Interval Kalman Filter enhanced by lowering the covariance matrix upper bound

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INTRODUCTION

- a variance upper bound based interval Kalman filter : Improved Minimum Upper Bound of Variance Interval Kalman Filter (iUBIKF)

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- an academic numerical example shows the efficiency of the proposed method w.r.t its precedent UBIKF
- a comparative analysis between the proposed method and the one in [Raka and Combastel, 2013] by simulating a two wheels vehicle model

A real interval, $[x]$: closed and connected subset (interval) of $\mathbb{R}$.

A real interval matrix of dimension $p \times q$, $p, q \in \mathbb{N}^*$:

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Define

$$\sup([X]) \triangleq (\sup([x_{ij}])) \equiv \overline{X} \quad , \quad \inf([X]) \triangleq (\inf([x_{ij}])) \equiv \underline{X},$$

$$\text{mid}([X]) \triangleq \frac{\overline{X} + \underline{X}}{2} \equiv X_m \quad , \quad \text{rad}([X]) \triangleq \frac{\overline{X} - \underline{X}}{2} \equiv X_r$$

as element-wise operators applying to $[X]$.

PRELIMINARY - Interval analysis

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as element-wise operators applying to $[X]$.
Denote also : $[X] = (\underline{X}, \overline{X})$.

PRELIMINARY - Interval analysis

Proposition 1 ([Tran et al., 2017])

Given an $m \times n$ real matrix M belonging to an interval matrix $[M] = ([m_{ij}])$, there exist mn real values $\alpha^{ij} \in [-1, 1]$ with $i \in \{1, \dots, m\}$, $j \in \{1, \dots, n\}$ such that :

$$M = M_m + \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha^{ij} M_r^{(i,j)}, \ ^{a} \tag{1}$$

where $M_r^{(i,j)}$ is an $m \times n$ matrix whose elements are zeros except that

$$entry\ (i, j) = rad([m_{ij}]).$$

a. This expression is a developed form of the Hadamard product.

Proposition 2 ([Tran, 2017])

Given an $n \times n$ real symmetric matrix M belonging to a symmetric interval matrix $[M] = ([m_{ij}])$, there exist $n(n+1)/2$ real values $\alpha^{ij} \in [-1, 1]$ such that :

$$M = M_m + diag\,(M_r) diag\,(\alpha^{ii}) + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \alpha^{ij} M_r^{((i,j),(j,i))}, \quad (2)$$

where

- *$diag\,(M_r)$ is a diagonal matrix containing the radius of diagonal elements of $[M]$,*
- *$diag\,(\alpha^{ii})$ is a diagonal matrix with α^{ii} issued from the $n(n+1)/2$ real values $\alpha^{ij} \in [-1, 1]$.*
- *$M_r^{((i,j),(j,i))}$ is a symmetric matrix whose elements are zero except that*

$$entry\,(i,j) = entry\,(j,i) = rad([m_{ij}]),$$

Positive semi-definite matrix : $M \succeq 0$

Partial order of real square matrices

Let M, N be two real square matrices of the same dimension. M is called an ***upper bound*** of N , denoted by $N \preceq M$, if and only if :

$$M - N \succeq 0.$$

PRELIMINARY - Upper bounds of matrices

Proposition 3 ([Tran et al., 2017])

Given two non null matrices M, N with the same size and an arbitrary real number $\beta > 0$, the following inequality holds :

$$MN^T + NM^T \preceq \beta^{-1}MM^T + \beta NN^T. \tag{3}$$

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Proposition 4 ([Combastel, 2016])

Let M be a symmetric matrix and $M = VDV^T$ be its spectral decomposition, where V is an orthogonal matrix and D is a diagonal matrix. Let $M^+ = V|D|V^T$, where $|.|$ is the element-by-element absolute value operator. Then

$$M^+ \succeq 0 \quad \text{and} \quad \forall \alpha \in [-1, 1],\ \alpha M \preceq M^+.$$

Proposition 5

Given an $m \times n$ real matrix M belonging to an interval matrix $[M]$ and a symmetric positive definite matrix P of order n, there exists a symmetric positive definite matrix S of order m such that

$$MPM^T \preceq S.$$

The matrix S can be determined by using Propositions 1 and 4.

