Interval Observers for Fault Detection and Estimation

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General context

- **Linear time-invariant** (LTI) and **linear parameter-varying** (LPV) **models**:

$$\begin{cases} x_{k+1} = A(\rho_k)x_k + B(\rho_k)u_k + D(\rho_k)w_k \\ y_k = C(\rho_k)x_k + E(\rho_k)v_k \end{cases} \qquad \begin{cases} \dot{x}_t = A(\rho_t)x_t + B(\rho_t)u_t + D(\rho_t)w_t \\ y_t = C(\rho_t)x_t + E(\rho_t)v_t \end{cases}$$

  with:

  → constant ρ_k/ρ_t in the LTI case

  → unknown but bounded (UBB) perturbation w_k/w_t and measurement noise v_k/v_t

- **Fault**: additive bias on the state/measurement equation, modification of A and/or B and/or C, ...
- **Unknown input**:

  → additive bias on the state equation

  → can be used to represent a fault signal

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Context

- Work published in Chevet et al. (2021b)
- **Continuous-time LPV system**:

$$\begin{cases} \dot{x}_t = A(\rho_t)x_t + B(\rho_t)u_t + D(\rho_t)w_t \\ y_t = C(\rho_t)x_t + E(\rho_t)v_t \end{cases}$$

  → good approximation of nonlinear systems (Shamma 2012)
  → support use of linear methods
- **Sensor fault**: additive bias on measured signal y_t
- **Pointwise observer**: risk of false positive due to uncertainties (Lamouchi et al. 2018)

Contribution

A *robust interval observer for sensor fault detection* for LPV systems subject to *bounded perturbations*

Considered model

LPV system subject to perturbations and additive sensor fault with constant output matrix:

$$\begin{cases} \dot{x}_t = A(\rho_t)x_t + B(\rho_t)u_t + D(\rho_t)w_t \\ y_t = Cx_t + f_t \end{cases} \tag{1}$$

- state $x_t \in \mathbb{R}^{n_x}$, input $u_t \in \mathbb{R}^{n_u}$, output $y_t \in \mathbb{R}^{n_y}$, perturbation $w_t \in \mathbb{R}^{n_w}$, fault $f_t \in \mathbb{R}^{n_f}$, parameter $\rho_t \in \mathbb{R}^{n_\rho}$
- x_0, w_t unknown but bounded:
  - $\rightarrow$ $\underline{x}_0 \leq x_0 \leq \overline{x}_0$, with $\underline{x}_0, \overline{x}_0 \in \mathbb{R}^{n_x}$, $\|\underline{x}_0\|, \|\overline{x}_0\| < \infty$
  - $\rightarrow$ $\underline{w}_t \leq w_t \leq \overline{w}_t$, with $\underline{w}_t, \overline{w}_t \in \mathbb{R}^{n_w}$, $\forall t \geq 0$, $\|\underline{w}\|_\infty = \sup\{\|w_t\| | t \geq 0\}, \|\overline{w}\|_\infty < \infty$
- ρ_t unknown and unmeasurable:
  - $\rightarrow$ $M(\rho_t) = M_0 + \Delta M(\rho_t)$, $\forall M \in \{A, B, D\}$
  - $\rightarrow$ $\Delta M(\rho_t)$ unknown but bounded, i.e. $\underline{\Delta M} \leq \Delta M(\rho_t) \leq \overline{\Delta M}$, $\forall M \in \{A, B, D\}$
- $\|x\|_\infty < \infty$, $\|u\|_\infty < \infty \Rightarrow \|y\|_\infty < \infty$ if $f_t \equiv 0$

Prerequisites on interval analysis

Positive decomposition of a matrix

Let $M \in \mathbb{R}^{n\times m}$. Then $M = M^+ - M^-$ where $M^+ = \max\{\mathbf{0}, M\}$ and $M^+, M^- \geq \mathbf{0}$.

Lemma 1 (Efimov et al. 2012)

Let $x, \underline{x}, \overline{x} \in \mathbb{R}^{n_x}$ such that $\underline{x} \leq x \leq \overline{x}$

(i) If $M \in \mathbb{R}^{m\times n}$ is a constant matrix, then $M^+\underline{x} - M^-\overline{x} \leq Mx \leq M^+\overline{x} - M^-\underline{x}$

(ii) If $\underline{M} \leq M \leq \overline{M}$, with $M, \underline{M}, \overline{M} \in \mathbb{R}^{m\times n}$, then:

$$\underline{M}^+\underline{x}^+ - \overline{M}^+\underline{x}^- - \underline{M}^-\overline{x}^+ + \overline{M}^-\overline{x}^- \leq Mx \leq \overline{M}^+\overline{x}^+ - \underline{M}^+\overline{x}^- - \overline{M}^-\underline{x}^+ + \underline{M}^-\underline{x}^-$$

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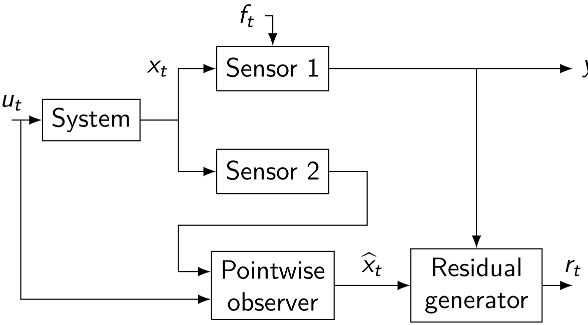
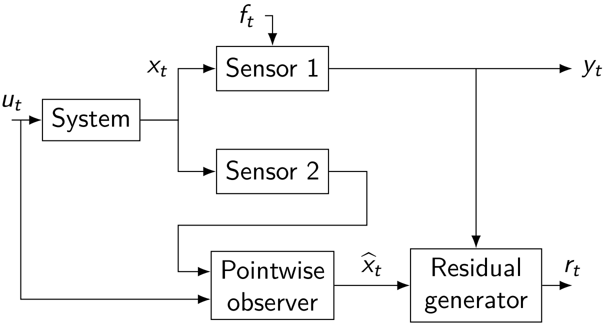
Pointwise strategy



