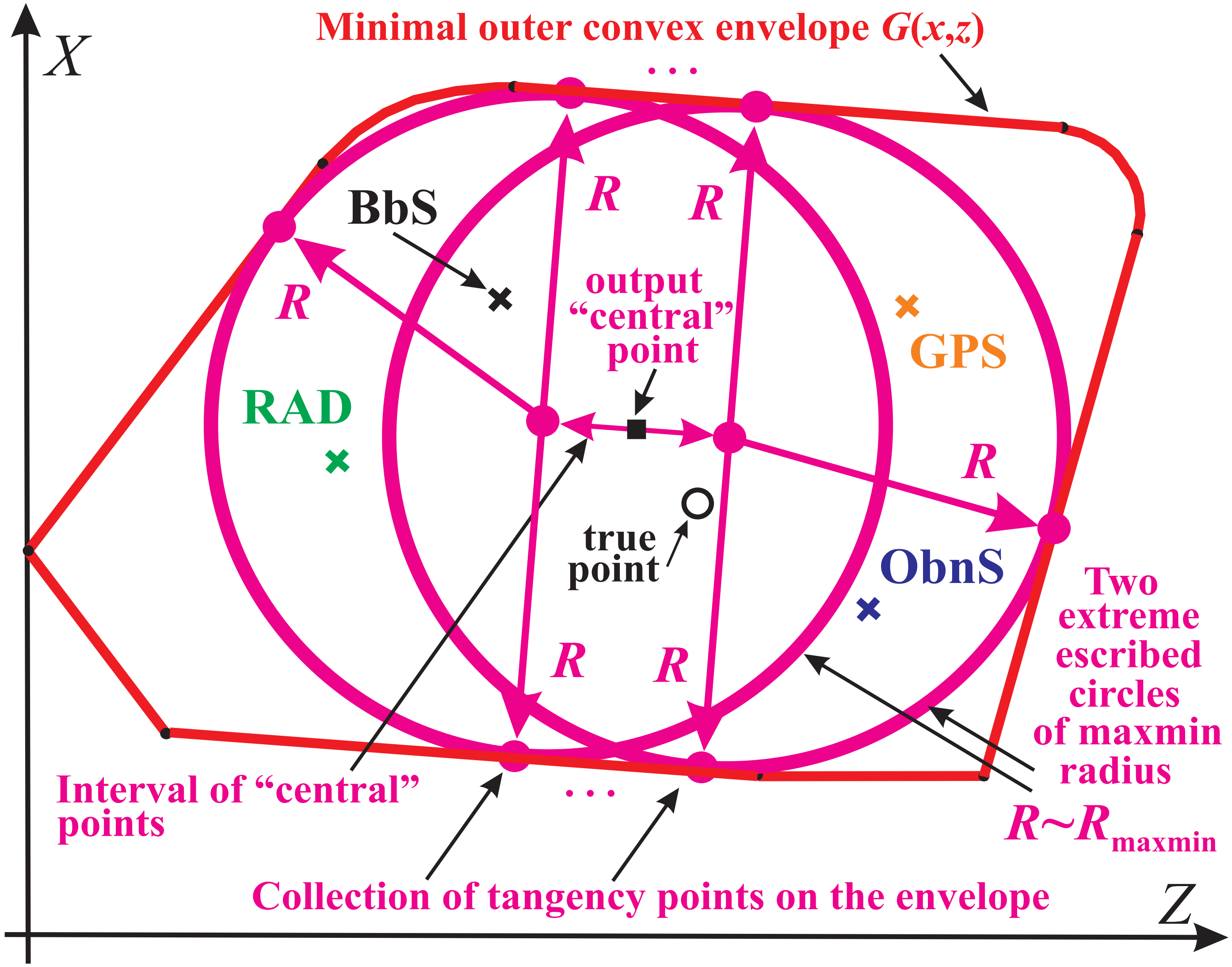
Procedure 2 (continue, Fig. 8)

The output "central" point can be found as the center of escribed (into the envelope) circle of the *maxmin* radius. The case is shown when such a point is unique, *i.e.*, when only one circle with such property can be escribed.



Procedure 2 (continue, Fig. 9)

The situation may be when there are a collection of the escribed circles and their centers compose an interval or, even, a curve. In this case, the output "central" point can be found as the center of this interval (the black square).



Procedure 3 (more fine dynamic filtration)

The procedure unites the idea of dynamic filration by means of more accurate reachable set, see Kostousova [5] of possible predicted positions of the aircraft (similarly to operation in Fig. 2) with the idea of using the uncertainty sets of measurements and the idea of the minimal outer convex envelope (Fig. 7).

