Set-membership estimation for discrete-time systems: the two-step method

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Presented by
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Existing methods

- **Geometrical methods:** Ellipsoids, polytopes, zonotopes, etc.;
- **Interval observer:** two sub-observers designed based on the monotone system theory.

Motivation

Existing problems

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- Design a set-membership estimation method for discrete-time systems with less computational complexity;
- Reduce the approximation error to increase the estimation accuracy.

Preliminaries

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- **Definition 1.** Given two sets $\mathbf{S}_1 \subset \mathbb{R}^n$ and $\mathbf{S}_2 \subset \mathbb{R}^n$, their Minkowski sum is defined as

$$\mathbf{S}_1 \oplus \mathbf{S}_2 = \{s \colon s = s_1 + s_2, \quad s_1 \in \mathbf{S}_1,\, s_2 \in \mathbf{S}_2\}.$$

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$$\mathbf{S}_1 \oplus \mathbf{S}_2 = \{s \colon s = s_1 + s_2, \quad s_1 \in \mathbf{S}_1,\, s_2 \in \mathbf{S}_2\}.$$
- **Definition 2.** For a set $\mathbf{S} \subset \mathbb{R}^n$, its *interval hull* is defined as the smallest interval vector containing it, which is denoted as
$$\mathbf{S} \subseteq \mathrm{Box}(\mathbf{S}) = [a, b],$$
where $a = [a_1, \ldots, a_n]^T$ and $b = [b_1, \ldots, b_n]^T$.

Definitions

Definition 3. An m-order zonotope $\mathcal{Z} \subset \mathbb{R}^n$ is an affine transformation of a hypercube $\mathbf{B}^m$, which is defined as

$$\mathcal{Z} = \langle p, H \rangle = \{p + Hz : z \in \mathbf{B}^m\},$$

where $p \in \mathbb{R}^n$ is the center of $\mathcal{Z}$ and $H \in \mathbb{R}^{n \times m}$ is called its generator matrix.

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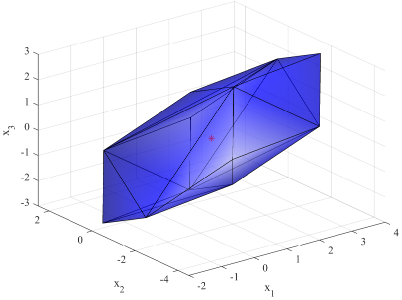

Figure: A 4-order zonotope.

Properties

Property 1. (Minkowski Sum) Given two zonotopes $\langle p_1, H_1\rangle \subset \mathbb{R}^n$ and $\langle p_2, H_2\rangle \subset \mathbb{R}^n$, their Minkowski sum satisfies

$$\langle p_1, H_1\rangle \oplus \langle p_2, H_2\rangle = \langle p_1 + p_2, \begin{bmatrix} H_1, & H_2 \end{bmatrix}\rangle.$$

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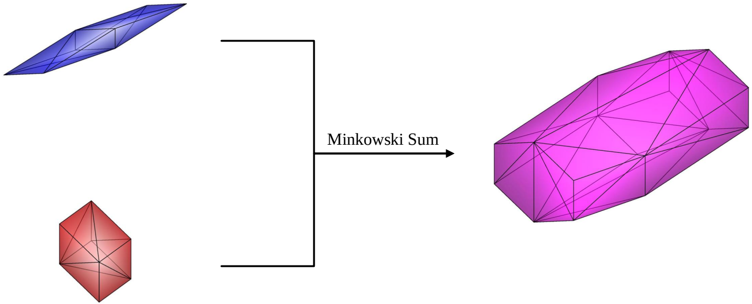

Figure: The Minkowski sum of two zonotopes.