Proposition 6

Given a symmetric matrix M belonging to an interval symmetric matrix $[M]$ of order n, there exists a symmetric positive definite matrix M^+ of order n such that

$$M \preceq M^+.$$

The matrix M^+ can be determined by using Propositions 2 and 4.

Problem formulation

Considered System :

$$\begin{cases} x_k = A_k x_{k-1} + B_k u_k + w_k \\ y_k = C_k x_k + v_k \end{cases} \tag{4}$$

where $x_k \in \mathbb{R}^{n_x}$ are state variables, $y_k \in \mathbb{R}^{n_y}$ measurements, $u_k \in \mathbb{R}^{n_u}$ inputs, $w_k \in \mathbb{R}^{n_x}$ state noises, $v_k \in \mathbb{R}^{n_y}$ measure noises.

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Assumptions (H)

- A_k, B_k, C_k : unknown, deterministic and belonging to known interval matrices $[A], [B], [C]$ respectively.
- $w_k \sim \mathcal{N}(0, Q_k)$, $v_k \sim \mathcal{N}(0, R_k)$ with Q_k and R_k belonging respectively to known interval matrices $[Q]$ and $[R]$.
- The initial state x_0 : centered Gaussian.
- x_0, $\{w_1 : w_k\}$ and $\{v_1 : v_k\}$: mutually independent.

Standard Kalman filter (SKF)

SKF $\longrightarrow$ the minimum variance estimate $\hat{\mathbf{x}}_{k|k}$ of $\mathbf{x}_k$ with the associated covariance matrix $P_{k|k}$.

Notations :

- $\hat{\mathbf{x}}_{k|k-1}$: the *a priori* state estimate vector at time k given observations up to time $k-1$,
- $\hat{\mathbf{x}}_{k|k}$: the *a posteriori* state estimate vector at time k given observations up to time k,
- $P_{k|k-1}$: the *a priori* error covariance matrix,
- $P_{k|k}$: the *a posteriori* error covariance matrix.

where

$$P_{k|k-i} = E\left[(\hat{\mathbf{x}}_{k|k-i} - \mathbf{x}_k)(\hat{\mathbf{x}}_{k|k-i} - \mathbf{x}_k)^T\right], \quad i \in \{0, 1\}. \tag{5}$$

Assuming that $P_{0|0} = P_0$ and $\mathbf{x}_{0|0} = \mathbf{x}_0$.

Improved Minimum Upper Bound of Variance Interval Kalman Filter (iUBIKF)

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- aim to reduce the overestimation on the state estimation error covariance, i.e. to obtain a less conservative upper bound $P^{+}_{k|k}$ on the state estimation error covariance :

$$E\left[(\hat{\mathbf{x}}_{k|k} - \mathbf{x}_k)(\hat{\mathbf{x}}_{k|k} - \mathbf{x}_k)^T\right] \preceq P^{+}_{k|k}. \quad (6)$$

than the one obtained by UBIKF.

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- can be designed in two steps : **prediction** and **correction**.

iUBIKF Prediction step

- assuming that $[\hat{\mathbf{x}}_{0|0}] \ni \mathbf{x}_0$ and $P_0 \preceq P^+_{0|0}$.
- performed similarly to the SKF using the natural interval extension, as follows :

$$\left[\hat{\mathbf{x}}_{k|k-1}\right] = [A]\left[\hat{\mathbf{x}}_{k-1|k-1}\right] + [B]\mathbf{u}_k. \tag{7}$$

