Guaranteed Tracking Controller for Wheeled Mobile Robot Based on Flatness and Interval Observer

Amine Abadi, Adnen El Amraoui, Hassen Mekki, Nacim Ramdani

University of Orléans, INSA CVL, PRISME EA 4229, F45072 Orléans, France.

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PLAN

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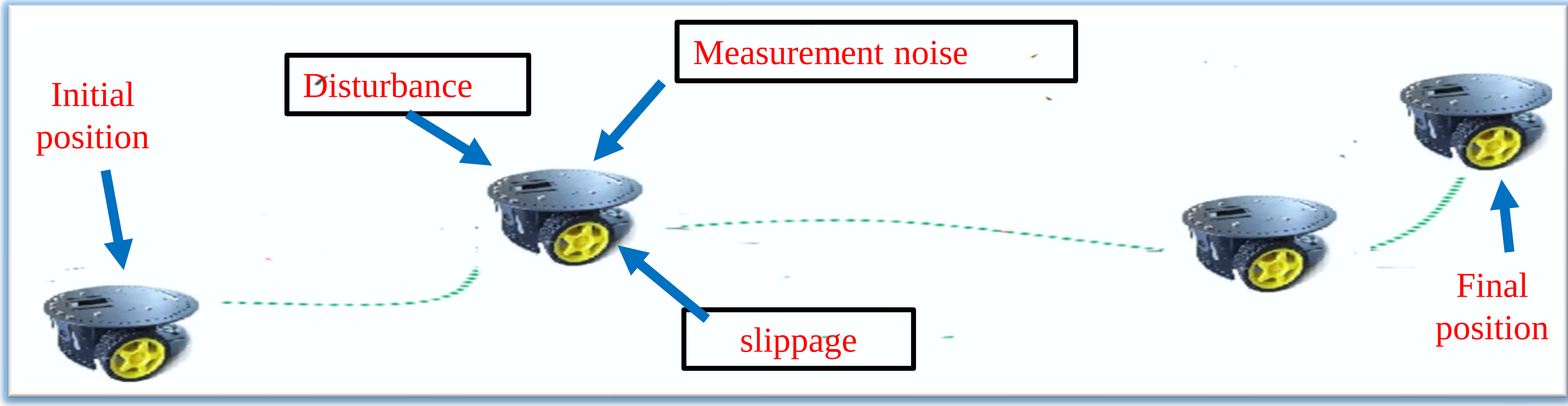
- Uncertain kinematic model
- Interval observer
- Guaranteed flatness based tracking control
- Simulation results
- Conclusion and perspective

PROBLEMATIC



Objective:

Our target consists in creating a performant guidance law for Wheeled Mobile Robot (WMR) enabling, its movement from an initial position to final one, despite the existence of un-measurable states and unknown but bounded uncertainties (slippage, external environmental disturbance and measurement noise).

Guaranteed flatness based tracking control

Flatness control

Flatness Control is used to transform the non linear WMR model into a canonical Brunovsky form, for which it is easier to create a state feedback controller

Interval observer

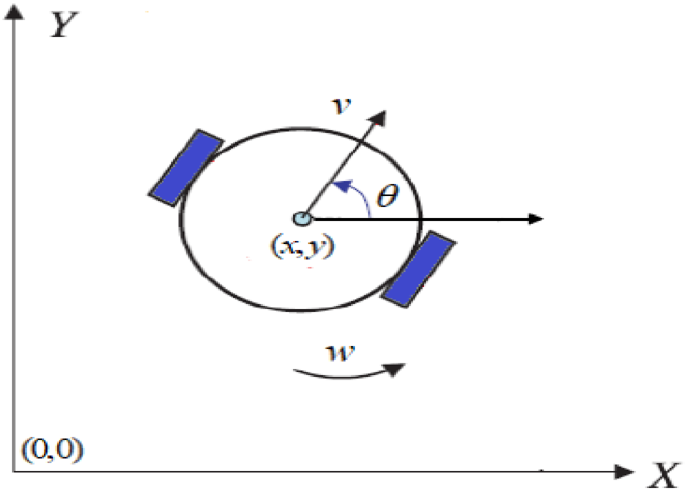
Interval observer is developed to generates an envelope enclosing every feasible state despite the existence of unknown but bounded uncertainties