Properties

Property 2. (Linear Transformation) Given a zonotope $\mathcal{Z} = \langle p, H \rangle \subset \mathbb{R}^n$, its linear transformation associated with the matrix $L \in \mathbb{R}^{m \times n}$ satisfies

$$L\mathcal{Z} = \langle Lp, LH \rangle.$$

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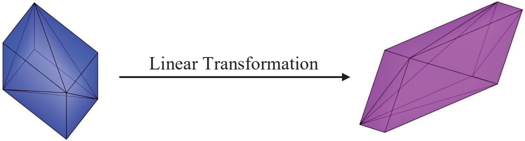

Figure: The linear transformation of a zonotope.

Properties

Property 3. (Interval Hull) For an m-order zonotope $\mathcal{Z} = \langle p, H \rangle \subset \mathbb{R}^n$, the components of $\mathrm{Box}(\mathcal{Z}) = [a, b]$ can be obtained from

$$\begin{cases} a_i = p_i - \sum_{j=1}^{m} |H_{i,j}|, & i = 1, \ldots, n \\ b_i = p_i + \sum_{j=1}^{m} |H_{i,j}|, & i = 1, \ldots, n \end{cases}$$

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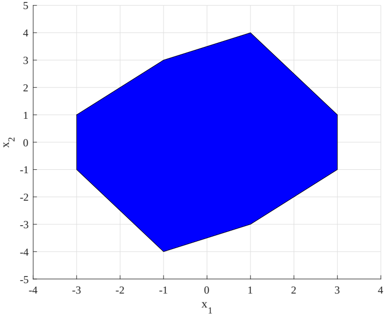

Figure: A zonotope.

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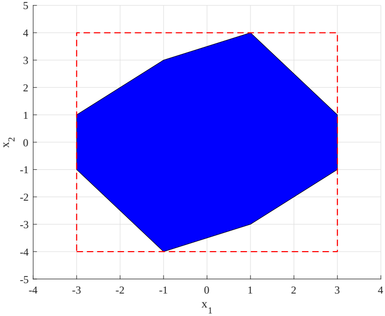

Figure: The interval hull of the zonotope.

Problem formulation

System description

Consider the following discrete-time system:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Ew_k \\ y_k = Cx_k + Fv_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$, $u_k \in \mathbb{R}^{n_u}$, $y_k \in \mathbb{R}^{n_y}$, $w_k \in \mathbb{R}^{n_w}$ and $v_k \in \mathbb{R}^{n_v}$.

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Assumption

x_0, w_k and v_k are unknown but bounded as follows

$$x_0 \in \langle p_0, H_0 \rangle, \quad w_k \in \mathbf{W} = \langle 0, H_w \rangle, \quad v_k \in \mathbf{V} = \langle 0, H_v \rangle,$$

where $p_0 \in \mathbb{R}^{n_x}$, $H_0 \in \mathbb{R}^{n_x \times n_x}$, $H_w \in \mathbb{R}^{n_w \times n_w}$ and $H_v \in \mathbb{R}^{n_v \times n_v}$.

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Objective

- We aim to estimate a convex $\mathbf{X}_k$ such that $x_k \in \mathbf{X}_k$ for all $k \geq 0$.

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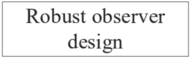

Figure: The schematic of the two-step method.

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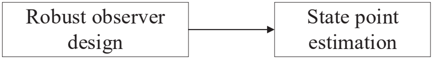

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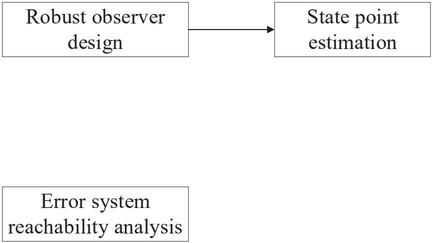

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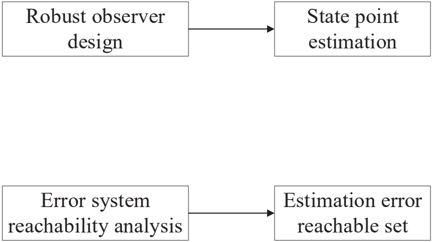