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- Find $P^+_{k|k-1}$ such that :

$$A_k P^+_{k-1|k-1} A_k^T + Q_k \preceq P^+_{k|k-1}, \quad \forall A_k \in [A], Q_k \in [Q], \tag{8}$$

where $P^+_{k-1|k-1}$ is the upper bound of the a posteriori covariance matrix at time $k-1$, by using Proposition 5 and 6 as follows :

$$\begin{aligned} P^+_{k|k-1} &= P^+_k + Q^+_k \\ A_k P^+_{k-1|k-1} A_k^T &\preceq P^+_k, \quad A_k \in [A] \\ Q_k &\preceq Q^+_k, \quad Q_k \in [Q] \end{aligned} \tag{9}$$

iUBIKF Correction step

- The **a posteriori state estimate** $\left[\hat{\mathbf{x}}_{k|k}\right]$ is computed by the natural interval extension of the SKF :

$$\left[\hat{\mathbf{x}}_{k|k}\right] = \left[\hat{\mathbf{x}}_{k|k-1}\right] + K_k \left(y_k - [C] \left[\hat{\mathbf{x}}_{k|k-1}\right]\right), \quad (10)$$

$$\left[\hat{\mathbf{x}}_{k|k}\right] = (I - K_k [C]) \left[\hat{\mathbf{x}}_{k|k-1}\right] + K_k y_k. \quad (11)$$

  given $\hat{\mathbf{x}}_{k|k-1} \in \left[\hat{\mathbf{x}}_{k|k-1}\right]$, $C_k \in [C]$ and K_k is a gain matrix.
  Equation (11) is used to reduce the effect of the dependency problem ([Jaulin et al., 2001]).

- The box $\left[\hat{\mathbf{x}}_{k|k}\right]$ encloses all possible values of $\hat{\mathbf{x}}_{k|k}$.

iUBIKF Correction step

The gain matrix K_k is determined as follows :

- Denote :

$$P_{k|k} = (I - K_k C_k) P^{+}_{k|k-1} (I - K_k C_k)^T + K_k R_k K_k^T, \quad (12)$$

for $C_k \in [C], R_k \in [R]$, as the error covariance matrix after the correction step.

- Find upper bound $P^{+}_{k|k}$ sucht that

$$P_{k|k} \preceq P^{+}_{k|k}, \quad \forall C_k \in [C], R_k \in [R]$$

- Find $K_k = \operatorname{argmin} \operatorname{tr}\{P^{+}_{k|k}\}$ where $P^{+}_{k|k}$ is a function of K_k.

iUBIKF Correction step

Find $P^+_{k|k}$ s.t. $P_{k|k} \preceq P^+_{k|k}, \forall C_k \in [C], R_k \in [R]$

For any $C_k \in [C]$, $C_k = C_m + \sum_{i,j} \alpha^{ij} C_r^{(i,j)}$ (**Proposition 1**).
Then

$$
\begin{aligned}
P_{k|k} &= K_k R_k K_k^T + (I - K_k C_m) P^+_{k|k-1} (I - K_k C_m)^T \\
&+ \sum_{i,j} \left(\alpha^{ij}\right)^2 \left[K_k C_r^{(i,j)} P^+_{k|k-1} (K_k C_m - I)^T \right. \\
&\qquad \left. + (K_k C_m - I) P^+_{k|k-1} \left(K_k C_r^{(i,j)}\right)^T \right] \\
&+ \sum_{i,j} \left(\alpha^{ij}\right)^2 K_k C_r^{(i,j)} P^+_{k|k-1} \left(C_r^{(i,j)}\right)^T K_k^T \\
&+ \frac{1}{2} \sum_{(m,l) \neq (i,j)} \alpha^{ij} \alpha^{ml} K_k \left[C_r^{(i,j)} P^+_{k|k-1} \left(C_r^{(m,l)}\right)^T \right. \\
&\qquad \left. + \left(C_r^{(m,l)}\right) P^+_{k|k-1} \left(C_r^{(i,j)}\right)^T \right] K_k^T \qquad (13)
\end{aligned}
$$

iUBIKF Correction step

Find $P^{+}_{k|k}$ s.t. $P_{k|k} \preceq P^{+}_{k|k}, \forall C_k \in [C], R_k \in [R]$