- Sensor 1: potentially affected by additive sensor fault
- Sensor 2: fault free
- Residual signal: $r_t = C\widehat{x}_t - y_t$

Pointwise strategy



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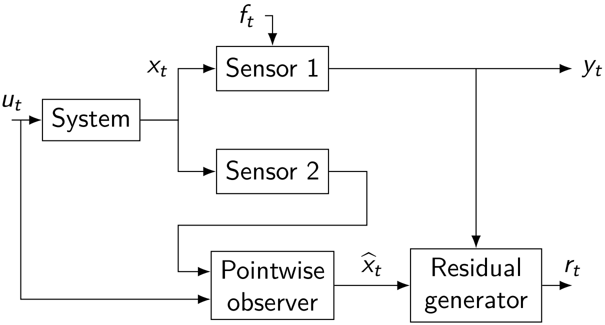
- Fault detection strategy:
  - $\rightarrow$ if $r_t = 0 \rightarrow$ sensor 1 is fault free
  - $\rightarrow$ if $r_t \neq 0 \rightarrow$ sensor 1 is affected by fault

Pointwise strategy



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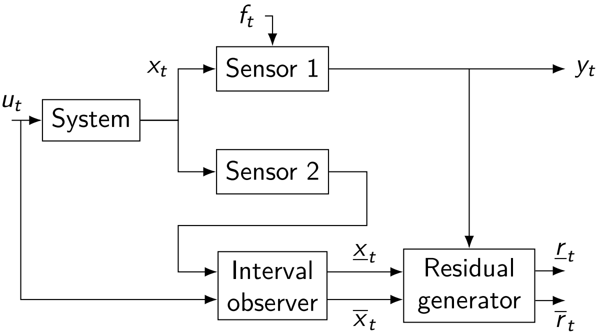
Limitation of pointwise approach

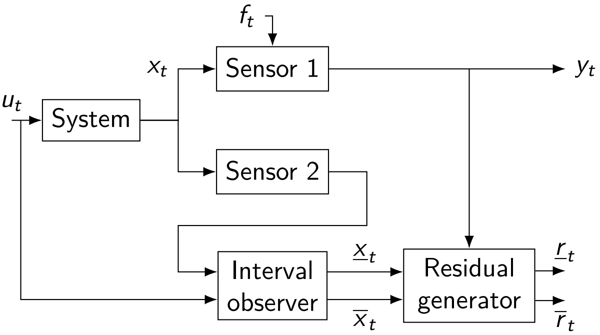
In the presence of perturbations (w_t) and system uncertainties (ΔM, $M \in \{A, B, D\}$), $r_t \neq 0$ even if sensor 1 is fault free $\Rightarrow$ **risk of false positive**

Interval strategy



- Sensor 1: potentially affected by additive sensor fault
- Sensor 2: fault free
- Residual bounds: $\overline{r}_t = \overline{Cx}_t - y_t$ and $\underline{r}_t = \underline{Cx}_t - y_t$

Interval strategy



- Sensor 1: potentially affected by additive sensor fault
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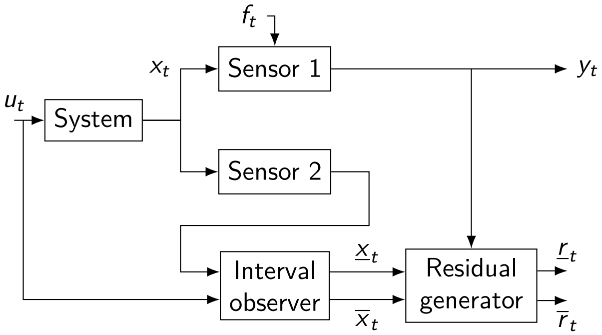
- Fault detection strategy:
  - $\rightarrow$ if $\mathbf{0} \in [\underline{r}_t, \overline{r}_t]$, sensor 1 is fault free or affected by undetectable low-magnitude fault
  - $\rightarrow$ if $\mathbf{0} \notin [\underline{r}_t, \overline{r}_t]$ sensor 1 is affected by fault

Interval strategy



- Sensor 1: potentially affected by additive sensor fault
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  - → if $\mathbf{0} \in [\underline{r}_t, \overline{r}_t]$, sensor 1 is fault free or affected by undetectable low-magnitude fault
  - → if $\mathbf{0} \notin [\underline{r}_t, \overline{r}_t]$ sensor 1 is affected by fault

Main requirement of the proposed interval observer

Attenuate effect of perturbations and system uncertainties on $[\underline{r}_t, \overline{r}_t]$ to detect low-magnitude faults

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State framer

Differential-algebraic system inspired by Li et al. (2019):

$$\begin{cases} \dot{\underline{\xi}}_t = (TA_0 - \underline{L}C)\underline{x}_t + TB_0 u_t + \underline{L}y_t + \underline{\phi}_t + \underline{\chi}_t + \underline{\omega}_t \\ \underline{x}_t = \underline{\xi}_t + Ny_t \\ \dot{\overline{\xi}}_t = (TA_0 - \overline{L}C)\overline{x}_t + TB_0 u_t + \overline{L}y_t + \overline{\phi}_t + \overline{\chi}_t + \overline{\omega}_t \\ \overline{x}_t = \overline{\xi}_t + Ny_t \end{cases} \tag{2}$$

where:

- $\underline{L}, \overline{L}$ observer gains
- T, N additional degrees of freedom (Wang et al. 2018):

$$T + NC = I \underset{\text{(Rao and Mitra 1972)}}{\Longrightarrow} \begin{bmatrix} T & N \end{bmatrix} = \begin{bmatrix} I \\ C \end{bmatrix}^{\dagger} + \underbrace{\Xi}_{\text{free matrix}} \left(I - \begin{bmatrix} I \\ C \end{bmatrix} \begin{bmatrix} I \\ C \end{bmatrix}^{\dagger} \right)$$

