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BARCELONATECH

Robust Fault Detection using Set-based Approaches for LPV Systems: Application to Autonomous Vehicles

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International Online Seminar on Interval Methods in Control Engineering
20, May, 2022

CONTENT

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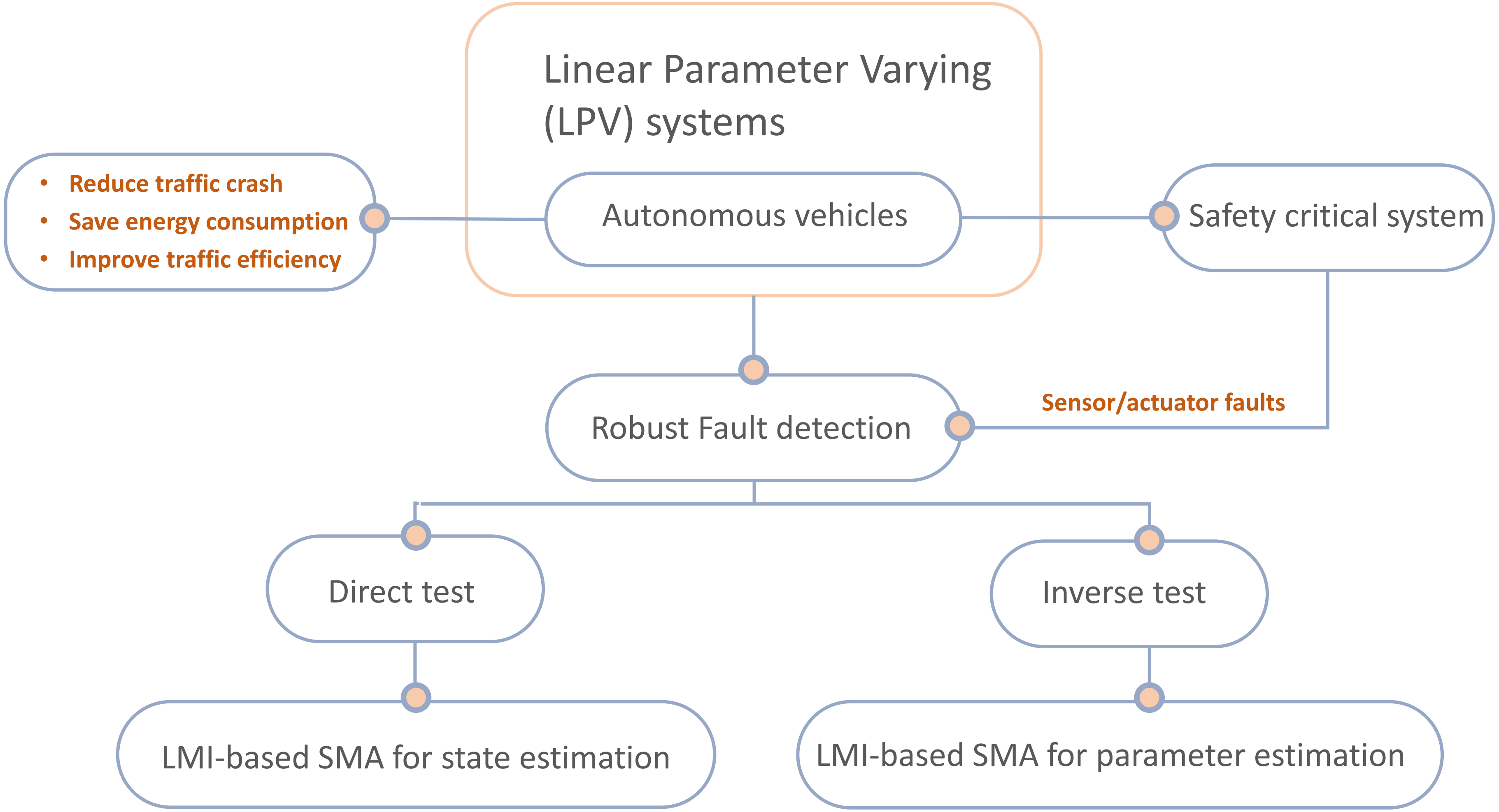
- Introduction &Problem Formulation
- Direct Test Implementation
- Inverse Test Implementation
- Application to Autonomous Vehicles
- Conclusions & Perspective

1. Introduction & Problem Formulation



1. Introduction & Problem Formulation

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- Vehicle Model

$$\begin{bmatrix}\dot{\beta}\\ \dot{\phi}\end{bmatrix}=\begin{bmatrix}-\frac{c_f+c_r}{mv_x} & \frac{c_r l_r - c_f l_f}{mv_x^2}-1\\ \frac{c_r l_r - c_f l_f}{I_z} & -\frac{c_r l_r^2 + c_f l_f^2}{I_z v_x}\end{bmatrix}\begin{bmatrix}\beta\\ \phi\end{bmatrix}+\begin{bmatrix}\frac{c_f}{mv_x}\\ \frac{c_f l_f}{I_z}\end{bmatrix}\delta_f$$

