A Geometric Approach to the Coverage Measure of the Area Explored by a Robot

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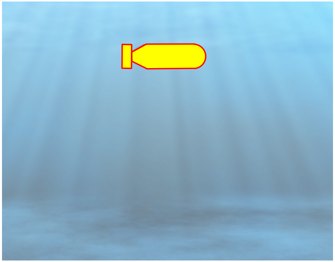
17/06/2022

Introduction

Case of Study

- Navigation sensors (Proprioceptive)
  - IMU, DVL ...



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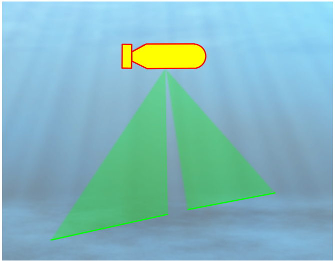

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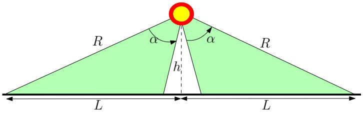
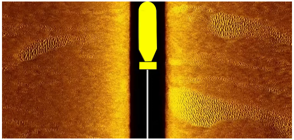
Side Scan Sonar





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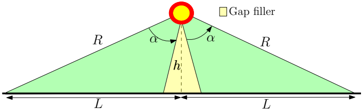
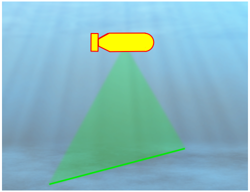
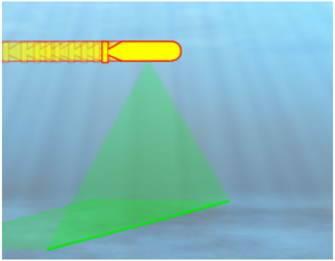




Introduction

Explored Area

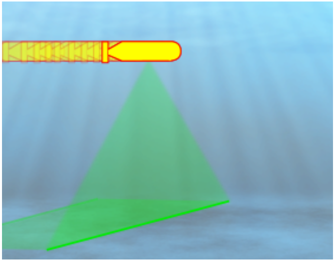
The explored area is the union of the visible areas over the whole trajectory.



Introduction

Objectives

- Compute the explored area.
- Compute the number of times each point in the environment has been explored.



Applications

Explored Area

- Assess area-covering missions,
- plan other missions to fill possible gaps.

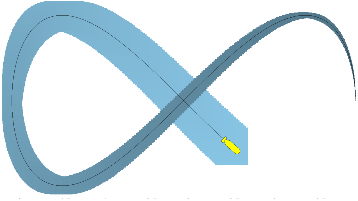
Coverage Measure

- Localization in homogeneous environments,
- assess revisiting missions,
- trajectory planning.

Applications

Coverage Measure

- Localization in homogeneous environments,



1. Proprioceptive estimation

S. Rohou, B. Desrochers, *et al.*, *The Codac library – Constraint-programming for robotics*, 2022

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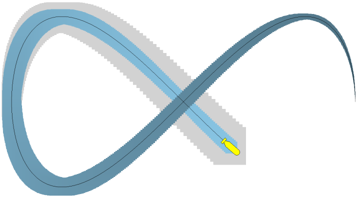
Coverage Measure

- Localization in homogeneous environments,



1. Proprioceptive estimation
2. Exteroceptive update

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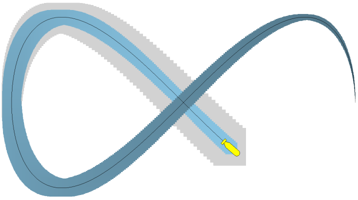

1. Proprioceptive estimation
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Figure 1

S. Rohou, B. Desrochers, *et al.*, *The Codac library – Constraint-programming for robotics*, 2022

Figure 1: S. Rohou, B. Desrochers, and L. Jaulin, "Set-membership state estimation by solving data association,",