Flatness Based Tracking Control

$$\dot{x} = f(x,u) \tag{1}$$

The nonlinear system (1) is differentially flat if there exists an output F such as:

$$x = \xi_1(F, \dot{F}, ..., F^r)$$
$$u = \xi_2(F, \dot{F}, ..., F^r, F^{r+1}) \tag{2}$$



Kinematic Model

$$\begin{cases} \dot{x} = \cos(\theta)v \\ \dot{y} = \sin(\theta)v \\ \dot{\theta} = \omega \\ U = [v, \omega] \quad X = [x, y, \theta] \end{cases} \tag{3}$$

Flat outputs → $\sigma = [x, y] = [\sigma_{11}, \sigma_{21}]$

$$v = \sqrt{\dot{\sigma}_{11}^2 + \dot{\sigma}_{21}^2} \tag{4}$$

$$\theta = \arctan\left(\frac{\dot{\sigma}_{21}}{\dot{\sigma}_{11}}\right) \tag{5}$$

$$w = \left(\frac{\dot{\sigma}_{11}\ddot{\sigma}_{21} - \ddot{\sigma}_{11}\dot{\sigma}_{21}}{\dot{\sigma}_{11}^2 + \dot{\sigma}_{21}^2}\right) \tag{6}$$

Flatness Based Tracking Control

New Kinematic Model

$$\begin{cases} \dot{x} = \cos(\theta)v \\ \dot{y} = \sin(\theta)v \\ \dot{v} = u_1 \\ \dot{\theta} = u_2 \\ Y_1 = x \quad Y_2 = y \end{cases} \qquad X_{new} = [x, y, v, \theta] \quad U_{new} = [u_1, u_2] \tag{7}$$

$$\sigma = [x, y] = [\sigma_{11}, \sigma_{21}] \qquad \begin{bmatrix} \ddot{\sigma}_{11} \\ \ddot{\sigma}_{21} \end{bmatrix} = B \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad B = \begin{bmatrix} \cos\theta & -v\sin\theta \\ \sin\theta & v\cos\theta \end{bmatrix} \tag{8}$$

The matrix B is not singular if $v \neq 0$. Under this assumption, the control can be defined as follows :

Flatness-based open loop control →

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = B^{-1} \begin{bmatrix} \ddot{\sigma}_{11} \\ \ddot{\sigma}_{21} \end{bmatrix} \tag{9}$$

Brunovky Form 1 $\begin{cases} \dot{\sigma}_{11} = \sigma_{12} \\ \dot{\sigma}_{12} = v_x \\ Y_1 = \sigma_{11} \end{cases}$ Brunovky Form 2 $\begin{cases} \dot{\sigma}_{21} = \sigma_{22} \\ \dot{\sigma}_{22} = v_y \\ Y_2 = \sigma_{21} \end{cases}$ (10)

Feedback Controller →

$$\begin{aligned} v_x &= \ddot{\sigma}_{xd} - K_{x2}\dot{e}_{rx1} - K_{x1}e_{rx1} \qquad e_{rx1} = \sigma_{11} - \sigma_{xd} \\ v_y &= \ddot{\sigma}_{yd} - K_{y2}\dot{e}_{ry1} - K_{y1}e_{ry1} \qquad e_{ry1} = \sigma_{21} - \sigma_{yd} \end{aligned} \tag{11}$$

Flatness Based Tracking Control →

$$\begin{bmatrix} u_{FBTC1} \\ u_{FBTC2} \end{bmatrix} = B^{-1} \begin{bmatrix} \ddot{\sigma}_{xd} - K_{x2}\dot{e}_{rx1} - K_{x1}e_{rx1} \\ \ddot{\sigma}_{yd} - K_{y2}\dot{e}_{ry1} - K_{y1}e_{ry1} \end{bmatrix} \tag{12}$$