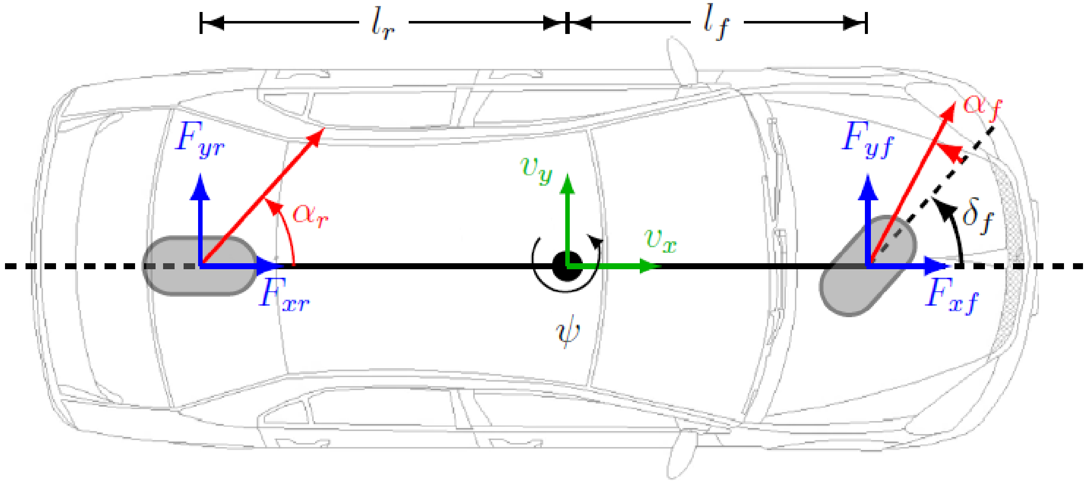

Figure. 2-DOF bicycle vehicle model.

- Linear Parameter-varying (LPV) Discrete-time Dynamic Model

$$x_{k+1} = A(\theta_k)x_k + B(\theta_k)u_k + Ew_k$$

$$y_k = Cx_k + Fv_k$$

$$A(\theta_k)=\begin{bmatrix}1-T\frac{c_f+c_r}{m\theta_k} & T\frac{c_r l_r - c_f l_f}{m\theta_k^2}-T\\ T\frac{c_r l_r - c_f l_f}{I_z} & 1-T\frac{c_r l_r^2 + c_f l_f^2}{I_z\theta_k}\end{bmatrix}, B(\theta_k)=\begin{bmatrix}T\frac{c_f}{m\theta_k}\\ T\frac{c_f l_f}{Iz}\end{bmatrix}, C=I.$$

- Polytopic LPV Model

$$x_{k+1}=\sum_{i=1}^{N}\mu_i(\theta_k)\left(A_i x_k + B_i u_k\right)$$

$$\sum_{i=1}^{N}\mu_i(\theta_k)=1,\quad \mu_i(\theta_k)\geq 0, \forall\theta_k\in\Theta$$

1. Introduction & Problem Formulation

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- Fault Detection
  - *Improve safety and reliability*
  - *Consistency check*
    - Estimated && observed behaviors
  - *Uncertainties*
    - *Modelling uncertainty*
    - *Noise*
    - *Disturbance*
- Robust Fault Detection
  - *Set-Membership*
  - Unknown but bounded uncertainties
  - Zonotopic sets

- Passive Approach
  - *Direct test*

$$\mathbf{r}(k) = \mathbf{y}_k - \mathbf{y}_k$$

$$\mathbf{y}_k \in [\mathbf{y}_k - \sigma, \mathbf{y}_k + \sigma]$$

↔ *State estimation*

  - *Inverse test*

$$\mathbf{r}(k) = \boldsymbol{\theta}_k - \boldsymbol{\theta}_k$$

$$\exists \hat{\theta}_k \in \Theta | \ \mathbf{y}(k, \theta_k) \in [\mathbf{y}_k - \sigma, \mathbf{y}_k + \sigma]$$

$$\Theta_{k+1} = \Theta_k \cap \mathbb{F}_k$$

$$\Theta_{k+1} = \varnothing$$

↔ *Parameter estimation*

$$\mathbb{F}_k = \left\{ \theta \in \mathbb{R}^{n_\theta} | \ \mathbf{y}_k - \sigma \le c_k^T \theta_k \le \mathbf{y}_k + \sigma \right\}$$

2. Direct Test Implementation

- Set-Membership Approach for State Estimation (for LTI)
  - *Prediction Step:*

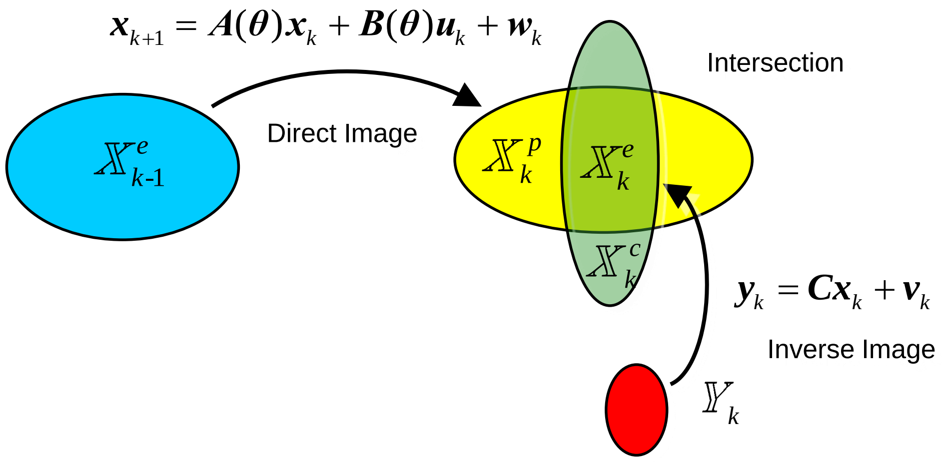
    Compute the uncertain state set $\mathcal{X}_k^p$
  - *Measurement Step:*

    Compute the measurement consistent state set $\mathcal{X}_k^c$ *with the measured output* y_k
  - *Correction Step:*