- $\underline{\phi}_t, \overline{\phi}_t, \underline{\chi}_t, \overline{\chi}_t, \underline{\omega}_t, \overline{\omega}_t$ obtained with Lemma 1, satisfying:

$$\underline{\phi}_t \leq T\Delta A(\rho_t)x_t \leq \overline{\phi}_t \qquad \underline{\chi}_t \leq T\Delta B(\rho_t)u_t \leq \overline{\chi}_t \qquad \underline{\omega}_t \leq TD(\rho_t)w_t \leq \overline{\omega}_t$$

Residual framer

Residual signal:

$$r_t = Cx_t - y_t$$

Residual framer:

$$\begin{cases} \underline{r}_t = C^+\underline{x}_t - C^-\overline{x}_t - y_t \\ \overline{r}_t = C^+\overline{x}_t - C^-\underline{x}_t - y_t \end{cases}$$

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Theorem 1

For the considered model, if $TA_0 - \underline{L}C$ and $TA_0 - \overline{L}C$ are Metzler matrices[a], then, in the fault-free case:

$$\underline{x}_t \leq x_t \leq \overline{x}_t, \forall t \geq 0$$

[a] A matrix $M \in \mathbb{R}^{n \times n}$ is Metzler if its off-diagonal elements are nonnegative (Chebotarev et al. 2015).

Residual framer

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With Theorem 1 and Lemma 1:

$$\underline{r}_t \leq r_t \leq \overline{r}_t$$

Stability

Interval observer (Dinh et al. 2020)

The state framer (2) is an interval observer if $\overline{e}_t = \overline{x}_t - x_t$ and $\underline{e}_t = \underline{x}_t - x_t$ are bounded (ideally input-to-state stable)

Based on input-to-state stability (ISS) condition (Sontag and Wang 1995)

$$\dot{V}_t \leq -\alpha V_t + \gamma \left\|\varepsilon_t\right\|^2$$

- $V_t = E_t^\top P E_t$ Lyapunov function
  $\rightarrow$ $P \in \mathbb{R}^{2n_x \times 2n_x}$, $P \succ 0$ diagonal
  $\rightarrow$ $E_t^\top = \begin{bmatrix} \underline{e}_t^\top & \overline{e}_t^\top \end{bmatrix}$
- $\gamma > 0$, $\alpha > 0$
- perturbation

$$\varepsilon_t = \begin{bmatrix} \underline{\chi}_t - T\Delta B(\rho_t)u_t + \underline{\omega}_t - TD(\rho_t)w_t \\ \overline{\chi}_t - T\Delta B(\rho_t)u_t + \overline{\omega}_t - TD(\rho_t)w_t \end{bmatrix}$$

Stability

Lyapunov function's time derivative

$$\begin{aligned}
\dot{V}_t &= \dot{E}_t P E_t + E_t^\top P \dot{E}_t \\
&= E_t^\top \left(S^\top + S + \alpha P\right) E_t + \Phi_t^\top P E_t + E_t^\top P \Phi_t + E_t^\top P \varepsilon_t \\
&\qquad + \varepsilon_t^\top P E_t - \alpha E_t^\top P E_t + \gamma \Phi_t^\top \Phi_t - \gamma \Phi_t^\top \Phi_t + \gamma \varepsilon_t^\top \varepsilon_t - \gamma \varepsilon_t^\top \varepsilon_t \\
&= \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix}^\top \begin{bmatrix} S + S^\top + \alpha P & P^\top & P^\top \\ P & -\gamma I_{2n_x} & \mathbf{0} \\ P & \mathbf{0} & -\gamma I_{2n_x} \end{bmatrix} \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix} - \alpha V_t + \gamma \left\|\varepsilon_t\right\|^2 + \gamma \Phi_t^\top \Phi_t
\end{aligned}$$

where

- $S = P(I_2 \otimes TA_0) - Y\Upsilon$, with $\Upsilon = I_2 \otimes C$, $Y = \mathrm{diag}(\underline{L}, \overline{L})$
- $\Phi_t^\top = \begin{bmatrix} (\underline{\phi}_t - T\Delta A(\rho_t) x_t)^\top & (\overline{\phi}_t - T\Delta A(\rho_t) x_t)^\top \end{bmatrix}$

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&= \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix}^\top \begin{bmatrix} S + S^\top + \alpha P & P^\top & P^\top \\ P & -\gamma I_{2n_x} & \mathbf{0} \\ P & \mathbf{0} & -\gamma I_{2n_x} \end{bmatrix} \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix} - \alpha V_t + \gamma \left\|\varepsilon_t\right\|^2 + \gamma \Phi_t^\top \Phi_t
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- $S = P(I_2 \otimes TA_0) - Y\Upsilon$, with $\Upsilon = I_2 \otimes C$, $Y = \mathrm{diag}(\underline{L}, \overline{L})$
- $\Phi_t^\top = \left[(\underline{\phi}_t - T\Delta A(\rho_t)x_t)^\top \quad (\overline{\phi}_t - T\Delta A(\rho_t)x_t)^\top\right]$

Problem: presence of Φ_t, nonlinear function of state

Stability

- Bound provided by Zheng et al. (2016):

$$\Phi_t^\top \Phi_t \leq E_t^\top Q E_t + \beta$$

where:

→ β positive constant

→ $Q = 6 \cdot \mathrm{diag}(\underline{l}_\phi^2, \bar{l}_\phi^2)$, with $\underline{l}_\phi = \left\|(T\overline{\Delta A})^-\right\| + \left\|(T\underline{\Delta A})^-\right\|$, $\bar{l}_\phi = \left\|(T\overline{\Delta A})^+\right\| + \left\|(T\underline{\Delta A})^+\right\|$

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- Majoration of Lyapunov function's time derivative:

$$\dot{V}_t \leq \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix}^\top \underbrace{\begin{bmatrix} S + S^\top + \alpha P + \gamma Q & P^\top & P^\top \\ P & -\gamma I_{2n_x} & \mathbf{0} \\ P & \mathbf{0} & -\gamma I_{2n_x} \end{bmatrix}}_{\Lambda} \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix} - \alpha V_t + \gamma(\|\varepsilon_t\|^2 + \beta) \quad (3)$$

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In terms of linear matrix inequalities