+ using **Proposition 4** to find $S^{(m,l)}_{(i,j)}$ such that

$$\left[C_r^{(i,j)} P^{+}_{k|k-1} \left(C_r^{(m,l)} \right)^T + \left(C_r^{(m,l)} \right) P^{+}_{k|k-1} \left(C_r^{(i,j)} \right)^T \right] \preceq S^{(m,l)}_{(i,j)}$$

+ using **Proposition 6** to find R^{+}_{k} such that

$$R_k \preceq R^{+}_{k}$$

iUBIKF Correction step
Find $P^+_{k|k}$ s.t. $P_{k|k} \preceq P^+_{k|k}, \forall C_k \in [C], R_k \in [R]$

+ Writing

$$
\begin{aligned}
K_k C_r^{(i,j)} P^+_{k|k-1} (K_k C_m - I)^T
&= \left(K_k C_r^{(i,j)} \sqrt{P^+_{k|k-1}} \right) \left((K_k C_m - I) \sqrt{P^+_{k|k-1}} \right)^T \\
&\triangleq XY^T
\end{aligned}
$$

then :

$$
\begin{aligned}
& \left[K_k C_r^{(i,j)} P^+_{k|k-1} (K_k C_m - I)^T + (K_k C_m - I) P^+_{k|k-1} \left(K_k C_r^{(i,j)} \right)^T \right] \\
= \;& XY^T + YX^T \\
\preceq \;& XX^T + YY^T \qquad \textbf{(Proposition 3 with } \beta = 1\textbf{)} \\
= \;& K_k C_r^{(i,j)} P^+_{k|k-1} \left(C_r^{(i,j)} \right)^T K_k^T + (I - K_k C_m) P^+_{k|k-1} (I - K_k C_m)^T
\end{aligned}
$$

iUBIKF Correction step
Find $P^+_{k|k}$ s.t. $P_{k|k} \preceq P^+_{k|k}, \forall C_k \in [C], R_k \in [R]$

Finally

$$\begin{aligned} P_{k|k} &\preceq K_k R_k^+ K_k^T \\ &+ (n_0+1)(I - K_k C_m) P^+_{k|k-1} (I - K_k C_m)^T \\ &+ 2\sum_{i,j} K_k C_r^{(i,j)} P^+_{k|k-1} \left(C_r^{(i,j)}\right)^T K_k^T \\ &+ \frac{1}{2} \sum_{(m,l)\neq(i,j)} K_k S_{(i,j)}^{(m,l)} K_k^T \\ &= P^+_{k|k}, \end{aligned} \tag{14}$$

where n_0 is the number of interval elements of the matrix $[C]$, i.e. $n_0 = n_y \times n_x$.

iUBIKF Correction step

Find $K_k = \operatorname{argmin} \operatorname{tr}\{P^+_{k|k}\}$

$$\frac{\partial \mathrm{tr}\left(\mathrm{P}^+_{\mathrm{k|k}}\right)}{\partial K_k} = 2K_k R_k^+ - 2(n_0+1)P^+_{k|k-1}C_m^T + 2(n_0+1)K_k C_m P^+_{k|k-1}C_m^T + 4\sum_{i,j} K_k C_r^{(i,j)} P^+_{k|k-1}\left(C_r^{(i,j)}\right)^T + \sum_{(m,l)\neq(i,j)} K_k S_{(i,j)}^{(m,l)} = 2K_k S_k - 2(n_0+1)P^+_{k|k-1}C_m^T$$

where

$$S_k = R_k^+ + (n_0+1)C_m P^+_{k|k-1}C_m^T + 2\sum_{i,j} C_r^{(i,j)} P^+_{k|k-1}\left(C_r^{(i,j)}\right)^T + \frac{1}{2}\sum_{(m,l)\neq(i,j)} S_{(i,j)}^{(m,l)}$$

$$\frac{\partial \mathrm{tr}\left(\mathrm{P}^+_{\mathrm{k|k}}\right)}{\partial K_k} = 0 \quad \Leftrightarrow \quad K_k = (n_0+1)P^+_{k|k-1}C_m^T S_k^{-1}$$