    Compute an outer approximation of the intersection $\mathcal{X}_k^e$ *between* $\mathcal{X}_k^p$ *and* $\mathcal{X}_k^c$

Algorithm 2 Fault Detection using Set Computations

1: $\mathcal{X}_k^e \Leftarrow \mathcal{X}_0$
2: **for** $k = 1$ to N **do**
3: Compute $\mathcal{X}_k^p$
4: Compute $\mathcal{X}_k^c$
5: Compute $\mathcal{X}_k^e = \mathcal{X}_k^p \cap \mathcal{X}_k^c$
6: **end for**



$$\mathcal{X}_k^p = \langle c_p^{sm}, R_p^{sm} \rangle \qquad \begin{aligned} c_p^{sm} &= Ac_{x,-}^{sm} + Bu_- \\ R_p^{sm} &= \begin{bmatrix} AR_{x,-}^{sm} & E \end{bmatrix} \end{aligned}$$

$$\mathcal{X}_k^e = \langle c_x^{sm}, R_x^{sm} \rangle \qquad \begin{aligned} c_x^{sm} &= c_p^{sm} + \lambda\left(y - Cc_p^{sm}\right) \\ R_x^{sm} &= \begin{bmatrix} (I - \lambda C) R_p^{sm} & -\lambda F \end{bmatrix} \end{aligned} \qquad \lambda^* : \min \left\| R_x^{sm} \right\|_F^2$$

2. Direct Test Implementation

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LMI-based SMA for State Estimation (for LPV)

- Prediction

$$c_p^{lmi} = \sum_{i=1}^{N} \mu_i(\theta_{k-1})(A_i c_{x,-}^{lmi} + B_i u_-)$$

$$R_p^{lmi} = \sum_{i=1}^{N} \mu_i(\theta_{k-1})\begin{bmatrix} A_i R_{x,-}^{lmi} & E \end{bmatrix}$$

- Correction

$$c_x^{lmi} = c_p^{lmi} + \sum_{i=1}^{N} \mu_i(\theta_{k-1})\lambda_i\left(y - C c_p^{lmi}\right)$$

$$R_x^{lmi} = \sum_{i=1}^{N} \mu_i(\theta_{k-1})\begin{bmatrix} (I - \lambda_i C) R_p^{lmi} & -\lambda_i F \end{bmatrix}$$

$$\lambda^* : \min \left\| R_x^{sm} \right\|_F^2, \quad \lambda^* = \frac{R_p^{lmi} R_p^{lmiT} C}{C R_p^{lmi} R_p^{lmiT} C^T + F F^T}$$

$$\lambda^* = \sum_{i=1}^{N} \mu_i(\theta_{k-1})\lambda_i$$

$$\lambda_i = \Upsilon^{-1} W_i : \quad \min \gamma, s.t., \quad \begin{bmatrix} \gamma I_{n_x} & I_{n_x} \\ I_{n_x} & \Upsilon \end{bmatrix} > 0, \quad \begin{bmatrix} -\Upsilon & \Upsilon A_i - W_i C & \Upsilon E & W_i \\ A_i^T \Upsilon - C^T W_i^T & -\Upsilon & 0 & 0 \\ E^T \Upsilon & 0 & -I & 0 \\ W_i^T & 0 & 0 & -R^{-1} \end{bmatrix} < 0$$

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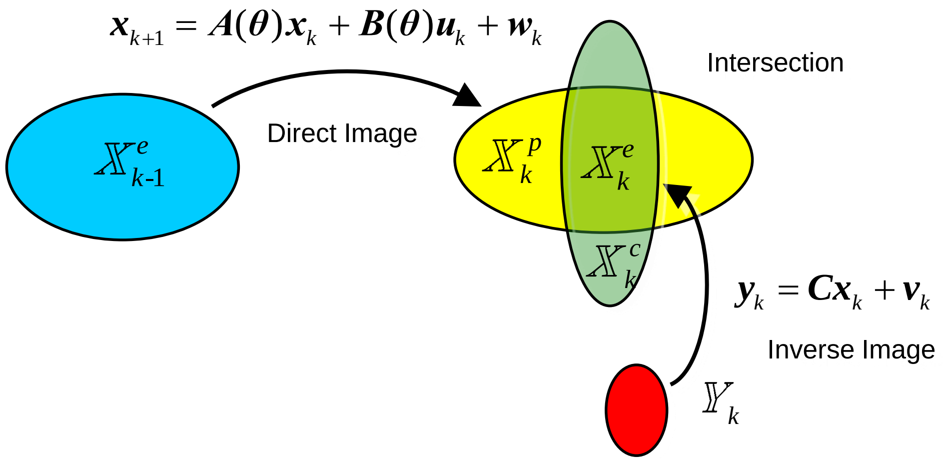
$$-P + (A_i - \lambda_i C) P (A_i - \lambda_i C)^T + Q + \lambda_i R \lambda_i^T < 0 \rightarrow \Upsilon = P^{-1}, W_i = \Upsilon \lambda_i$$