The following steps are performed:

— the auxiliary sets are formed by intersection of the uncertainty sets with the predicted set of possible positions of the aircraft (Fig. 10);

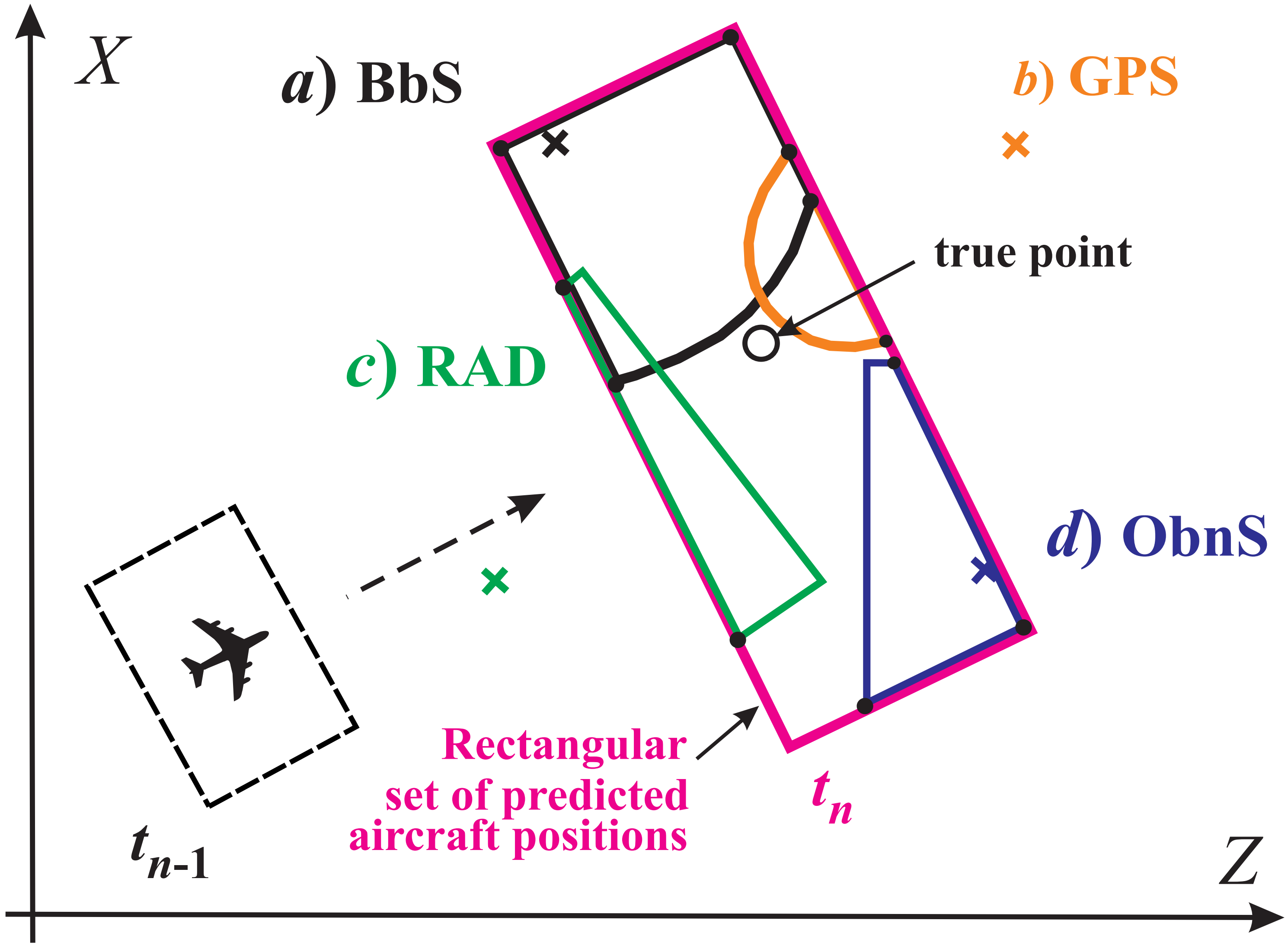
— the minimal outer convex envelope is built for possible improving the possible set of positions (Fig. 11);

— by means of the escribed circles of the *maxmin* radii, the interval of possible centers of these circles is found (Fig. 11);

— as the output "central" point and the center of this interval are given out (Fig. 11).