iUBIKF Correction step

With $K_k = (n_0 + 1) P^+_{k|k-1} C_m^T S_k^{-1}$,

$$P^+_{k|k} = (n_0 + 1)\left(I - K_k C_m\right) P^+_{k|k-1}. \tag{15}$$

SIMULATION - Academic example

Consider the system

$$\begin{cases} \mathbf{x}_{k+1} = A_k \mathbf{x}_k + \mathbf{w}_k, \\ \mathbf{y}_k = C_k \mathbf{x}_k + \mathbf{v}_k, k \in \mathbb{N}. \end{cases}$$

and assuming that parameter matrices belong respectively to :

$$[A] = \begin{pmatrix} [2.55, 2.65] & [-1.43 - 1.37] & [0.26, 0.28] \\ [6.57, 6.83] & [-3.47, -3.33] & [2.55, 2.65] \\ [-0.77, -0.73] & [0.29, 0.31] & [0.09, 0.11] \end{pmatrix},$$

$$[C] = \begin{pmatrix} [-8.24, -7.76] & [-4.12, -3.88] & [1.94, 2.06] \\ [-2.06, -1.94] & [-2.06, -1.94] & [-6.18, -5.82] \\ [-0.41, -0.39] & [15.52, 16.48] & [6.79, 7.21] \end{pmatrix},$$

$$[Q] = \begin{pmatrix} [8, 12] & [-6, -4] & [3.2, 4.8] \\ [-6, -4] & [8, 12] & [1.6, 2.4] \\ [3.2, 4.8] & [1.6, 2.4] & [8.12] \end{pmatrix},$$

$$[R] = \begin{pmatrix} [8, 12] & [-6, -4] & [3.2, 4.8] \\ [-6, -4] & [8, 12] & [1.6, 2.4] \\ [3.2, 4.8] & [1.6, 2.4] & [8.12] \end{pmatrix}.$$

- Comparison : iUBIKF v.s. UBIKF ([Tran et al., 2017])
- **Performance evaluation criteria** :
  - + **Root mean square error upper bound** :

$$\overline{RMSE} = \sup \left(\sqrt{\left(\sum_{k=1}^{L} \left(\mathbf{x}_k - [\widehat{\mathbf{x}}_{k|k}] \right)^2 \right) / L} \right),$$

where $(.)^2$ and $\sqrt{(.)}$ are element-wise operators and L is the number of iterations.

  - + **The percentage** O where confidence intervals $[I_{c_k}]$ contain the corresponding real states $\mathbf{x}_k$:

$$O = \sum_{k=1}^{L} \mathbf{1}(\mathbf{x}_k \in [I_{c_k}])/L,$$

$$[I_{c_k}] = [\widehat{\mathbf{x}}_{k|k}] + \left[-3\sqrt{diag\left(P^{+}_{k|k}\right)}, \; 3\sqrt{diag\left(P^{+}_{k|k}\right)} \right],$$

where $diag(M)$ is the vector of diagonal elements of matrix M.

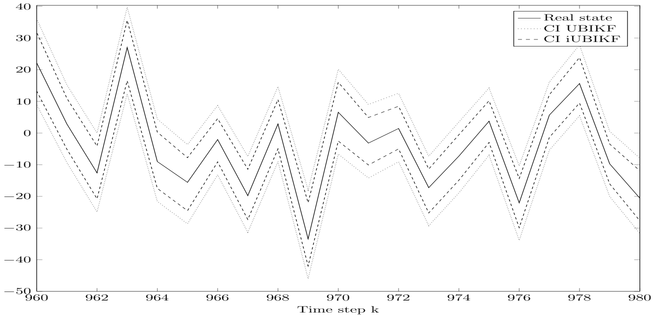

Figure 1 – Real x_1 state component and the 3σ confidence intervals $[I_{c_k}]$ obtained by the UBIKF and the iUBIKF