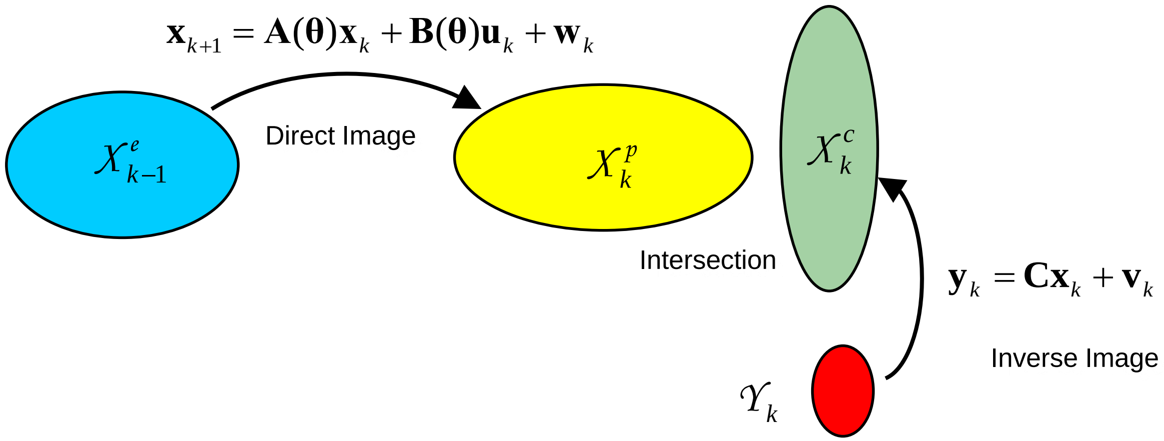
2. Direct Test Implementation

- Fault Detection by Direct Test

Algorithm 1 Fault Detection using Set Computations

```
1: X_k^e ⇐ X_0
2: for k = 1 to N do
3:     Compute X_k^p
4:     Compute X_k^c
5:     Compute X_k^e = X_k^p ∩ X_k^c
6:     if X_k^e = ∅ then
7:         Exit (Fault detected)
8:     end if
9: end for
```





3. Inverse Test Implementation

- LPV model for Parameter Estimation

$$y_k = c_k^T \theta_k + v_k = \hat{y}_k + v_k$$

$$y_k = \phi_k, \theta_k = \left[A_{21}, A_{22}, B_2\right]^\top,$$
$$c_k^\top = \left[\beta_{k-1}, \phi_{k-1}, \delta_{f\,k-1}\right],$$

$$A = I, B = 0, C = c_k^T, E = 0$$

$$\begin{bmatrix} \beta_{k+1} \\ \phi_{k+1} \end{bmatrix} = \begin{bmatrix} 1 - T\dfrac{c_f + c_r}{m\theta_k} & T\dfrac{c_r l_r - c_f l_f}{m{\theta_k}^2} - T \\ T\dfrac{c_r l_r - c_f l_f}{I_z} & 1 - T\dfrac{c_r l_r^2 + c_f l_f^2}{I_z \theta_k} \end{bmatrix} \begin{bmatrix} \beta_k \\ \phi_k \end{bmatrix} + \begin{bmatrix} T\dfrac{c_f}{m\theta_k} \\ T\dfrac{c_f l_f}{Iz} \end{bmatrix} \delta_{f_k}$$

$$\theta_k \in \Theta_k = P_k \oplus H_k B^n = \left\{P_k + H_k \mathbf{z} : \mathbf{z} \in B^n\right\}$$

3. Inverse Test Implementation

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- Set-Membership Approach for Parameter Estimation

$$P_+ = P + \lambda\left(y_+ - c^T P\right)$$

$$H_+ = \left[\left(I - \lambda c^T\right)H, -\lambda F\right]$$

$$\lambda^* = \frac{HH^T c}{c^T HH^T c + FF^T}$$

$A = I, B = 0, C = c_k^T, E = 0$

SMA for State Estimation

$$c_p^{sm} = Ac_{x,-}^{sm} + Bu_-$$

$$R_p^{sm} = \left[AR_{x,-}^{sm} \quad E\right]$$

$$c_x^{sm} = c_p^{sm} + \lambda\left(y - Cc_p^{sm}\right)$$

$$R_x^{sm} = \left[\left(I - \lambda C\right)R_p^{sm} \quad -\lambda F\right]$$

- LMI-based Set-Membership Approach for Parameter Estimation

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$$\lambda^* : \min \gamma, s.t., \quad \begin{bmatrix} \gamma I_{n_x} & I_{n_x} \\ I_{n_x} & \Upsilon_k \end{bmatrix} > 0, \quad \begin{bmatrix} -P^{-1} & \Upsilon I_{n_x} - WC & W \\ I_{n_x}\Upsilon - C^T W^T & -\Upsilon & 0 \\ W^T & 0 & -R^{-1} \end{bmatrix} < 0$$

$$P = (I - \lambda C)\bar{P}_+(I - \lambda C)^T + \lambda FF^T\lambda^T \rightarrow \Upsilon = P_+^{-1}, P = HH^T$$

$A = I, B = 0, C = c_k^T, E = 0$

$$\lambda_i = \Upsilon^{-1}W_i : \min \gamma, s.t., \quad \begin{bmatrix} \gamma I_{n_x} & I_{n_x} \\ I_{n_x} & \Upsilon \end{bmatrix} > 0, \quad \begin{bmatrix} -\Upsilon & \Upsilon A_i - W_i C & \Upsilon E & W_i \\ A_i^T \Upsilon - C^T W_i^T & -\Upsilon & 0 & 0 \\ E^T \Upsilon & 0 & -I & 0 \\ W_i^T & 0 & 0 & -R^{-1} \end{bmatrix} < 0$$

$$-P + (A_i - \lambda_i C)P(A_i - \lambda_i C)^T + Q + \lambda_i R \lambda_i^T < 0 \rightarrow \Upsilon = P^{-1}, W_i = \Upsilon\lambda_i$$

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Fault Detection by Inverse Test