Uncertain Kinematic Model

Uncertain Kinematic Model

$$\begin{cases} \dot{x} = \cos(\theta)v + v_t \cos\theta + v_s \sin\theta + d_x \\ \dot{y} = \sin(\theta)v + v_t \sin\theta - v_s \cos\theta + d_y \\ \dot{v} = u_1 \\ \dot{\theta} = u_2 + w_s + d_\theta \\ Y_1 = x + \eta_x \quad Y_2 = y + \eta_y \end{cases}$$

$$X = [x, y, v, \theta]$$

$$U = [u_1, u_2]$$

The slip component

$$v_t = v_s = w_s = \gamma_1 v$$

$$\begin{bmatrix} \ddot{\sigma}_{11} \\ \ddot{\sigma}_{21} \end{bmatrix} = B \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + C + D \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (13)$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = B^{-1} \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

Flatness control

Uncertain Brunovsky Form 1

$$\begin{cases} \dot{\sigma}_{11} = \sigma_{12} \\ \dot{\sigma}_{12} = v_x + \Delta_x \\ Y_1 = \sigma_{11} + \eta_x \end{cases}$$

Uncertain Brunovsky Form 2

$$\begin{cases} \dot{\sigma}_{21} = \sigma_{22} \\ \dot{\sigma}_{22} = v_y + \Delta_y \\ Y_2 = \sigma_{21} + \eta_y \end{cases} \qquad (14)$$

Lumped Disturbances

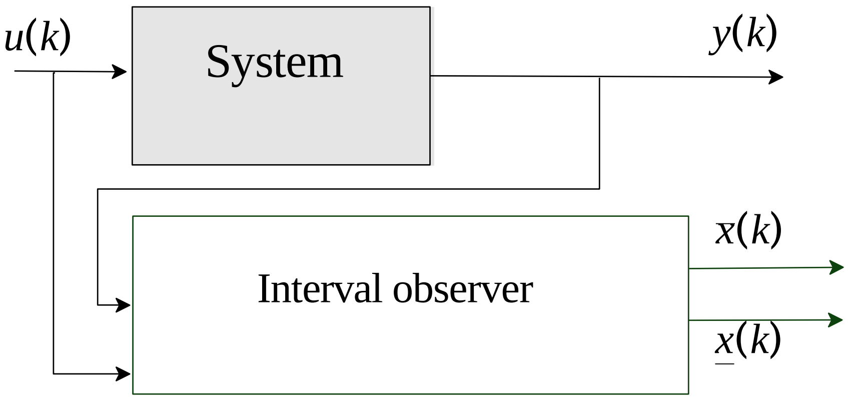
$$\Delta = \left[\Delta_x, \Delta_y\right]^T = DB^{-1}v + C$$

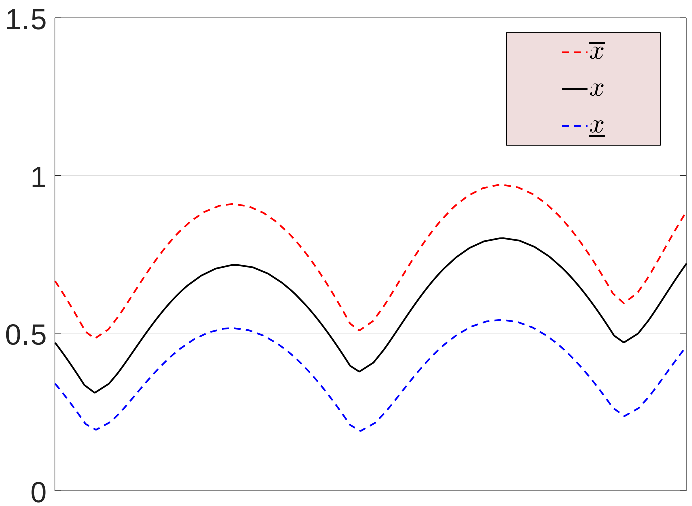
$$v = [v_x, v_y]^T$$

Interval Observer

In the presence of uncertainties

- In some cases, point estimation (classical observer) cannot converge to the real states.
- Interval observers
  - compute the set of admissible values,
  - provide the lower and upper bounds of state vector.