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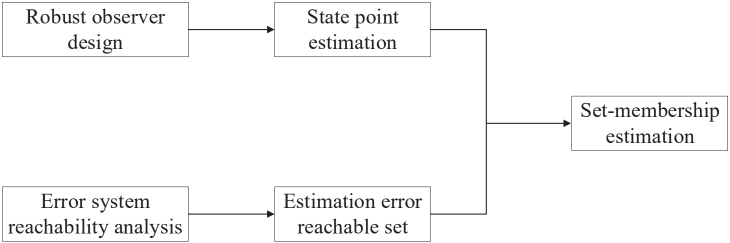

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Two-step set-membership estimation

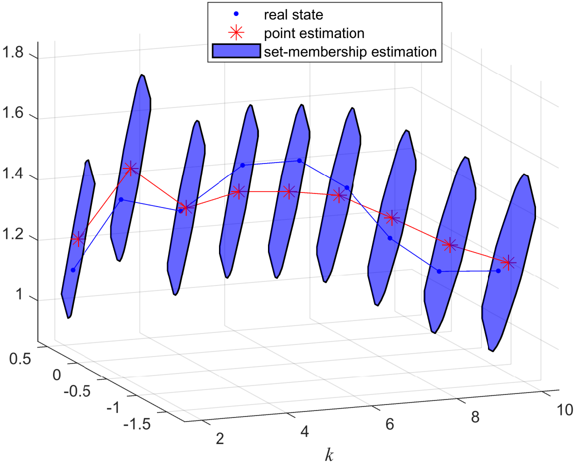

Figure: The demonstration of the two-step method.

Two-step set-membership estimation

Observer

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L(y_k - C\hat{x}_k) \longrightarrow \text{state point estimation}$$

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$$e_{k+1} = (A - LC)e_k + Ew_k - LFv_k \xrightarrow{\text{reachability analysis}} e_k \in \mathbf{E}_k$$

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Set-membership estimation

$$x_k \in \mathbf{X}_k = \hat{x}_k \oplus \mathbf{E}_k$$

Robust observer design

Size of $\mathbf{E}_k$ $\longrightarrow$ estimation accuracy

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Robust observer design based on H_∞ technique

Given a scalar $\gamma > 0$, if there exist a positive definite matrix $P \in \mathbb{R}^{n_x \times n_x}$ and a matrix $Y \in \mathbb{R}^{n_x \times n_y}$ such that

$$\begin{bmatrix} I_{n_x} - P & * & * & * \\ 0 & -\gamma^2 I_{n_w} & * & * \\ 0 & 0 & -\gamma^2 I_{n_v} & * \\ PA - YC & PE & -YF & -P \end{bmatrix} \prec 0$$

and $L = P^{-1}Y$, then the transfer function $G_{ed}(z) = (zI_{n_x} - A_e)^{-1}B_e$ satisfies $\|G_{ed}(z)\|_\infty < \gamma$. Moreover, e_k is bounded.

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Minimize γ to increase estimation accuracy

Set-membership estimation

Reachability analysis

$$e_{k+1} \in (A - LC)\langle 0,\ \tilde{H}_k \rangle \oplus E\mathbf{W} \oplus (-LF)\mathbf{V} \quad \longrightarrow \quad e_k \in \langle 0,\ H_k \rangle$$

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$$H_{k+1} = \begin{bmatrix} (A - LC)\tilde{H}_k & EH_w & -LFH_v \end{bmatrix}$$

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Order reduction

Limit the dimension of $\langle 0,\ H_k \rangle$:

$$\tilde{H}_k = \begin{cases} H_k, & m \leq s; \\ \mathscr{R}_s(H_k), & m > s. \end{cases}$$