E_t bounded if $\Lambda \preceq 0$

Performance

- By integration, condition (3) equivalent to:

$$V_t \leq V_0 \mathrm{e}^{-\alpha t} + \gamma(\|\varepsilon\|_\infty^2 + \beta) \tag{4}$$

- Residual framer dynamics:

$$\underbrace{\begin{bmatrix} \underline{r}_t \\ \overline{r}_t \end{bmatrix}}_{R_t} = \underbrace{\begin{bmatrix} C^+ & -C^- \\ -C^- & C^+ \end{bmatrix}}_{\mathcal{C}} E_t$$

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- If:

$$\|R_t\|^2 \leq \mu \left(V_t + (\mu - \gamma)(\|\varepsilon\|_\infty^2 + \beta) \right) \tag{5}$$

with $\mu > 0$, then, from (4), $\|R_t\|^2 \leq \mu V_0 \mathrm{e}^{-\alpha t} + \mu^2(\|\varepsilon\|_\infty^2 + \beta)$

Performance

- By integration, condition (3) equivalent to:

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In terms of linear matrix inequalities

(5) is true if $\begin{bmatrix} P & \mathbf{0} & \mathcal{C}^\top \\ \mathbf{0} & \mu - \gamma & \mathbf{0} \\ \mathcal{C} & \mathbf{0} & \mu I_{2n_y} \end{bmatrix} \succeq 0$

Interval observer

Theorem 2

For the proposed model and given $\alpha > 0$, $\eta > 0$, if there exists $\gamma > 0$, $\mu > 0$, $P \in \mathbb{R}^{2n_x \times 2n_x}$, with $P \succ 0$ diagonal, and $Y \in \mathbb{R}^{2n_x \times 2n_y}$ such that:

$$S + \eta P \geq \mathbf{0} \qquad \text{(Cooperativity)}$$

$$\begin{bmatrix} S + S^\top + \alpha P + \gamma Q & P^\top & P^\top \\ P & -\gamma I_{2n_x} & \mathbf{0} \\ P & \mathbf{0} & -\gamma I_{2n_x} \end{bmatrix} \preceq 0 \qquad \text{(Stability)}$$

$$\begin{bmatrix} P & \mathbf{0} & \mathcal{C}^\top \\ \mathbf{0} & \mu - \gamma & \mathbf{0} \\ \mathcal{C} & \mathbf{0} & \mu I_{2n_y} \end{bmatrix} \succeq 0 \qquad \text{(Performance)}$$

then (2) is a robust interval observer for (1) with performance $\|R_t\|^2 \leq \mu V_0 \mathrm{e}^{-\alpha t} + \mu^2(\|\varepsilon\|_\infty^2 + \beta)$

- First inequality ensures $TA_0 - \underline{L}C$, $TA_0 - \overline{L}C$ Metzler (Chebotarev et al. 2015)
- Gain matrices $\underline{L}, \overline{L}$ obtained as $\text{diag}(\underline{L}, \overline{L}) = P^{-1}Y$

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Simulation parameters

- Dampened mass-spring system (Scherer 2012):

$$\begin{cases} \dot{x}_t = \begin{bmatrix} 0 & 1 \\ 2+\rho_t & -1 \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_t + w_t \\ y_t = \begin{bmatrix} 1 & 0 \end{bmatrix} x_t + f_t \end{cases}$$

  $\rightarrow$ $x_t^\top = \begin{bmatrix} p_t & \dot{p}_t \end{bmatrix}$, with p_t horizontal position of the mass, $\overline{x}_0 = -\underline{x}_0 = 0.1 \cdot \mathbf{1}_2$

  $\rightarrow$ $\rho_t = \sin(0.3t)$, $u_t = \mathrm{sgn}(\sin(t))$, $w_t^\top = 0.1 \begin{bmatrix} \cos(2t) & \sin(3t) \end{bmatrix}$, $\overline{w}_t = -\underline{w}_t = 0.1 \cdot \mathbf{1}_2$
- $\Delta B(\rho_t) = \mathbf{0}$, $\Delta D(\rho_t) = \mathbf{0}$, $D_0 = I_2$, $\Delta D(\rho_t) = \mathbf{0}$ and:

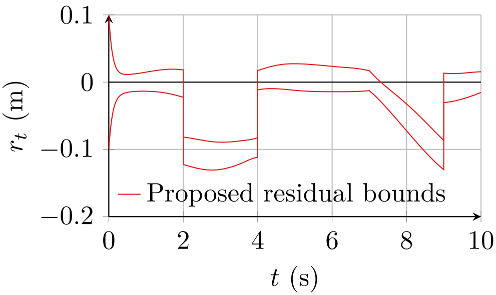
$$A_0 = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} \qquad \overline{\Delta A} = -\underline{\Delta A} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \qquad B_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- $\underline{l}_\phi = \overline{l}_\phi = 1$ with:

$$T = \begin{bmatrix} 0.6 & 0 \\ -3 & 1 \end{bmatrix} \qquad N = \begin{bmatrix} 0.4 \\ 3 \end{bmatrix}$$

- $\alpha = 0.1$, $\eta = 10$ so that $\mu = \gamma = 0.3384$ and $\underline{L} = \overline{L} = \begin{bmatrix} 10 & -2 \end{bmatrix}^\top$

Simulation results



- Sensor fault signal:

$$f_t = \begin{cases} 0.1 & \text{if } 2 \leq t \leq 4 \\ 0.05 \cdot (t-7) & \text{if } 7 \leq t \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

- Fault detected between $t = 2\,\mathrm{s}$ and $t = 4\,\mathrm{s}$ since $\mathbf{0} \notin [\underline{r}_t, \overline{r}_t]$
- Fault appearing at $t = 7\,\mathrm{s}$ not detected before $t = 7.3\,\mathrm{s}$ since $\mathbf{0} \in [\underline{r}_t, \overline{r}_t]$ between $t = 7\,\mathrm{s}$ and $t = 7.3\,\mathrm{s}$
- No false positive between $t = 0\,\mathrm{s}$ and $t = 2\,\mathrm{s}$, $t = 4\,\mathrm{s}$ and $t = 7\,\mathrm{s}$ and for $t > 9\,\mathrm{s}$