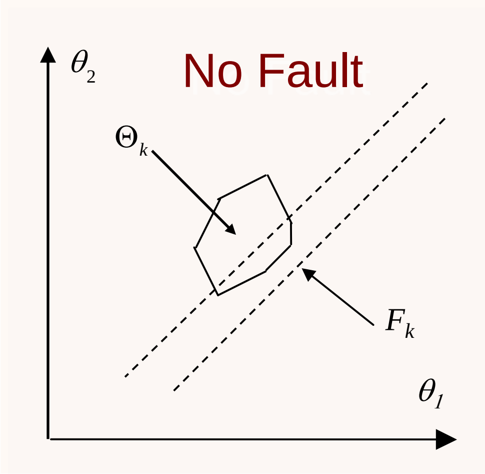
$$\exists \hat{\theta}_k \in \Theta | \; \mathbf{y}(k, \theta_k) \in [\mathbf{y}_k - \sigma, \mathbf{y}_k + \sigma]$$

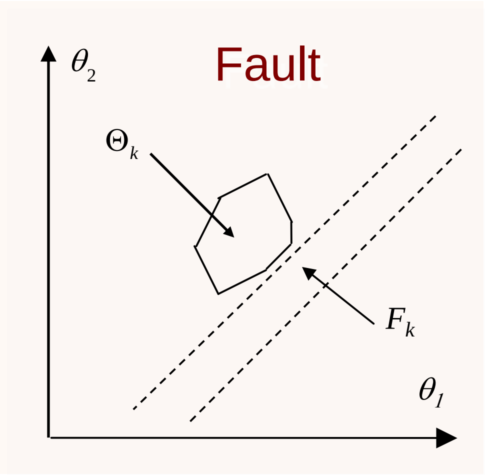
$$\mathbb{F}_k = \left\{ \theta \in \mathbb{R}^{n_\theta} \mid \mathbf{y}_k - \sigma \le c_k^T \theta_k \le \mathbf{y}_k + \sigma \right\}$$

$$\Theta_k \cap \mathbb{F}_k = \varnothing$$

Algorithm 1 Fault Detection using Inverse Test.

```
Obtain a sequence of rehressor vector values c_k and measurement values y_k
Θ_k ← Θ_0
for k = 1 to N do
    Compute Θ_k
    Compute F_k
    Compute Θ_{k+1} = Θ_k ⋂ F_k
    if Θ_{k+1} = ∅ then
        Exit (Fault)
    end if
end for
```





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State Estimation

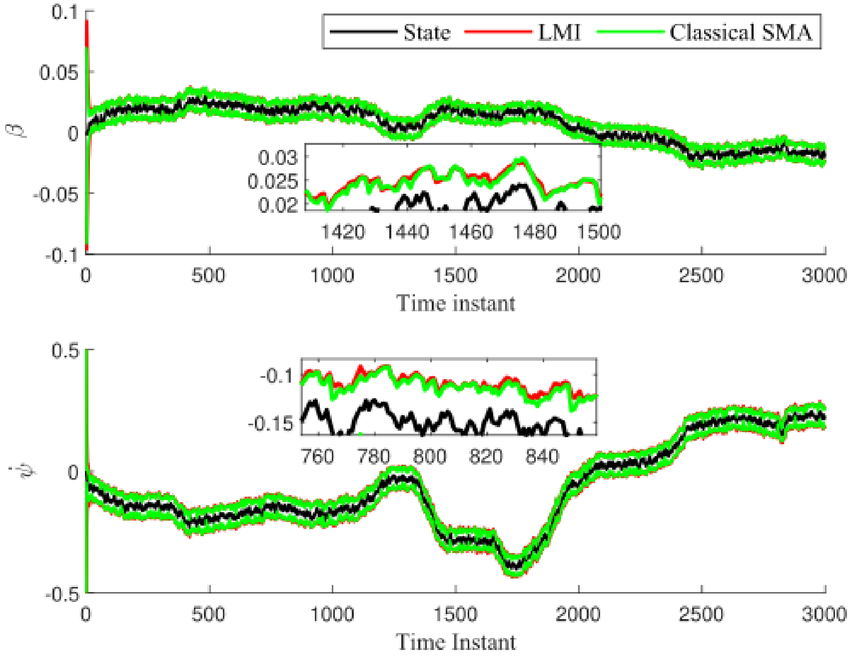


Figure. State estimation using classical SMA and LMI-based SMA

$$\begin{bmatrix}\beta_{k+1}\\ \phi_{k+1}\end{bmatrix} = \begin{bmatrix} 1-T\dfrac{c_f+c_r}{m\theta_k} & T\dfrac{c_r l_r - c_f l_f}{m\theta_k^{\,2}} - T \\ T\dfrac{c_r l_r - c_f l_f}{I_z} & 1-T\dfrac{c_r l_r^2 + c_f l_f^2}{I_z\theta_k}\end{bmatrix}\begin{bmatrix}\beta_k\\ \phi_k\end{bmatrix} + \begin{bmatrix} T\dfrac{c_f}{m\theta_k}\\ T\dfrac{c_f l_f}{Iz}\end{bmatrix}\delta_{f_k}$$