Applications

Coverage Measure

- assess revisiting missions,

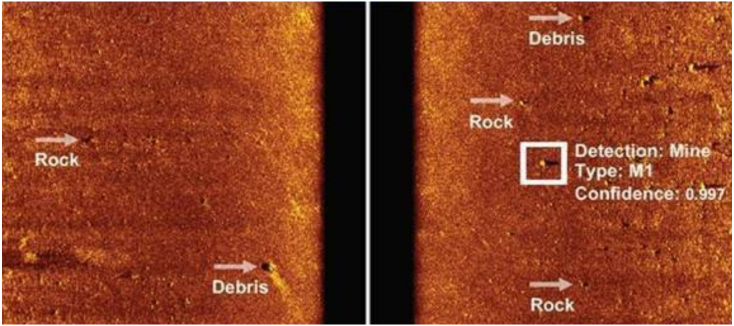


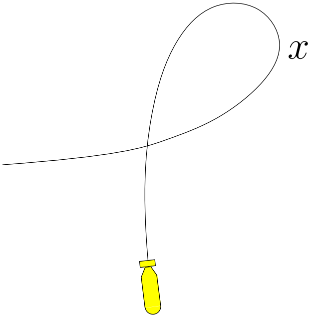
Figure 2

Figure 2: *How marine roboticists are turning auv sight into perception*, www.maritimemagazines.com/

Problem Statement

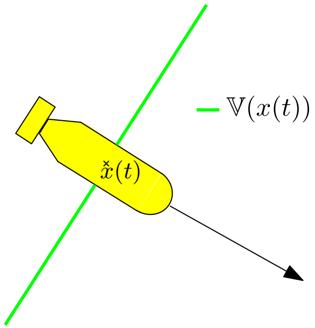
Hypothesis

- $x : \mathbb{R} \to \mathbb{R}^2$,
- $T = [0, T_{max}]$,
- x is continuous in T.



Problem Statement

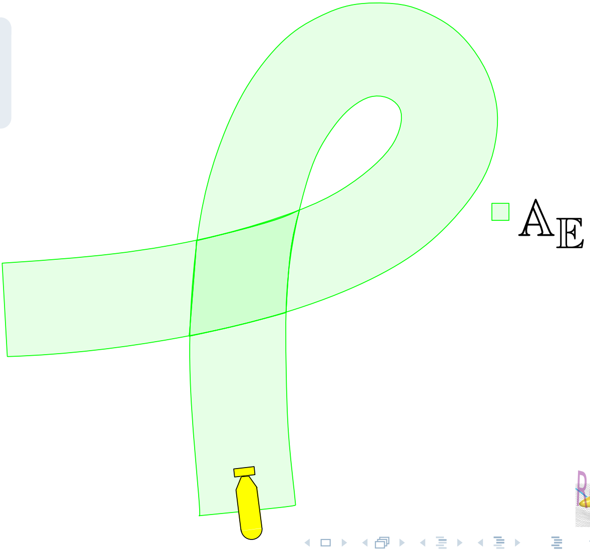
- $\mathbb{V}(x(t))$ is the visible area at time t.



Problem Statement

$\mathbb{A}_{\mathbb{E}}$ is the explored area

$$\mathbb{A}_{\mathbb{E}} = \bigcup_{t \in T} \mathbb{V}(t)$$



Problem Statement

Entries

- x, the robot's trajectory,
- T, time interval.
- $\mathbb{V}$, visible area

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Desired Output

- Explored area $\mathbb{A}_{\mathbb{E}}$,
- coverage measure of all the points in the plane, $\mathbb{C}_{\mathbb{M}}$.

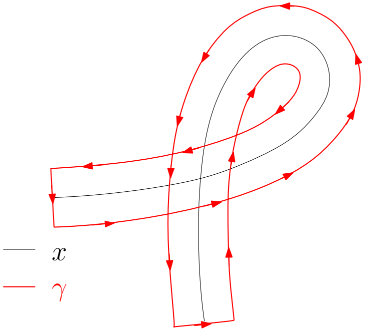
$$\mathbb{A}_{\mathbb{E}} = \{p \in \mathbb{R}^2 | \mathbb{C}_{\mathbb{M}}(p) > 0\}$$

1. Introduction
2. Problem Statement
3. **Problem Approach**
4. Computing the Winding Number
5. Results
6. Conclusions

Problem Approach

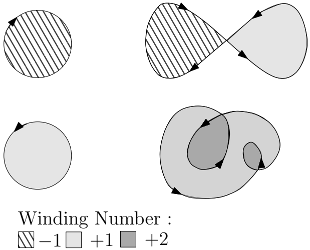
The sonar's contour

- $\gamma : [0, 1] \to \mathbb{R}^2$,
- γ is continuous,
- $\gamma(0) = \gamma(1)$.