Interval estimation

Interval estimation

$$\underline{e}_k \leq e_k \leq \overline{e}_k \quad \longrightarrow \quad \underline{e}_k + \hat{x}_k \leq x_k \leq \hat{x}_k + \overline{e}_k$$

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Estimation error

$$e_k = (A - LC)^k e_0 + \sum_{i=0}^{k-1} (A - LC)^i E w_{k-1-i} + \sum_{i=0}^{k-1} (A - LC)^i (-LF v_{k-1-i})$$

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Reachable set

$$\mathbf{E}_k = (A - LC)^k \mathbf{E}_0 \oplus \bigoplus_{i=0}^{k-1} (A - LC)^i E\mathbf{W} \oplus \bigoplus_{i=0}^{k-1} (A - LC)^i (-LF\mathbf{V})$$

Interval hull

Properties

- **Property 1.** Give sets $\mathbf{S}_i \subset \mathbb{R}^n$ $(i = 1, \ldots, m)$, the interval hull of their Minkowski sum satisfies

$$\mathrm{Box}(\bigoplus_{i=1}^{m} \mathbf{S}_i) = \bigoplus_{i=1}^{m} \mathrm{Box}(\mathbf{S}_i)$$

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- **Property 2.** Given two interval vectors $[a,\ b] \subset \mathbb{R}^n$ and $[c,\ d] \subset \mathbb{R}^n$, their Minkowski sum satisfies

$$[a,\ b] \oplus [c,\ d] = [a + c,\ b + d]$$

Interval estimation

Interval hull of error set

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Interval estimation

- Interval estimation of e_k: $[\underline{e}_k, \overline{e}_k] = \mathrm{Box}(\mathbf{E}_k)$

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Interval estimation

- Interval estimation of e_k: $[\underline{e}_k, \overline{e}_k] = \mathrm{Box}(\mathbf{E}_k)$
- Interval estimation of x_k: $\begin{cases} \underline{x}_k = \hat{x}_k + \underline{e}_k, \\ \overline{x}_k = \hat{x}_k + \overline{e}_k \end{cases}$

Interval estimation

Algorithm 1 Interval estimation based on reachability analysis

Input: u_k, y_k

Output: $\overline{x}_k$, $\underline{x}_k$

Initialization:

$\hat{x}_0 = p_0$, $\mathbf{D}_{w_0} = E\mathbf{W}$, $\mathbf{D}_{v_0} = -LF\mathbf{V}$

$\mathbf{S}_{x_0} = \langle 0, H_0 \rangle$, $\mathbf{S}_{w_0} = \varnothing$, $\mathbf{S}_{v_0} = \varnothing$

for $k \geq 0$ **do**

$\quad [\underline{e}_k, \overline{e}_k] = \mathrm{Box}(\mathbf{S}_{x_k}) \oplus \mathbf{S}_{w_k} \oplus \mathbf{S}_{v_k}$

$\quad \overline{x}_k = \hat{x}_k + \overline{e}_k$

$\quad \underline{x}_k = \hat{x}_k + \underline{e}_k$

$\quad \hat{x}_{k+1} = A\hat{x}_k + Bu_k + L(y_k - C\hat{x}_k)$

$\quad \mathbf{S}_{x_{k+1}} = (A - LC)\mathbf{S}_{x_k}$

$\quad \mathbf{S}_{w_{k+1}} = \mathbf{S}_{w_k} \oplus \mathrm{Box}(\mathbf{D}_{w_k})$

$\quad \mathbf{S}_{v_{k+1}} = \mathbf{S}_{v_k} \oplus \mathrm{Box}(\mathbf{D}_{v_k})$

$\quad \mathbf{D}_{w_{k+1}} = (A - LC)\mathbf{D}_{w_k}$

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Comparison with other methods

Comparison (theoretically provable)

Under the same conditions:

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- The interval estimation obtained by Algorithm 1 is more accurate than that by the zonotope-based method[1];

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Comparison (theoretically provable)

Under the same conditions:

- The interval estimation obtained by Algorithm 1 is more accurate than that by the zonotope-based method[1];
- The interval estimation obtained by Algorithm 1 is more accurate than that by the regular interval observer[2];

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[2] Efimov D, Rassi T. Design of interval observers for uncertain dynamical systems[J]. Automation and Remote Control, 2016, 77(2):191–225.