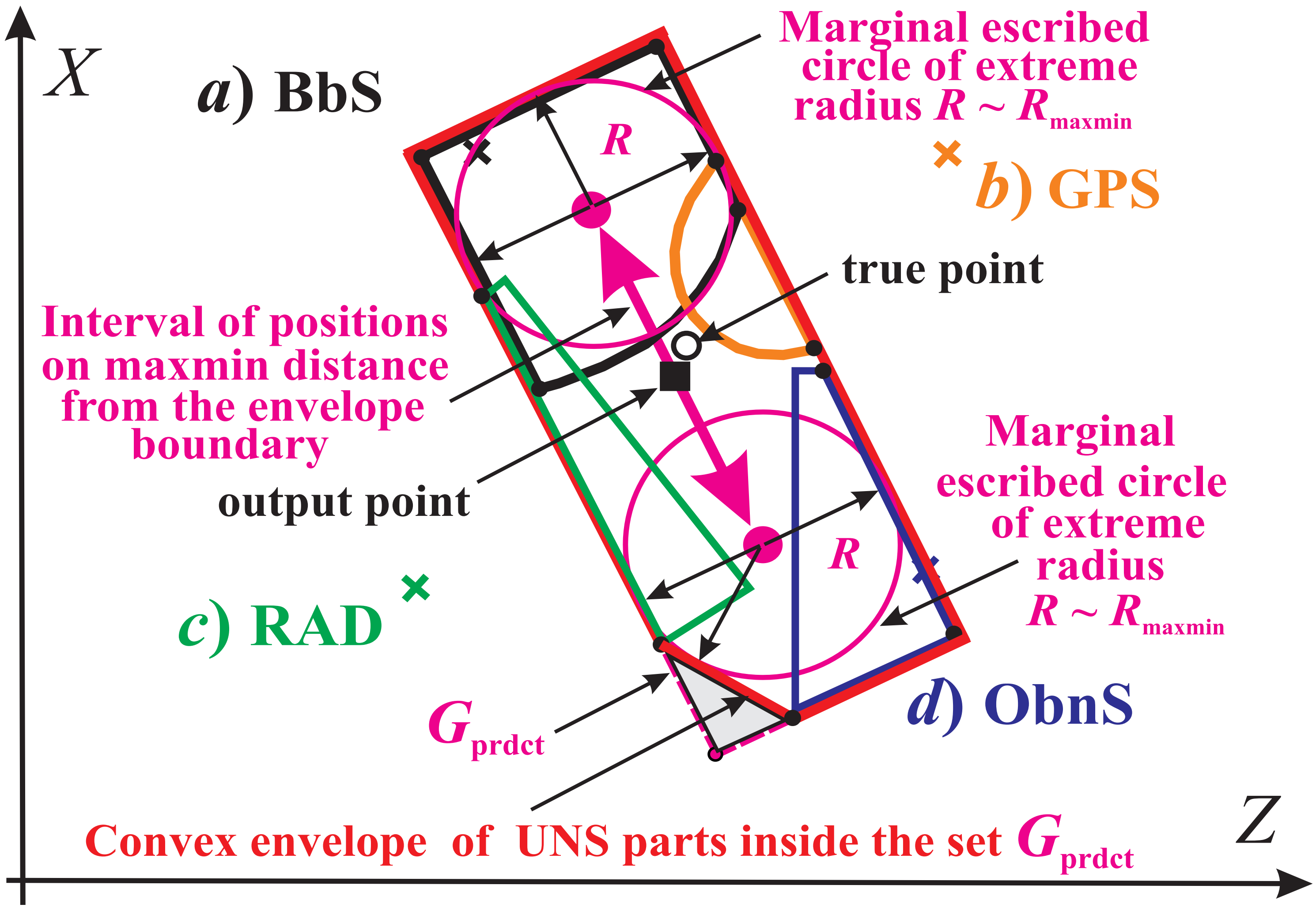
Procedure 3 (continue, Fig. 10)

Taking into account the uncertainty sets of measurements by intersecting them with the predicted reachable set of possible positions of the aircraft.



Procedure 3 (continue, Fig. 11)

The predicted set G_{prdct} of possible positions, the minimal outer convex envelope, the escribed circles of the *maxmin* radii, the interval (or a curve) of possible centers, and the output "central" point.



Conclusions

Implementation and simulation of the elaborated interval approach to processing noised information have shown, for example, "Patsko V.S., Fedotov A.A., et.al, Informational Sets..." and "Kumkov S.I., Patsko V.S. Informational Sets..." [6,7] that application of the Interval Analysis methods (in contrast to the standard statistical ones) allow us:

— to use constructively information of the measurements' uncertainty sets from each of various information sources;

— to obtain (in the cases of all closing measurements) the accurate set that with guarantee contains the true aircraft position;

— in the cases when some measurements are not closing but the information set is not empty, the set is placed most closely to the true point;

— in the cases when all the measurements in the sample are non-closing, the interval methods give more validated estimates, since using additional information of the measuring results.

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Thanks for attention