Problem Approach

Winding Number

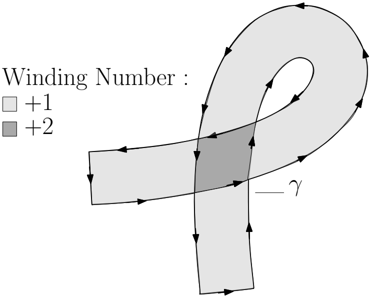


- γ is a closed curve in $\mathbb{R}^2$,
- $p \in \mathbb{R}^2$,
- $\eta(\gamma, p) \in \mathbb{Z}$.

Problem Approach

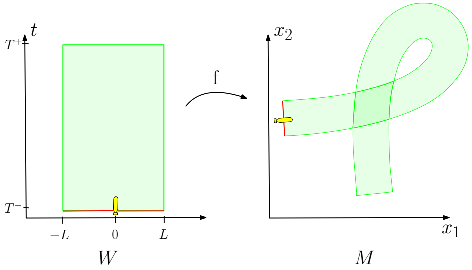
$$\mathbb{C}_{\mathbb{M}}(p) = \eta(\gamma, p)$$

$$\mathbb{A}_{\mathbb{E}} = \{p \in \mathbb{R}^2 | \eta(\gamma, p) > 0\}$$



Problem Approach

Waterfall and Mosaic spaces [1]



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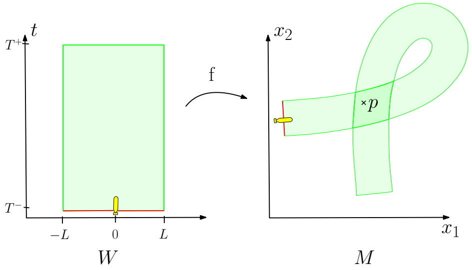

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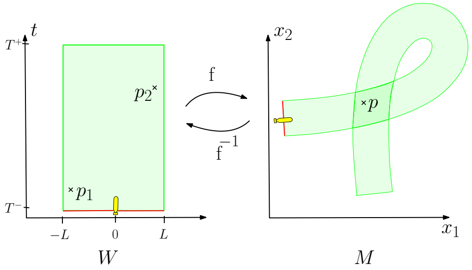

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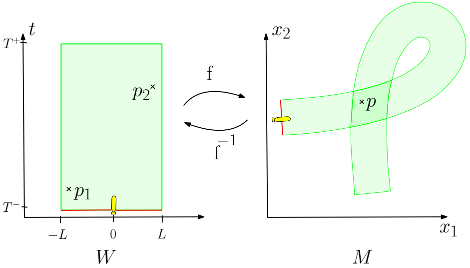

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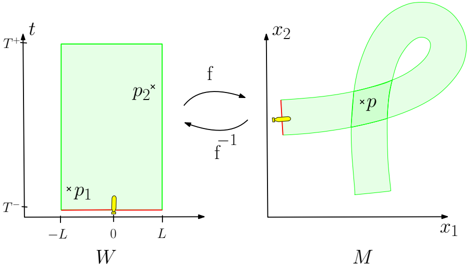

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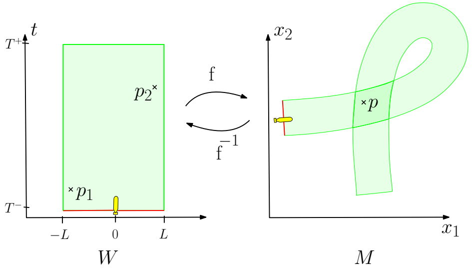

$$f_p = f - p$$

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From [Milnor 1965] [2], if f_p preserves orientation,

$$deg(f_p, W) = \sum_{i=1}^{k} sign(det\ Df_p(p_i)) = k$$

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[2] J. Milnor, *Topology from the Differentiable Viewpoint*. 1965

Problem Approach

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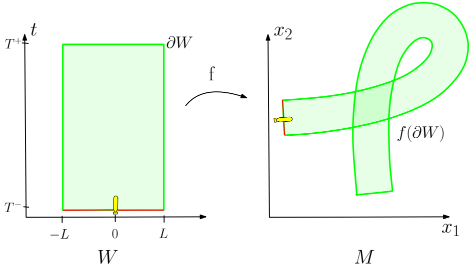