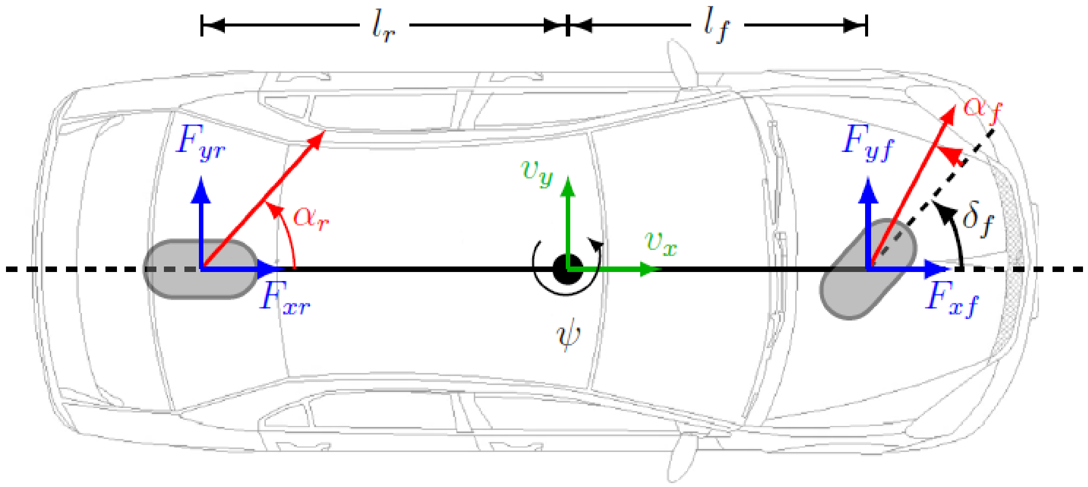


Figure. 2-DOF bicycle vehicle model.

Parameter Estimation

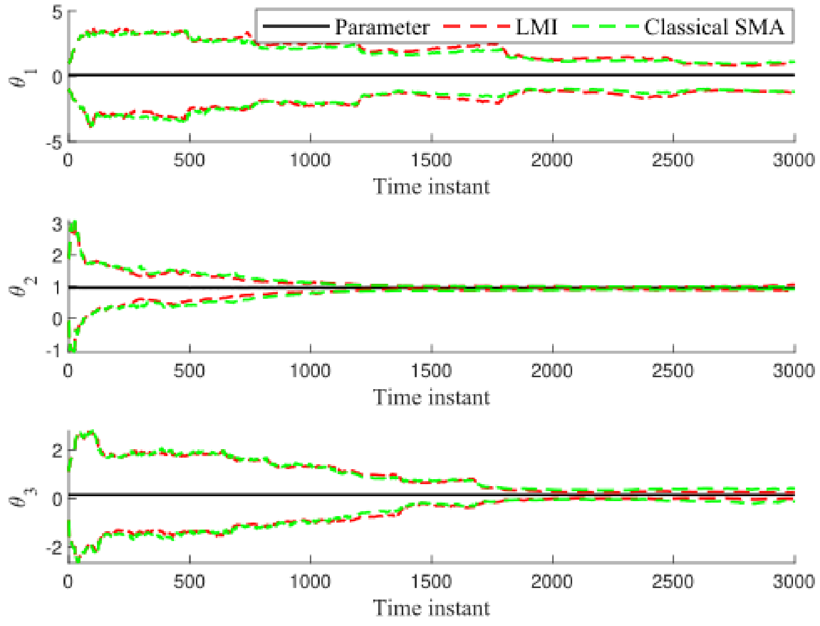


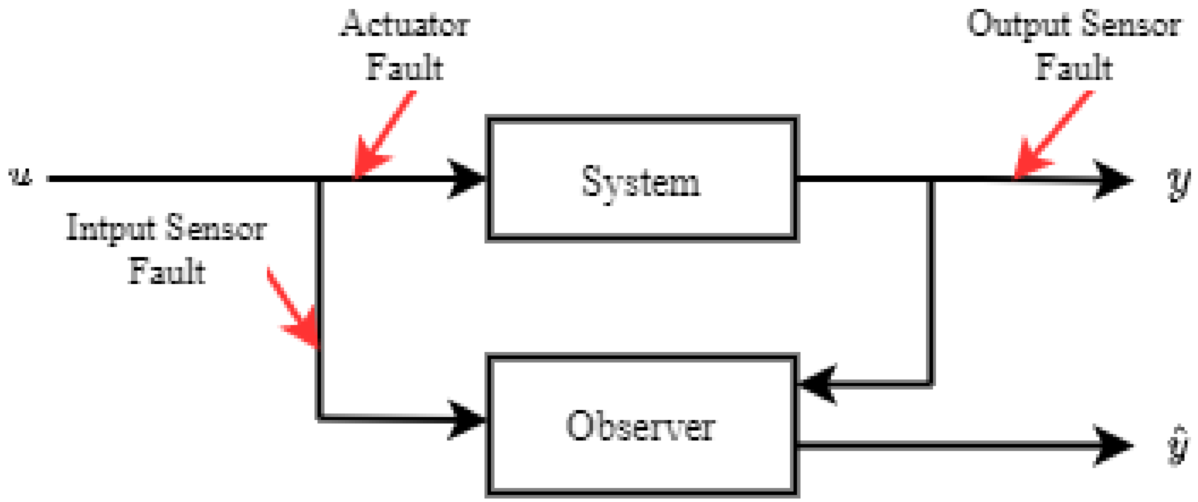
Figure. Parameter estimation using classical SMA and LMI-based SMA

$$y_k = c_k^T \theta_k + v_k = \hat{y}_k + v_k$$

$$y_k = \phi_k, \theta_k = \left[A_{21}, A_{22}, B_2\right]^\top,$$
$$c_k^\top = \left[\beta_{k-1}, \phi_{k-1}, \delta_{f\,k-1}\right],$$

$$A(\theta_k) = \begin{bmatrix} 1 - T\dfrac{c_f + c_r}{m\theta_k} & T\dfrac{c_r l_r - c_f l_f}{m{\theta_k}^2} - T \\ T\dfrac{c_r l_r - c_f l_f}{I_z} & 1 - T\dfrac{c_r l_r^2 + c_f l_f^2}{I_z \theta_k} \end{bmatrix}, B(\theta_k) = \begin{bmatrix} T\dfrac{c_f}{m\theta_k} \\ T\dfrac{c_f l_f}{Iz} \end{bmatrix}$$