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Context

- Work published in Chevet et al. (2022) in collaboration with Zhenhua Wang, Harbin Institute of Technology
- **Discrete-time LTI system**:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Dw_k \\ y_t = Cx_k + Ev_k \end{cases}$$

- **Unknown input**: additive bias d_k on state equation
- **Pointwise observer**: significant uncertainty due to bounded perturbations and measurement noise

Contribution

A *zonotopic Kalman filter-based interval observer for joint estimation of state and unknown inputs* for LTI systems subject to *bounded perturbations and unknown inputs*

Considered model

LTI system subject to bounded perturbations, bounded measurement noise and unknown input:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + D_d d_k + D_w w_k \\ y_k = Cx_k + D_v v_k \end{cases} \tag{6}$$

- state $x_k \in \mathbb{R}^{n_x}$, input $u_k \in \mathbb{R}^{n_u}$, output $y_k \in \mathbb{R}^{n_y}$, perturbation $w_k \in \mathbb{R}^{n_w}$, measurement noise $v_k \in \mathbb{R}^{n_v}$, unknown input $d_k \in \mathbb{R}^{n_d}$
- $x_0 \in \widehat{\mathcal{X}}_0 = \langle \widehat{x}_0, \widehat{G}_0 \rangle$ zonotope with center $\widehat{x}_0 \in \mathbb{R}^{n_x}$ and generator matrix $\widehat{G}_0$
- $|w_k| \leq \bar{w}_k$, with $\bar{w}_k \geq \mathbf{0}$ so that $w_k \in \mathcal{W}_k = \langle \mathbf{0}, W_k \rangle$ zonotope with center $\mathbf{0}$ and generator matrix $W_k = \text{diag}(\bar{w}_k)$
- $|v_k| \leq \bar{v}_k$, with $\bar{v}_k \geq \mathbf{0}$ so that $v_k \in \mathcal{V}_k = \langle \mathbf{0}, V_k \rangle$ zonotope with center $\mathbf{0}$ and generator matrix $V_k = \text{diag}(\bar{v}_k)$

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Descriptor dynamics

- Several "classical" approaches for unknown input estimation:
  - → definition of evolution model for d_k as $d_{k+1} = A_d d_k + B_d b_k$, with b_k a noise signal, and state augmentation
  - → separation of the model into two subsystems, one free of the unknown input for state estimation, the other used for unknown input estimation (Robinson et al. 2020)

Descriptor dynamics

- Several “classical” approaches for unknown input estimation:
  - → definition of evolution model for d_k as $d_{k+1} = A_d d_k + B_d b_k$, with b_k a noise signal, and state augmentation
  - → separation of the model into two subsystems, one free of the unknown input for state estimation, the other used for unknown input estimation (Robinson et al. 2020)
- **Considered approach**: addition of d_{k-1} to state vector and rewriting of the system into descriptor form (Li et al. 2020):

$$\begin{cases} Ez_{k+1} = Fz_k + Gu_k + Dw_k \\ y_k = Hz_k + D_v v_k \end{cases}$$

  - → $z_k^\top = \begin{bmatrix} x_k^\top & d_{k-1}^\top \end{bmatrix}$, $d_{-1} = \mathbf{0}$
  - → $z_0 \in \widehat{\mathcal{Z}}_0 = \langle \widehat{z}_0^\top = \begin{bmatrix} \widehat{x}_0^\top & \mathbf{0} \end{bmatrix}, \widehat{Z}_0 = \mathrm{diag}(\widehat{G}_0, \mathbf{0}) \rangle$
  - → the matrices:

$$E = \begin{bmatrix} I & -D_d \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \qquad F = \begin{bmatrix} A & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \qquad G = \begin{bmatrix} B \\ \mathbf{0} \end{bmatrix} \qquad D = \begin{bmatrix} D_w \\ \mathbf{0} \end{bmatrix} \qquad H = \begin{bmatrix} C & \mathbf{0} \end{bmatrix}$$

Rewriting as state-space dynamics

Assumption

$$\text{rank} \begin{bmatrix} I & -D_d \\ C & \mathbf{0} \end{bmatrix} = n_x + n_d = n_z$$

There exists T, N satisfying:

$$TE + NH = I \underset{\text{(Rao and Mitra 1972)}}{\Longrightarrow} \begin{bmatrix} T & N \end{bmatrix} = \begin{bmatrix} E \\ H \end{bmatrix}^{\dagger} + \underbrace{\Xi}_{\text{free matrix}} \left(I - \begin{bmatrix} E \\ H \end{bmatrix} \begin{bmatrix} E \\ H \end{bmatrix}^{\dagger} \right)$$

Dynamics for observer design:

$$\begin{cases} z_{k+1} = TFz_k + TGu_k + TDw_k + Ny_{k+1} - ND_v v_{k+1} \\ y_k = Hz_k + D_v v_k \end{cases}$$

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Prediction step

Theorem 3

If, at time k, $z_k \in \widehat{\mathcal{Z}}_k = \langle \widehat{z}_k, \widehat{Z}_k \rangle$, then $z_{k+1} \in \widetilde{\mathcal{Z}}_{k+1} = \langle \widetilde{z}_{k+1}, \widetilde{H}_{k+1} \rangle$ where

$$\widetilde{z}_{k+1} = TF\widehat{z}_k + TGu_k + Ny_{k+1}$$
$$\widetilde{Z}_{k+1} = \begin{bmatrix} TF \downarrow_q \widehat{Z}_k & TDW_k & -ND_v V_k \end{bmatrix}$$

- Obtained from results on usual operations on zonotopes
- $\downarrow_q \widehat{Z}_k$: order reduction operation (Combastel 2003)
  - $\rightarrow$ sorting of the generators in $\widehat{Z}_k \in \mathbb{R}^{n_z \times r}$ by decreasing norm
  - $\rightarrow$ if $r \leq q$, $\downarrow_q H = H$
  - $\rightarrow$ otherwise, $\downarrow_q H = \begin{bmatrix} H_> & \operatorname{diag}(|H_<|\,\mathbf{1}) \end{bmatrix}$, with $H_>$ first $q-n$ columns of H, $H_>$ last $r-q+n$ columns of H