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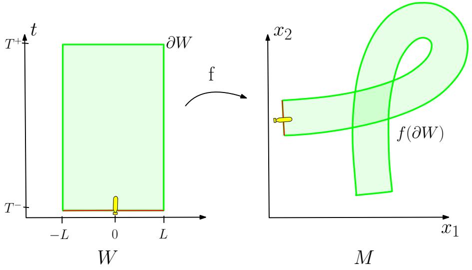

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$$deg(f_p, W) = \eta(f_p(\partial W), 0) = \eta(f(\partial W), p) = \mathbb{C}_{\mathbb{M}}(p)$$

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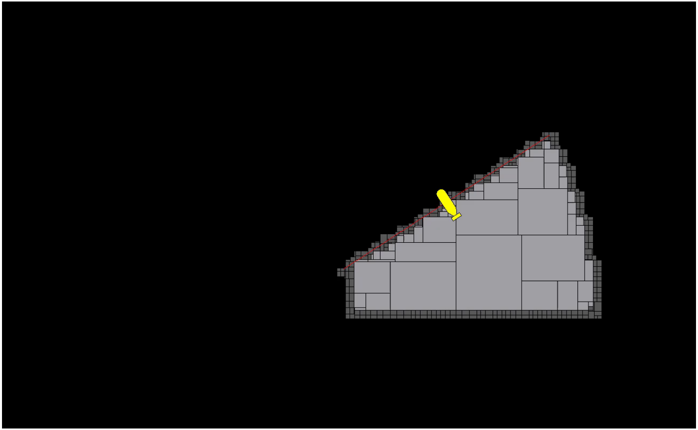
Problem Approach

What if f does not preserve orientation?



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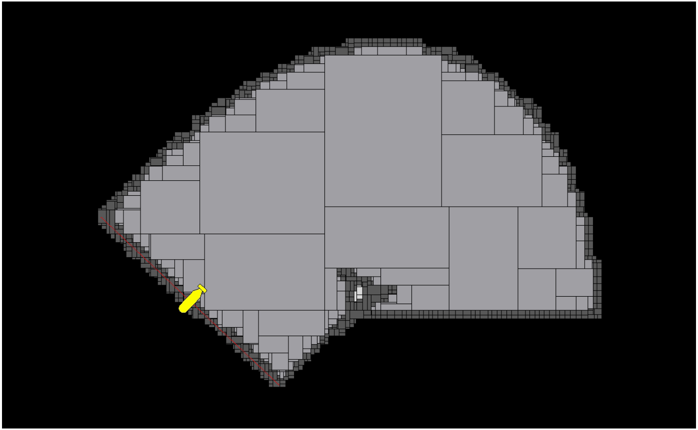

Problem Approach

Entries

- γ, the sonar's contour.

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Desired Output

- Explored area $\mathbb{A}_{\mathbb{E}}$,
- coverage measure of all the points in the plane, $\mathbb{C}_{\mathbb{M}}$.

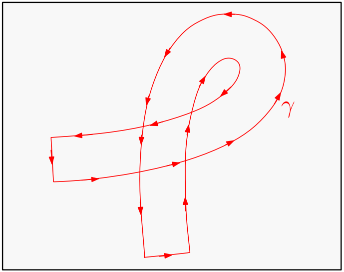
Proposed solution

$$\forall p \in \mathbb{R}^2, \text{ calculate } \eta(\gamma, p)$$

Computing the Winding Number

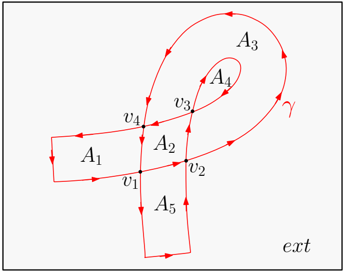
Hypothesis on γ:

1. continuous,
2. finite number of crossing points,
3. only passes through each crossing point twice.



Computing the Winding Number

- Construction of a CW-complex $C(\gamma)$ from the cycle γ



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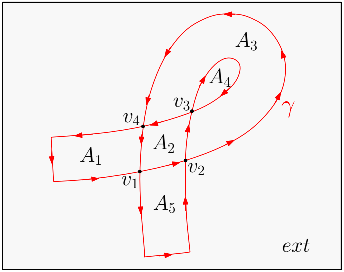

0-cell

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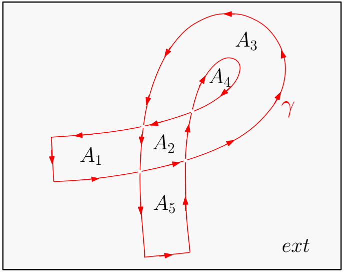