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- The interval estimation obtained by Algorithm 1 is more accurate than that by the zonotope-based method[1];
- The interval estimation obtained by Algorithm 1 is more accurate than that by the regular interval observer[2];
- The interval estimation obtained by Algorithm 1 is more accurate than that by the interval observer based on coordinate transformation[3]:

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[1] Combastel C. Zonotopes and Kalman observers: Gain optimality under distinct uncertainty paradigms and robust convergence[J]. Automatica, 2015, 55:265–273.
[2] Efimov D, Rassi T. Design of interval observers for uncertain dynamical systems[J]. Automation and Remote Control, 2016, 77(2):191–225.
[3] Efimov D, Perruquetti W, Rassi T, et al. On interval observer design for timeinvariant discrete-time systems[C]. Proceedings of 2013 European Control Conference, Zurich, Switzerland: IEEE, 2013: 2651–2656.

Simulation results

Consider a DC motor[1]:

$$\begin{bmatrix} \dot{\theta} \\ \dot{n} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{\mu}{J} & \frac{K_t}{J} \\ 0 & -\frac{K_e}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \theta \\ n \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u.$$

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The discrete-time model parameters:

$$A = \begin{bmatrix} 1 & 0.1 & 0 \\ 0 & 0.8495 & 0.4977 \\ 0 & -0.0357 & 0.9995 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0.0729 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, E = I_3, F = I_2.$$

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$$|w_k| \le \begin{bmatrix}0.0225\\ 0.0225\\ 0.0404\end{bmatrix},\quad |v_k| \le \begin{bmatrix}0.0564\\ 0.0564\end{bmatrix}.$$

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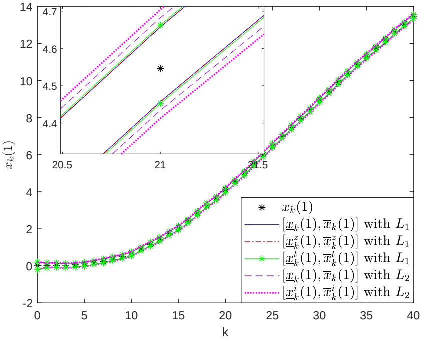

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L_1–the observer gain designed by the proposed method;
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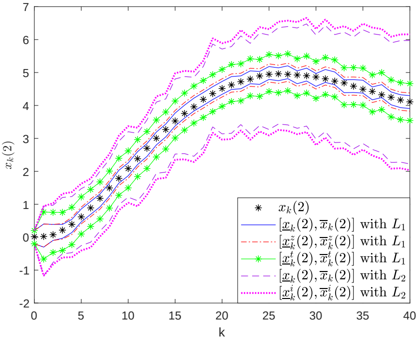

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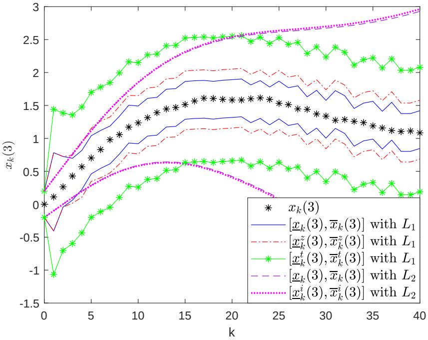

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Problem formulation

System description

Consider the following descriptor system:

$$\begin{cases} Ex_{k+1} = Ax_k + Bu_k + D_w w_k \\ y_k = Cx_k + D_v v_k \end{cases} \tag{2}$$