Interval Observer

Uncertain Brunovsky Form 1
$$\begin{cases}\dot{\sigma}_1 = A_1\sigma_1 + B_1 v_x + \Delta_1 \\ Y_1 = x + \eta_x\end{cases}$$

Uncertain Brunovsky Form 2
$$\begin{cases}\dot{\sigma}_2 = A_1\sigma_2 + B_2 v_y + \Delta_2 \\ Y_2 = y + \eta_y\end{cases} \quad (15)$$

$$\sigma_1 = [\sigma_{11}, \sigma_{12}] \quad \sigma_2 = [\sigma_{21}, \sigma_{22}] \quad A_1 = A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad B_1 = B_2 = [0 \quad 1]^T \quad \Delta_1 = [0 \quad \Delta_x]^T \quad \Delta_2 = [0 \quad \Delta_y]^T$$

The main idea of designing interval observers of the uncertain system (15) is to create lower and upper bounds of the real states σ_1 and σ_2 which allows us to guarantee that both latter belong to a specific interval. The interval observer design of system (15) requires the following assumptions :

A1. Pairs (A_1, C_1) and (A_2, C_2) are detectable.

A2. There exist L_1 and L_2 gains such that matrices $(A_1 - L_1C_1)$ and $(A_2 - L_2C_2)$ are Hurwitz and Metzler (off diagonal elements are positive).

A3. The lumped uncertainties and the measurements noise are unknown but bounded.

Interval observer 1
$$\begin{cases}\dot{\bar{\sigma}}_1 = A_1\bar{\sigma}_1 + B_1 v_x + L_1C_1(\sigma_1 - \bar{\sigma}_1) + \bar{\Delta}_1 + L_1\bar{\eta}_x \\ \dot{\underline{\sigma}}_1 = A_1\underline{\sigma}_1 + B_1 v_x + L_1C_1(\sigma_1 - \underline{\sigma}_1) + \underline{\Delta}_1 + L_1\underline{\eta}_x \\ \underline{\sigma}_1(0) \le \sigma_1(0) \le \bar{\sigma}_1(0)\end{cases}$$

Interval observer 2
$$\begin{cases}\dot{\bar{\sigma}}_2 = A_2\bar{\sigma}_2 + B_2 v_y + L_2C_2(\sigma_2 - \bar{\sigma}_2) + \bar{\Delta}_2 + L_2\bar{\eta}_y \\ \dot{\underline{\sigma}}_2 = A_2\underline{\sigma}_2 + B_2 v_y + L_2C_2(\sigma_2 - \underline{\sigma}_2) + \underline{\Delta}_2 + L_2\underline{\eta}_y \\ \underline{\sigma}_2(0) \le \sigma_2(0) \le \bar{\sigma}_2(0)\end{cases}$$

$$Z_1 = G_1\sigma_1 \qquad Z_2 = G_2\sigma_2 \qquad E_1 = G_1(A_1 - L_1C_1)G_1^{-1} \quad E_2 = G_2(A_2 - L_2C_2)G_2^{-1}$$

Raissi T., Efimov D., Zolghadri, A.," Interval state estimation for a class of nonlinear systems", IEEE Transactions on Automatic Control, 2012, 57(1), 260-265.

Guaranteed Flatness Based Tracking Control

$\sigma = [x, y] = [\sigma_{11}, \sigma_{21}]$

Robust Estimation → $$\hat{\sigma}_1 = \frac{\underline{\sigma}_1 + \overline{\sigma}_1}{2}, \hat{\sigma}_2 = \frac{\underline{\sigma}_2 + \overline{\sigma}_2}{2}, \hat{\theta} = \frac{\underline{\theta} + \overline{\theta}}{2}, \hat{v} = \frac{\underline{v} + \overline{v}}{2} \quad (16)$$