1-cell

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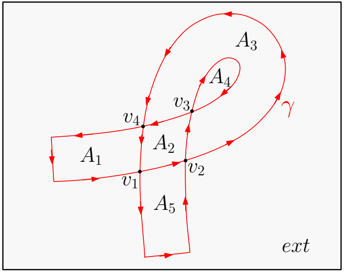
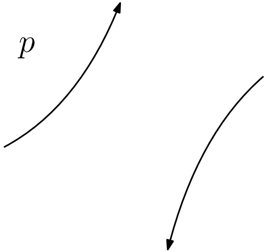


2-cell

Computing the Winding Number

Möbius's combinatorial rule for defining the winding number [1]



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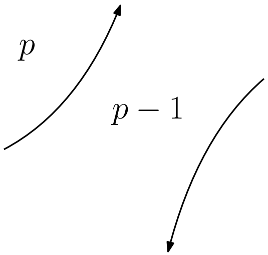

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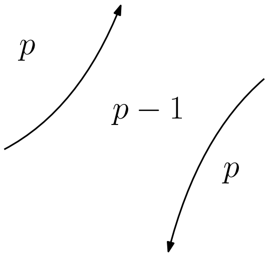

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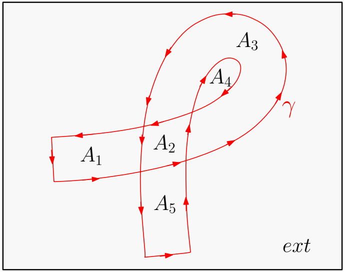

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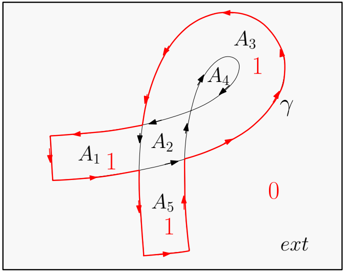

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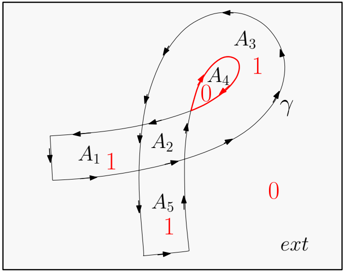

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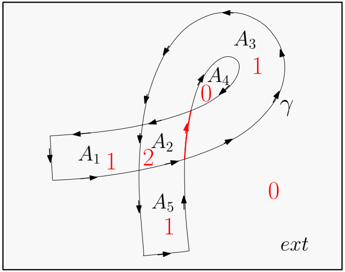

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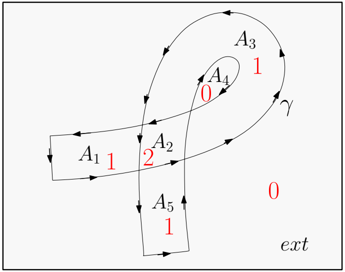

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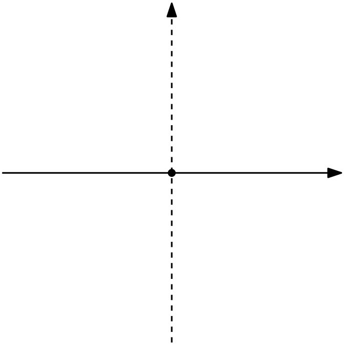

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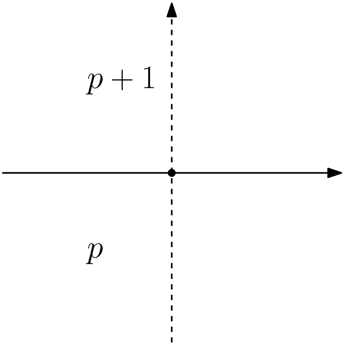

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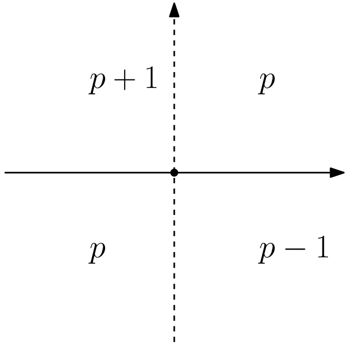