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Assumptions

- **Assumption 1.** (UBB, Unknown But Bounded):

$$x_0 \in \langle \hat{x}_0,\ H_0 \rangle, \quad w_k \in \mathbf{W} = \langle 0,\ H_w \rangle, \quad v_k \in \mathbf{V} = \langle 0,\ H_v \rangle.$$

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- **Assumption 2.** (Observable):

$$\operatorname{rank} \begin{bmatrix} E \\ C \end{bmatrix} = n_x, \quad \operatorname{rank} \begin{bmatrix} zE - A \\ C \end{bmatrix} = n_x, \quad z \in \mathbb{C}.$$

Two-step set-membership estimation

Observer

$$\hat{x}_k = TA\hat{x}_{k-1} + TBu_{k-1} + L(y_{k-1} - \hat{x}_{k-1}) + Ny_k \longrightarrow \text{state point estimation}$$

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Error system

$$e_k = (TA - LC)e_{k-1} + TD_w w_{k-1} - LD_v v_{k-1} - ND_v v_k \longrightarrow e_k \in \mathbf{E}_k = \langle 0,\ H_k \rangle$$

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Set-membership estimation

$$x_k \in \mathbf{X}_k = \hat{x}_k \oplus \mathbf{E}_k$$

Robust observer design

Robust observer design based on H_∞ technique

Given a scalar $\gamma > 0$, if there exist a positive definite matrix $P \in \mathbb{R}^{n_x \times n_x}$ and matrices $W \in \mathbb{R}^{n_x \times n_y}$, $Y \in \mathbb{R}^{n_x \times (n_x+n_y)}$ such that

$$\begin{bmatrix} I_{n_x} - P & * & * & * & * \\ 0 & -\gamma^2 I_{n_w} & * & * & * \\ 0 & 0 & -\gamma^2 I_{n_v} & * & * \\ 0 & 0 & 0 & -\gamma^2 I_{n_v} & * \\ \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & -P \end{bmatrix} \prec 0,$$

$$\begin{aligned}
\Omega_1 &= P\Theta^\dagger \alpha_1 A + Y\Psi\alpha_1 A - WC, \\
\Omega_2 &= P\Theta^\dagger D_w + Y\Psi\alpha_1 D_w, \\
\Omega_3 &= -WD_v, \\
\Omega_4 &= -P\Theta^\dagger \alpha_2 D_v - Y\Psi\alpha_2 D_v, \\
\Theta &= \begin{bmatrix} E \\ C \end{bmatrix}, \quad \alpha_1 = \begin{bmatrix} I_{n_x} \\ 0 \end{bmatrix}, \quad \alpha_2 = \begin{bmatrix} 0 \\ I_{n_y} \end{bmatrix}, \\
\Psi &= I_{n_x+n_y} - \Theta\Theta^\dagger
\end{aligned}$$

Robust observer design

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and T, N, L satisfy

$$\begin{aligned} T &= \Theta^\dagger \alpha_1 + P^{-1} Y \Psi \alpha_1, \\ N &= \Theta^\dagger \alpha_2 + P^{-1} Y \Psi \alpha_2, \\ L &= P^{-1} W \end{aligned}$$

then, estimation error is robust against disturbance and noise, and satisfies $\|G_{ed}(z)\|_\infty < \gamma$.