Measurement step

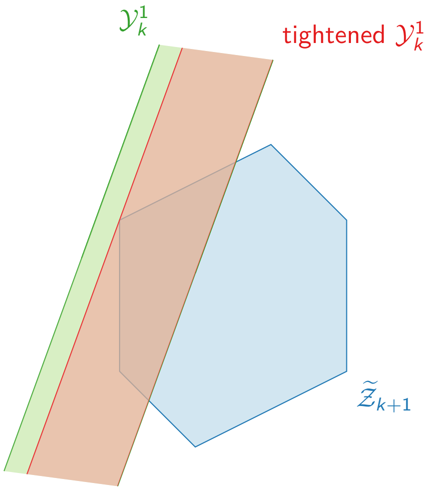
- At time $k+1$, $z_{k+1} \in \mathcal{Y}_{k+1}$

$$\mathcal{Y}_{k+1} = \bigcap_{i=1}^{n_y} \mathcal{Y}_{k+1}^i$$

with the strips[a] $\mathcal{Y}_{k+1}^i$

$$\mathcal{Y}_{k+1}^i = \left\{ z \in \mathbb{R}^{n_z} \,\middle|\, \left| H^i z - y_{k+1}^i \right| \leq \left(|D_v| \, \bar{v} \right)^i \right\}$$

- If necessary, tightening (Bravo et al. 2006) of strip $\mathcal{Y}_{k+1}^1$ with respect to zonotope $\widetilde{\mathcal{Z}}_{k+1}$



[a] exponent i denotes i-th component in case of vector, i-th row in case of matrix

Correction step

Assuming $\widetilde{Z}_{k+1} \in \mathbb{R}^{n_z \times r}$, denoting $\widetilde{\mathcal{Z}}^0_{k+1} = \widetilde{\mathcal{Z}}_{k+1}$:

1. computation of r zonotopes $\mathcal{T}^j_{k+1} = \langle \tau^j_{k+1}, T^j_{k+1} \rangle$, $j \in \overline{1, r}$ (Chai et al. 2013), satisfying

$$\widetilde{\mathcal{Z}}^0_{k+1} \cap \mathcal{Y}^1_{k+1} \subseteq \mathcal{T}^j_{k+1}, \forall j \in \overline{1, r}$$

2. select $\widetilde{\mathcal{Z}}^1_{k+1} = \mathcal{T}^{j^*}_{k+1}$ with

$$j^* = \arg\min_{j \in \overline{0, r}} \operatorname{tr}\left(T^j_{k+1} {T^j_{k+1}}^\top \right)$$

3. repeat steps 1 and 2 with $\widetilde{\mathcal{Z}}^{i-1}_{k+1}$, $\mathcal{Y}^i_{k+1}$ for $i \in \overline{2, n_y}$ (if necessary, tightening of $\mathcal{Y}^i_{k+1}$ with respect to $\widetilde{\mathcal{Z}}^{i-1}_{k+1}$)
4. corrected zonotope: $\widehat{\mathcal{Z}}_{k+1} = \widetilde{\mathcal{Z}}^{n_y}_{k+1}$

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Simulation parameters

- LTI system

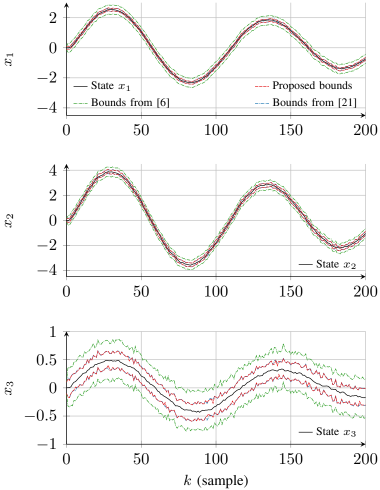
$$\begin{cases} x_{k+1} = \begin{bmatrix} 0.2 & 0.4 & 0.1 \\ 0 & 0.7 & 0.2 \\ 0 & 0 & 0.5 \end{bmatrix} x_k + \begin{bmatrix} 0.3 \\ 0.8 \\ 0.1 \end{bmatrix} u_k + \begin{bmatrix} 0.5 \\ 1 \\ 0.5 \end{bmatrix} d_k + \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.3 \end{bmatrix} w_k \\ y_k = \begin{bmatrix} 0.3 & 0.1 & 0 \\ 0 & 0.2 & 0.1 \end{bmatrix} x_k + \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix} v_k \end{cases}$$

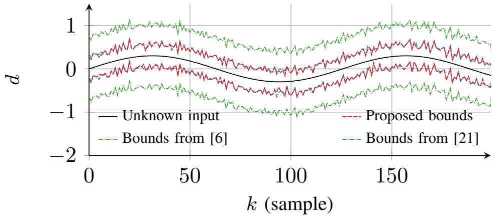
- $x_0 \in \widehat{\mathcal{X}}_0 = \langle \mathbf{0}, \mathrm{diag}(0.1 \cdot \mathbf{1}) \rangle$, $w_k \in \mathcal{W} = \langle \mathbf{0}, \mathrm{diag}(0.06 \cdot \mathbf{1}) \rangle$, $v_k \in \mathcal{V} = \langle \mathbf{0}, \mathrm{diag}(0.06 \cdot 1) \rangle$
- $u_k = \sin(0.02\pi k)$, $d_k = 0.3 \sin(0.05k)$
- $\Xi = \mathbf{0}$ so that

$$T = \begin{bmatrix} 0.6645 & -0.2882 & -0.0882 & 0 \\ -0.5716 & 0.3905 & -0.2095 & 0 \\ -0.2787 & -0.3071 & 0.8929 & 0 \\ -0.5858 & -0.6047 & -0.2047 & 0 \end{bmatrix} \qquad N = \begin{bmatrix} 1.1185 & 0.8815 \\ 1.9052 & 2.0948 \\ 0.9289 & 1.0711 \\ 1.9526 & 2.0474 \end{bmatrix}$$