- Consistency Test
  - Additive faults
    - Actuator fault
    - Input sensor fault
    - Output sensor fault
  - Multiplicative faults
    - C_f fault
    - I_z fault



$$A(\theta_k) = \begin{bmatrix} 1 - T\dfrac{c_f + c_r}{m\theta_k} & T\dfrac{c_r l_r - c_f l_f}{m\theta_k^{\ 2}} - T \\ T\dfrac{c_r l_r - c_f l_f}{I_z} & 1 - T\dfrac{c_r l_r^2 + c_f l_f^2}{I_z \theta_k} \end{bmatrix}, B(\theta_k) = \begin{bmatrix} T\dfrac{c_f}{m\theta_k} \\ T\dfrac{c_f l_f}{Iz} \end{bmatrix}$$

4. Application to Autonomous Vehicles

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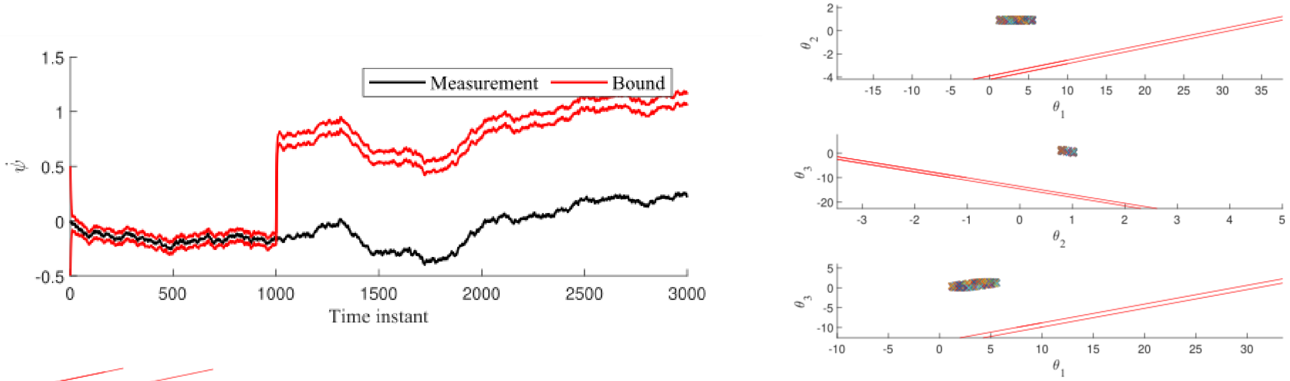
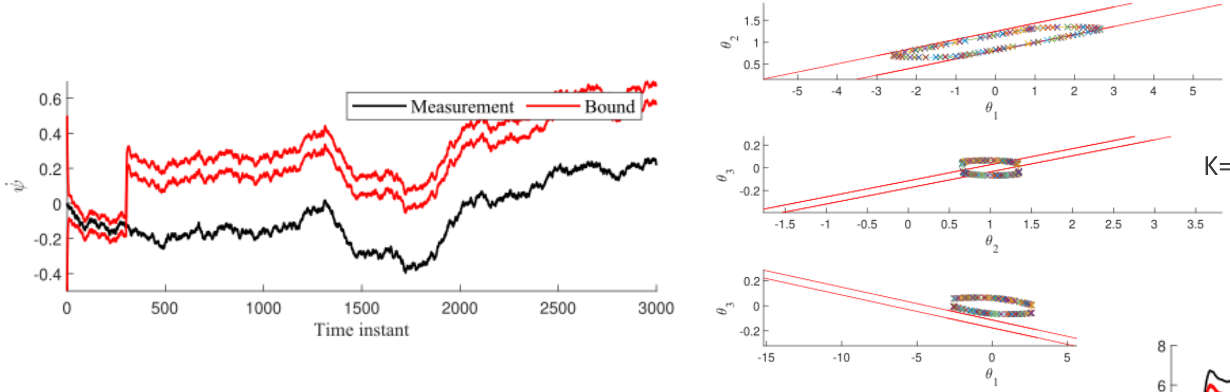
- Consistency Test for Additive Fault
  - Direct test is effective for all additive faults,
  - Inverse test is partially valid for additive faults.



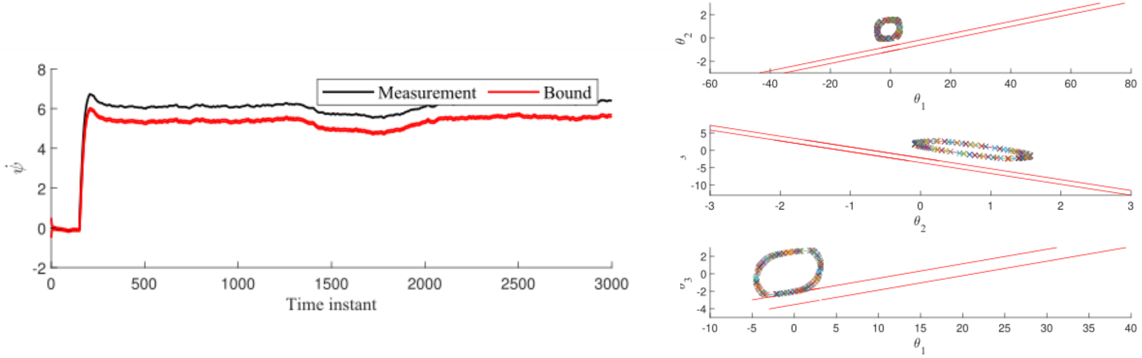
K=1002

Figure. Output fault detection by direct and inverse test



K=302

Figure. Input fault detection by direct and inverse test



K=152

Figure. Actuator fault detection by direct and inverse test

4. Application to Autonomous Vehicles

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- Consistency Test for Multiplicative Fault
  - Direct test is partially effective for multiplicative faults,
  - Inverse test is valid for all multiplicative faults.