Table 1 – UBIKF and iUBIKF comparative evaluation

		UBIKF	iUBIKF
x_1	$\overline{RMSE}$	**3.64**	**3.55**
	$O(\%)$	100	100
x_2	$\overline{RMSE}$	**3.60**	**3.49**
	$O(\%)$	100	100
x_3	$\overline{RMSE}$	**2.88**	**2.83**
	$O(\%)$	100	100
Time (s)		15	30

SIMULATION - Academic example

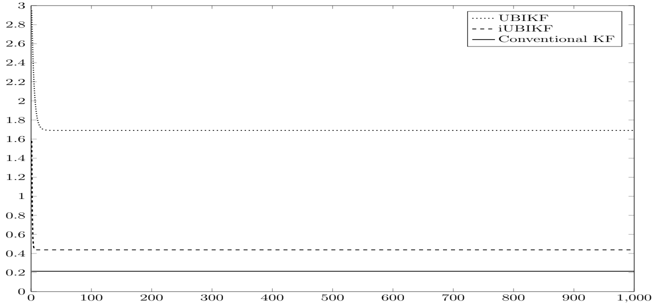

Figure 2 – $P^{+11}_{k|k}$ for the UBIKF, the iUBIKF and $max(P^{11}_{k|k})$ for the conventional Kalman filter

- based on the continuous-time non-linear model of the dynamics of a two wheels vehicle that has been linearized and discretized.

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- the resulting state space model has two states :
  + x_1 : the angular speed of the sideslip angle,
  + x_2 : the acceleration of the vehicle yaw.
- **comparison** : iUBIKF - UBIKF - the Interval Observer (Int.Obs) proposed in [Raka and Combastel, 2013]

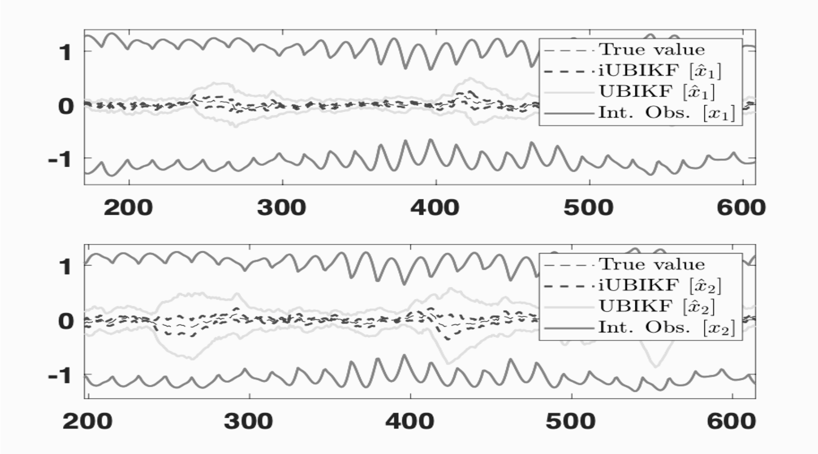

Figure 3 – Estimation results for the UBIKF, the iUBKF and the interval observer for the two wheels vehicle model – angular speed of the sideslip angle x_1 (top) and acceleration of the vehicle yaw x_2 (bottom)

Table 2 – UBIKF, iUBIKF, and Int.Obs comparative evaluation

		UBIKF	iUBIKF	Int. Obs
x_1	$\overline{RMSE}$	0.17585	0.051212	1.1276
x_2	$\overline{RMSE}$	0.291	0.080989	1.1274
Time (s)		2.3916	7.6362	0.40902

CONCLUSION

1. The improved Minimum Upper Bound of Variance Interval Kalman Filter (iUBIKF)
   - provides a lower error covariance upper bound
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$\longrightarrow$ This work shows that the integration of statistical and bounded uncertainties in a same model can be successfully achieved, which opens wide perspectives from a practical point of view.

Combastel, C. (2016).
An Extended Zonotopic and Gaussian Kalman Filter (EZGKF) merging set-membership and stochastic paradigms : Toward non-linear filtering and fault detection.
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THANKS FOR YOUR ATTENTION

AND

Q & A !