Estimated feedback controller → $$\begin{aligned} \hat{v}_x &= \ddot{\sigma}_{xd} - K_{x2}\dot{\hat{e}}_{rx1} - K_{x1}\hat{e}_{rx1} \quad e_{rx1} = \hat{x} - \sigma_{xd} \\ \hat{v}_y &= \ddot{\sigma}_{yd} - K_{y2}\dot{\hat{e}}_{ry1} - K_{y1}\hat{e}_{ry1} \quad e_{ry1} = \hat{y} - \sigma_{yd} \end{aligned} \quad (17)$$

Guaranteed Flatness-Based Tracking Control (GFBTC) → $$\begin{bmatrix} u_{GFBTC1} \\ u_{GFBTC2} \end{bmatrix} = \hat{B}^{-1} \begin{bmatrix} \ddot{\sigma}_{xd} - K_{x2}\dot{\hat{e}}_{rx1} - K_{x1}\hat{e}_{rx1} \\ \ddot{\sigma}_{yd} - K_{y2}\dot{\hat{e}}_{ry1} - K_{y1}\hat{e}_{ry1} \end{bmatrix} \quad (18)$$

Flatness Based Tracking Control (FBTC) → $$\begin{bmatrix} u_{FBTC1} \\ u_{FBTC2} \end{bmatrix} = B^{-1} \begin{bmatrix} \ddot{\sigma}_{xd} - K_{x2}\dot{e}_{rx1} - K_{x1}e_{rx1} \\ \ddot{\sigma}_{yd} - K_{y2}\dot{e}_{ry1} - K_{y1}e_{ry1} \end{bmatrix} \quad (19)$$

Simulation Results

Simulation 1:

$\sigma_1 = [x, \dot{x}] \quad \sigma_2 = [y, \dot{y}]$

Initial condition → $\underline{\sigma}_1(0) = [-2, 0.2]^T, \underline{\sigma}_2(0) = [-2, 0.2]^T$

$\bar{\sigma}_1(0) = [4, 4.2]^T, \bar{\sigma}_2(0) = [4, 7.47]^T$

Uncertain parameters → $\underline{\gamma}_1 = \underline{\gamma}_2 = \underline{\gamma}_3 = -0.4 \quad \bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}_3 = 0.4$

External disturbance → $d_1 = \underline{d}_2 = \underline{d}_4 = \underline{d}_5 = -0.3 \quad \underline{d}_3 = \underline{d}_6 = -0.25$

$\bar{d}_1 = \bar{d}_2 = \bar{d}_4 = \bar{d}_5 = 0.3 \quad \bar{d}_3 = \bar{d}_6 = 0.25$

Uncertain initial condition → $\hat{X}(0) = [\hat{x}, \hat{y}, \hat{v}, \hat{\theta}]^T = [1, 1, 7.47, 60]^T$

Simulation 2:

Initial condition → $\underline{\sigma}_1(0) = [-2, 0.2]^T, \underline{\sigma}_2(0) = [-2, 0.2]^T$

$\bar{\sigma}_1(0) = [4, 4.2]^T, \bar{\sigma}_2(0) = [4, 7.47]^T$

Uncertain parameters → $\underline{\gamma}_1 = \underline{\gamma}_2 = \underline{\gamma}_3 = -0.8 \quad \bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}_3 = 0.8$