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Optimization

Minimize γ to increase estimation accuracy

Set-membership estimation

Reachability analysis

$$e_{k+1} \in (TA - LC)\langle 0, \tilde{H}_k \rangle \oplus TD_w \langle 0, H_w \rangle \oplus (-LD_v)\langle 0, H_v \rangle \oplus (-ND_v)\langle 0, H_v \rangle$$

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Set-membership estimation

$$x_k \in \mathbf{X}_k = \langle \hat{x}_k, H_k\rangle, \quad \forall k \geq 0$$

Interval estimation

Interval estimation

$$\underline{e}_k \le e_k \le \overline{e}_k \quad \longrightarrow \quad \hat{x}_k + \underline{e}_k \le x_k \le \hat{x}_k + \overline{e}_k$$

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Estimation error

$$e_k = (TA - LC)^k e_0 + \sum_{i=0}^{k-1} (TA - LC)^i TD_w w_{k-1-i} + \sum_{i=0}^{k-1} (TA - LC)^i (-LD_v v_{k-1-i}) + \sum_{j=0}^{k-1} (TA - LC)^j (-ND_v v_{k-j})$$

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\begin{aligned}
e_k = (TA - LC)^k e_0 + \sum_{i=0}^{k-1} (TA - LC)^i TD_w w_{k-i} - (TA - LC)^{k-1} LD_v v_0 \\
- ND_v v_k + \sum_{i=0}^{k-2} (TA - LC)^i [(LC - TA)ND_v - LD_v] v_{k-1-i}, \quad k \geq 2 \\
e_1 = (TA - LC)e_0 + TD_w w_0 - LD_v v_0 - ND_v v_1
\end{aligned}
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Reachable sets

$$\mathbf{E}_k = (TA - LC)^k \mathbf{E}_0 \oplus \bigoplus_{i=0}^{k-1} (TA - LC)^i TD_w \mathbf{W} \oplus [-(TA - LC)^{k-1} LD_v] \mathbf{V}$$

$$\oplus (-ND_v)\mathbf{V} \oplus \bigoplus_{i=0}^{k-2} (TA - LC)^i [(LC - TA)ND_v - LD_v] \mathbf{V}, \quad k \geq 2$$

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$$\begin{cases} \underline{x}_k = \hat{x}_k + \underline{e}_k \\ \overline{x}_k = \hat{x}_k + \overline{e}_k \end{cases}$$

Algorithm 1 Interval estimation based on reachability analysis

Input: u_k, y_k

Output: $\overline{x}_k$, $\underline{x}_k$

Initialization:

$\mathbf{X}_0 = \langle 0, H_0 \rangle$, $\hat{x}_0 = p_0$, $\mathbf{M}_0 = D_w \mathbf{W}$, $\mathbf{D}_0 = \varnothing$, $\mathbf{N}_0 = \varnothing$, $\mathbf{S}a_0 = -LD_v\mathbf{V}$, $\mathbf{S}b_0 = -ND_v\mathbf{V}$, $\mathbf{S}c_0 = \varnothing$, $[\overline{e}_0, \underline{e}_0] = \text{Box}(\mathbf{X}_0)$, $\overline{x}_0 = \hat{x}_0 + \overline{e}_0$, $\underline{x}_0 = \hat{x}_0 + \underline{e}_0$.

for $k \geq 0$ **do**

- $\mathbf{X}_k = (TA - LC)\mathbf{X}_{k-1}$
- $\mathbf{D}_k = \mathbf{D}_{k-1} \oplus \text{Box}(\mathbf{M}_{k-1})$
- $\mathbf{N}_k = \mathbf{N}_{k-1} \oplus \text{Box}(\mathbf{S}a_{k-1}) \oplus \text{Box}(\mathbf{S}b_{k-1}) \oplus \text{Box}(\mathbf{S}c_{k-1})$
- $\mathbf{M}_k = (TA - LC)\mathbf{M}_{k-1}$
- $\mathbf{S}a_k = (TA - LC)\mathbf{S}a_{k-1}$
- $\mathbf{S}b_k = (TA - LC)\mathbf{S}b_{k-1}$
- **if** $k = 1$ **then**
  - $\mathbf{S}c_k = \big((LC - TA)ND_v - LD_v\big)\mathbf{V}$
- $\mathbf{S}c_k = (TA - LC)\mathbf{S}c_{k-1}$
- $[\overline{e}_k, \underline{e}_k] = \text{Box}(\mathbf{X}_k) \oplus \mathbf{D}_k \oplus \mathbf{N}_k$
- $\hat{x}_k = TA\hat{x}_{k-1} + TBu_{k-1} + L(y_{k-1} - C\hat{x}_{k-1}) + Ny_k$
- $\overline{x}_k = \hat{x}_k + \overline{e}_k$
- $\underline{x}_k = \hat{x}_k + \underline{e}_k$