- zonotope reduction order $q = 20$

Simulation results





- upper and lower bounds:

$$\underline{z}_k = \widehat{z}_k - |\widehat{Z}_k|\mathbf{1}, \qquad \overline{z}_k = \widehat{z}_k + |\widehat{Z}_k|\mathbf{1}$$

- intervals containing each state component and unknown input
- on this example, better performance than Robinson et al. (2020) (reference [6] on figures), performance on par with Zhang et al. (2020) (reference [21] on figures)
- potential improvement of performance by tuning T, N with respect to criterion to be selected

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Context

- Work published in Chevet et al. (2021a)
- **Discrete-time LPV system**:

$$\begin{cases} x_{k+1} = A(\rho_k)x_k + B(\rho_k)u_k + D(\rho_k)w_k \\ y_t = C(\rho_k)x_k + E(\rho_k)v_k \end{cases}$$

- **Unknown input**: additive bias d_k on state equation
- **Pointwise observer**: significant uncertainty due to bounded perturbations, measurement noise and model uncertainties

Contribution

A *robust interval observer for joint estimation of state and unknown inputs* for LPV systems subject to *bounded perturbations and unknown inputs*

Considered model

LPV system subject to bounded perturbations, bounded measurement noise and unknown input:

$$\begin{cases} x_{k+1} = A(\rho_k)x_k + B(\rho_k)u_k + D_d d_k + D_w(\rho_k)w_k \\ y_k = Cx_k + D_v v_k \end{cases} \tag{7}$$

- state $x_k \in \mathbb{R}^{n_x}$, input $u_k \in \mathbb{R}^{n_u}$, output $y_k \in \mathbb{R}^{n_y}$, perturbation $w_k \in \mathbb{R}^{n_w}$, measurement noise $v_k \in \mathbb{R}^{n_v}$, unknown input $d_k \in \mathbb{R}^{n_f}$, parameter $\rho_k \in \mathbb{R}^{n_\rho}$
- x_0, w_k unknown but bounded:
  - $\rightarrow$ $\underline{x}_0 \leq x_0 \leq \overline{x}_0$, with $\underline{x}_0, \overline{x}_0 \in \mathbb{R}^{n_x}$, $\|\underline{x}_0\|, \|\overline{x}_0\| < \infty$
  - $\rightarrow$ $\underline{w}_k \leq w_k \leq \overline{w}_k$, with $\underline{w}_k, \overline{w}_k \in \mathbb{R}^{n_w}$, $\forall k \geq 0$, $\|\underline{w}\|_\infty = \sup\{\|w_k\| | k \geq 0\}$, $\|\overline{w}\|_\infty < \infty$
  - $\rightarrow$ $\underline{v}_k \leq v_k \leq \overline{v}_k$, with $\underline{v}_k, \overline{v}_k \in \mathbb{R}^{n_v}$, $\forall k \geq 0$, $\|\underline{v}\|_\infty = \sup\{\|v_k\| | k \geq 0\}$, $\|\overline{v}\|_\infty < \infty$
- ρ_k unknown and unmeasurable:
  - $\rightarrow$ $M(\rho_k) = M_0 + \Delta M(\rho_k)$, $\forall M \in \{A, B, D_w\}$
  - $\rightarrow$ $\Delta M(\rho_k)$ unknown but bounded, i.e. $\underline{\Delta M} \leq \Delta M(\rho_k) \leq \overline{\Delta M}$, $\forall M \in \{A, B, D_w\}$
- $\|x\|_\infty < \infty$, $\|u\|_\infty < \infty$

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Descriptor dynamics

Same approach as LTI case:

- addition of d_{k-1} to state vector and rewriting of the system into descriptor form (Li et al. 2020)

$$\begin{cases} Ez_{k+1} = F(\rho_k)z_k + G(\rho_k)u_k + D(\rho_k)w_k \\ y_k = Hz_k + D_v v_k \end{cases}$$

$\rightarrow$ $z_k^\top = \begin{bmatrix} x_k^\top & d_{k-1}^\top \end{bmatrix}$, $d_{-1} = \mathbf{0}$
$\rightarrow$ the matrices

$$E = \begin{bmatrix} I & -D_d \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad F(\rho_k) = \begin{bmatrix} A(\rho_k) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad G(\rho_k) = \begin{bmatrix} B(\rho_k) \\ \mathbf{0} \end{bmatrix}$$

$$D(\rho_k) = \begin{bmatrix} D_w(\rho_k) \\ \mathbf{0} \end{bmatrix} \quad H = \begin{bmatrix} C & \mathbf{0} \end{bmatrix}$$

- M_0, $\Delta M(\rho_k)$, $\forall M \in \{F, G, D\}$ obtained from $A(\rho_k)$, $B(\rho_k)$, $D_w(\rho_k)$

Rewriting as state-space dynamics

Assumption

$$\text{rank}\begin{bmatrix} I & -D_d \\ C & \mathbf{0} \end{bmatrix} = n_x + n_d = n_z$$

There exists T, N satisfying:

$$TE + NH = I \underset{\substack{\text{(Rao and} \\ \text{Mitra 1972)}}}{\Longrightarrow} \begin{bmatrix} T & N \end{bmatrix} = \begin{bmatrix} E \\ H \end{bmatrix}^{\dagger} + \underbrace{\Xi}_{\text{free matrix}} \left(I - \begin{bmatrix} E \\ H \end{bmatrix} \begin{bmatrix} E \\ H \end{bmatrix}^{\dagger} \right)$$

Dynamics for observer design:

$$\begin{cases} z_{k+1} = TF(\rho_k)z_k + TG(\rho_k)u_k + TD(\rho_k)w_k + Ny_{k+1} - ND_v v_{k+1} \\ y_k = Hz_k + D_v v_k \end{cases}$$

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State framer

$$\begin{cases} \underline{z}_{k+1} = (TF_0 - \underline{L}H)\underline{z}_k + TG_0 u_k + Ny_{k+1} + \underline{L}y_k + \underline{\phi}_k + \underline{\chi}_k + \underline{\psi}_k + \underline{\omega}_k \\ \overline{z}_{k+1} = (TF_0 - \overline{L}H)\overline{z}_k + TG_0 u_k + Ny_{k+1} + \overline{L}y_k + \overline{\phi}_k + \overline{\chi}_k + \overline{\psi}_k + \overline{\omega}_k \end{cases} \quad (8)$$