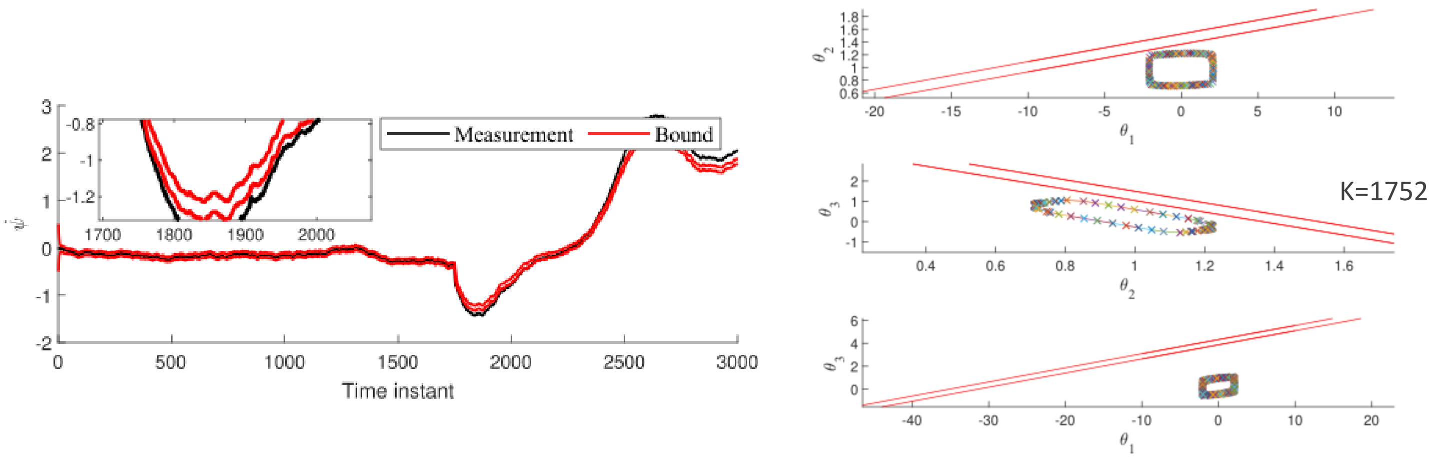

Figure. f_{cf} fault detection by direct and inverse test

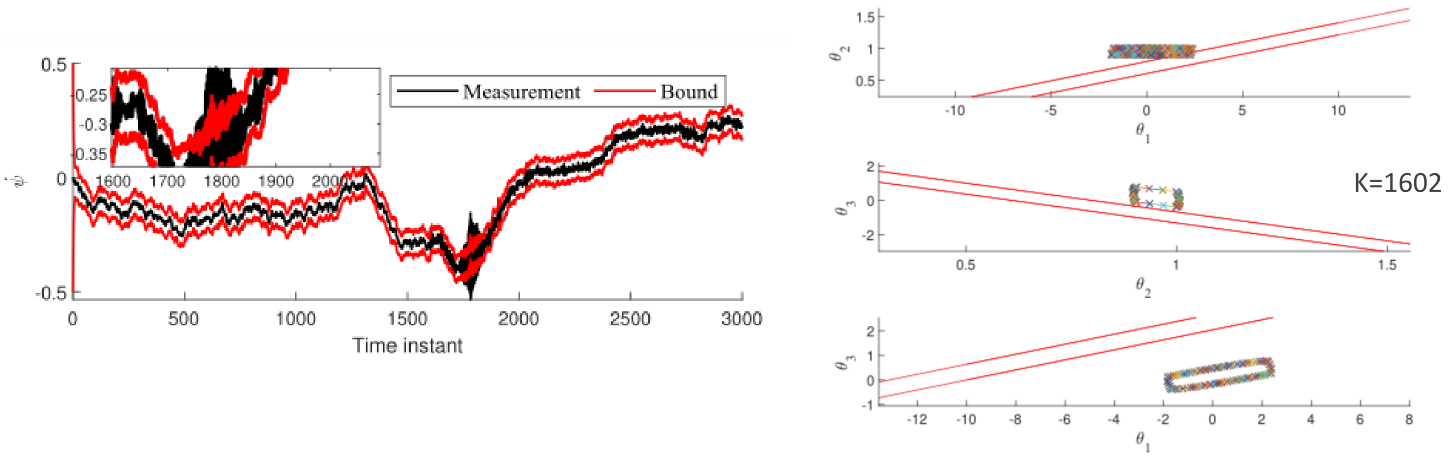

Figure. f_{I_z} fault detection by direct and inverse test

5. Conclusions and Perspective

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- Conclusions
  - LMI-based state/parameter estimation methods for LPV systems,
  - LMI-based SMA is a good alternative to the classical SMA,
  - LMI-based SMA for parameter estimation has kind of low efficiency but high response with quick convergence,
  - Direct test has better performance on additive faults, inverse test has better performance on multiplicative faults.
- Perspective
  - Find the minimum detectable fault to describe the detectability,
  - Research on a switched LPV system to adapt the whole range of parameters.

Thank you!