Simulation results

Consider a numerical system with parameters as follows:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.8 & 0.95 & 0 \\ -1 & 0.5 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}, \quad D_w = I_3, \quad D_v = I_2.$$

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$$T = \begin{bmatrix} 0.8941 & 0.1059 & 0.3783 \\ 0.6901 & 0.3099 & 0.5364 \\ -0.8941 & -0.1059 & -0.3783 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & 0.1059 \\ 0 & -0.6901 \\ 1 & -0.1059 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.1546 & -0.1744 \\ 0.2156 & -0.3246 \\ -0.1545 & 0.1743 \end{bmatrix}.$$

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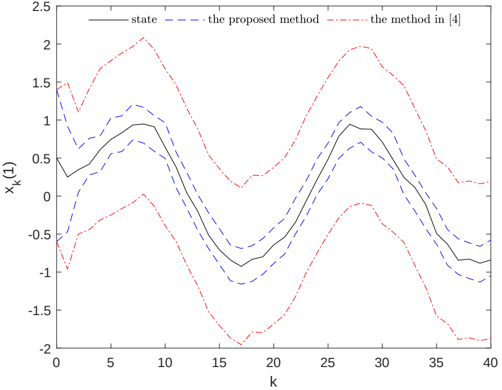

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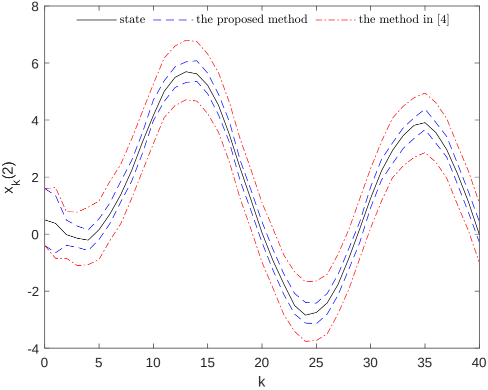

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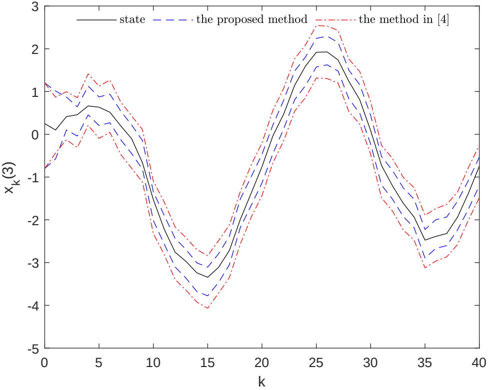

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Outlook

- Extend the proposed two-step method to nonlinear systems;
- Apply the two-step method to robust control for instance the constrained MPC.

Related work

[1] Wentao Tang, Zhenhua Wang, Ye Wang, Tarek Rassi, Yi Shen, Interval estimation methods for discrete-time linear time-invariant systems, IEEE Transactions on Automatic Control, 2019, 64(11): 4717-4724.

[2] Wentao Tang, Zhenhua Wang, Yi Shen, Interval estimation for discrete-time linear systems: A two-step method, Systems & Control Letters, 2019, 123: 69-74.

[3] Wentao Tang, Zhenhua Wang, Qinghua Zhang, Yi Shen, Set-membership estimation for linear time-varying descriptor systems, Automatica, 2020, 115, DOI: 10.1016/j.automatica.2020.108867.

Thanks for your attention!