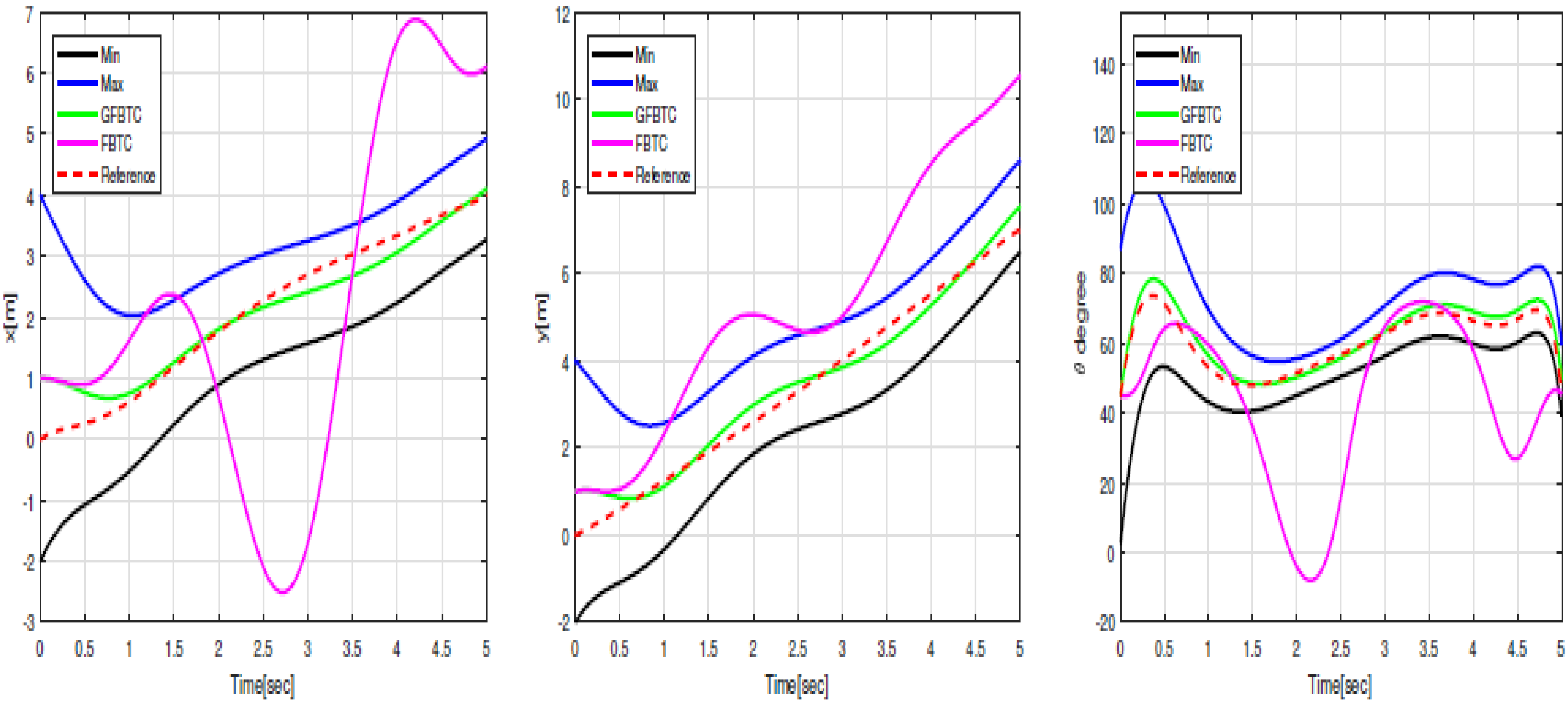
External disturbances → $d_1 = \underline{d}_2 = \underline{d}_4 = \underline{d}_5 = -0.6 \quad \underline{d}_3 = \underline{d}_6 = -0.5$

$\bar{d}_1 = \bar{d}_2 = \bar{d}_4 = \bar{d}_5 = 0.6 \quad \bar{d}_3 = \bar{d}_6 = 0.5$

Uncertain initial condition → $\hat{X}(0) = [\hat{x}, \hat{y}, \hat{v}, \hat{\theta}]^T = [1, 1, 7.47, 60]^T$