- $\underline{L}$, $\overline{L}$ observer gains
- $\underline{\phi}_k, \overline{\phi}_k, \underline{\chi}_k, \overline{\chi}_k, \underline{\psi}_k, \overline{\psi}_k, \underline{\omega}_k, \overline{\omega}_k$ obtained with Lemma 1, satisfying

$$\underline{\phi}_k \leq T\Delta A(\rho_k)x_k \leq \overline{\phi}_k \qquad \underline{\chi}_k \leq T\Delta B(\rho_k)u_k \leq \overline{\chi}_k$$

$$\underline{\omega}_k \leq TD(\rho_k)w_k \leq \overline{\omega}_k \qquad \underline{\psi}_k \leq TD_v v_k \leq \overline{\psi}_k$$

Theorem 4

For the considered model, if $TF_0 - \underline{L}H$ and $TF_0 - \overline{L}H$ are positive matrices[a], then

$$\underline{z}_k \leq z_k \leq \overline{z}_k, \ \forall k \geq 0$$

[a] A matrix $M \in \mathbb{R}^{n \times n}$ is positive if all its elements are nonnegative.

Interval observer

Same approach and notations as in the continuous-time case

Theorem 5

For the proposed model and given $\alpha > 0$, if there exists $\gamma > 0$, $P \in \mathbb{R}^{2n_z \times 2n_z}$, with $P \succ 0$ diagonal, and $Y \in \mathbb{R}^{2n_z \times 2n_y}$ such that:

$$S \geq \mathbf{0} \qquad \text{(Cooperativity)}$$

$$\begin{bmatrix} (\alpha - 1)P + \gamma Q & \mathbf{0} & \mathbf{0} & S^\top \\ \mathbf{0} & -\gamma I_{2n_z} & \mathbf{0} & P^\top \\ \mathbf{0} & \mathbf{0} & -\gamma I_{2n_z} & P^\top \\ S & P & P & -P \end{bmatrix} \preceq 0 \qquad \text{(Stability)}$$

$$P \succeq \alpha I_{2n_z} \qquad \text{(Performance)}$$

then (8) is a robust interval observer for (7) with performance $\|E_k\|^2 \leq \frac{(1-\alpha)^k}{\alpha} V_0 + \frac{\gamma}{\alpha^2}(\|\varepsilon\|_\infty^2 + \beta)$

- Gain matrices $\underline{L}, \overline{L}$ obtained as $\text{diag}(\underline{L}, \overline{L}) = P^{-1} Y$

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Simulation parameters

- Matrices $D_{w0} = I_3$, $E = I_2$,

$$A_0 = 0.1 \begin{bmatrix} -6 & 5 & 4 \\ 7 & 5 & 2 \\ 1 & 5 & 3 \end{bmatrix} \qquad B_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad D_d = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

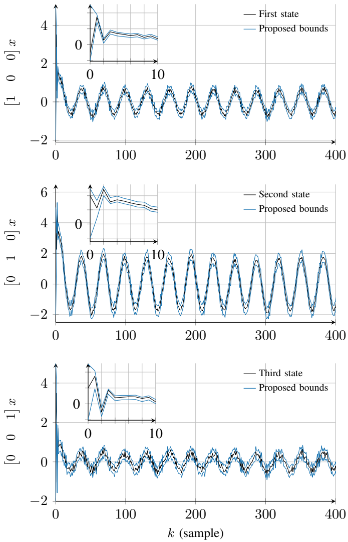
- $\Delta B(\rho_k) = \mathbf{0}$, $\Delta D_w(\rho_k) = \mathbf{0}$ and

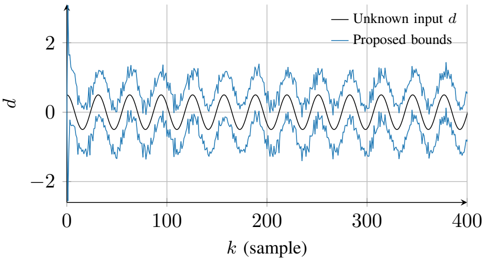
$$\Delta A(\rho_k) = 0.02 \cdot \begin{bmatrix} 0.1\sin(\omega_1 k) & \sin(\omega_2 k) & \cos(\omega_1 k) \\ \cos(\omega_2 k) & \sin(2\omega_1 k) & 0.1\cos(2\omega_1 k) \\ \sin(\omega_1 k/2) & 0.1\cos(\omega_2 k/2) & \sin(\omega_1 k)\cos(\omega_2 k) \end{bmatrix}$$

- $-2 \cdot \mathbf{1}_3 \leq x_0 \leq 5 \cdot \mathbf{1}_3$, $-0.1 \cdot \mathbf{1}_3 \leq w_k \leq 0.1 \cdot \mathbf{1}$, $-0.1 \cdot \mathbf{1}_2 \leq v_k \leq 0.1 \cdot \mathbf{1}_2$
- $u_k = -\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} y_k$, $d_k = 0.5\cos(0.2k)$
- $\Xi = \mathbf{0}$ so that

$$T = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{bmatrix} \qquad N = \begin{bmatrix} 0 & 0.5 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Simulation results





- intervals containing each state component and unknown input
- potential improvement of performance by tuning T, N with respect to criterion to be selected

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General conclusion

- Interval observer-based sensor fault detection and unknown input estimation strategies for linear parameter-varying systems
  - $\rightarrow$ linear matrix inequality-based design allowing for inclusion of additional constraints
- Zonotopic Kalman filter-based unknown input estimation strategy for linear time-invariant systems
- Future work
  - $\rightarrow$ optimal tuning of weighting matrices T, N
  - $\rightarrow$ adapt fault detection strategy to detection of actuator/input sensor faults
  - $\rightarrow$ adapt zonotopic Kalman filter to linear parameter-varying systems

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