Simulation Results

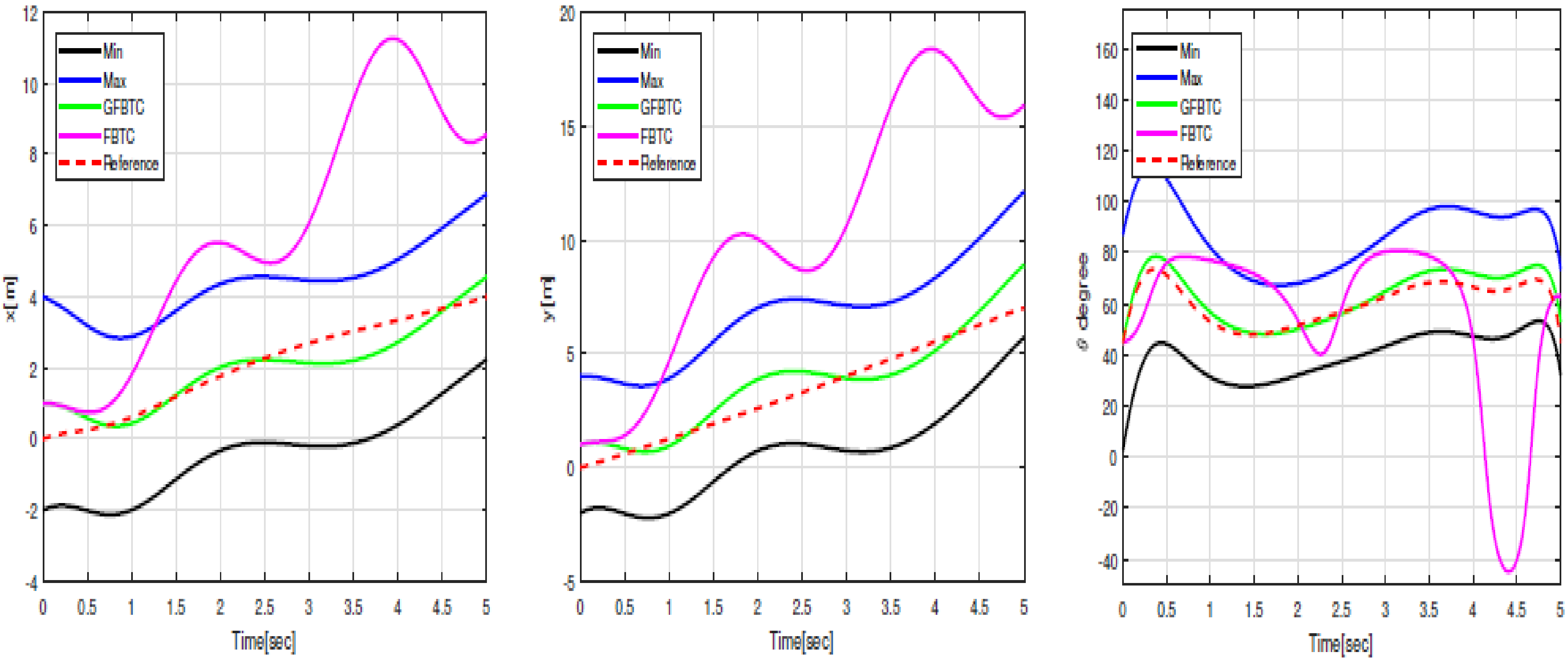
Simulation 1:



From Simulation 1, it can be firstly observed that the interval observers allows guaranteeing the estimation result of a WMR despite the presence of bounded uncertainties. In addition, it can be demonstrated that GFBTC permits the WMR to move in a precise interval containing the desired reference trajectory. Whereas, when applying FBTC to the uncertain WMR system, this latter diverges strongly from the reference. As a result, the controllers that are not based on an uncertain model, even if they are feedback controllers, may not work correctly

Simulation Results

Simulation 2:



From Simulation 2, the effectiveness of GFBTC can be seen compared to FBTC. Furthermore, the width of the estimated interval increases compared the interval width obtained in simulation 1, and this is due to the augmentation of the value of uncertainties. In spite of this disadvantage, it can be observed that the WMR still moves in a precise interval containing the reference trajectory.

Conclusion

The combination of the interval observer and the flatness theory permits to design a guaranteed tracking controller for a WMR despite the presence of un-measurable states and unknown but bounded uncertainties(slippage, external environmental disturbances, measurement noise)

Perspectives

Interval observer gains need to be further optimized in order to reduce the effect of uncertainties on the interval estimation accuracy of the WMR .

Thanks for your attention!