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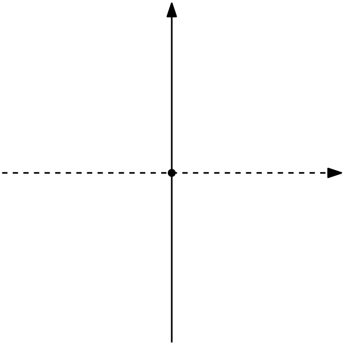

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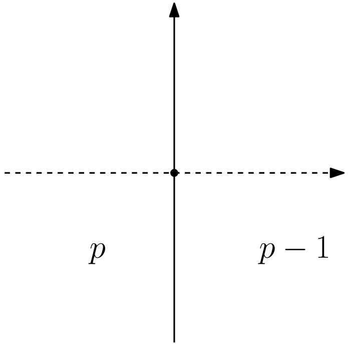

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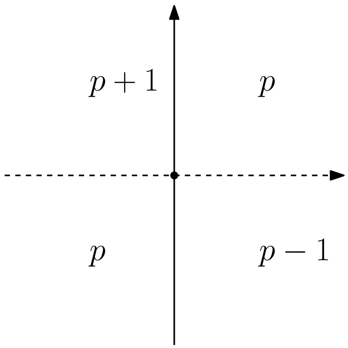

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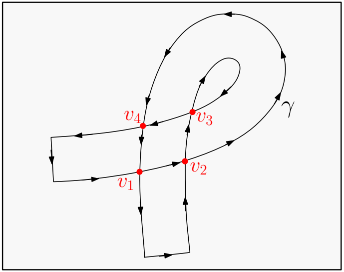
Computing the Winding Number

Algorithm:

1. Detection of self-intersections (0-cell) ,

$$V = \{v_1, v_2, v_3, v_4\}$$

2. Alexander numbering the regions (2-cell).



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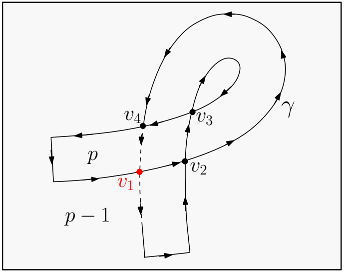

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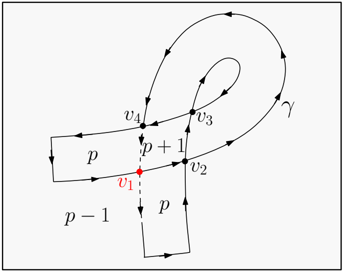

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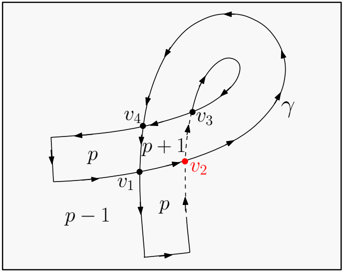

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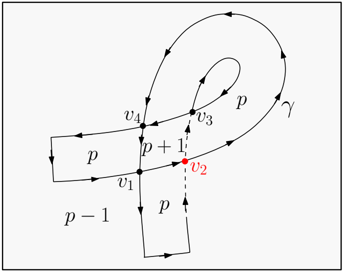

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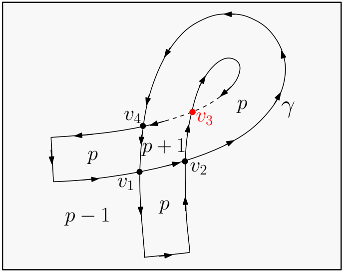

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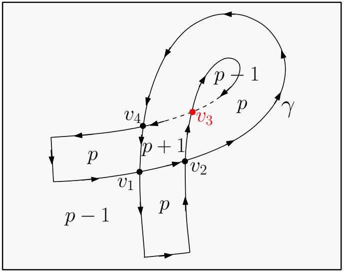

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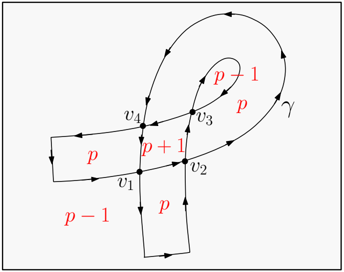

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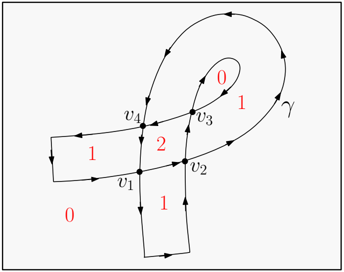
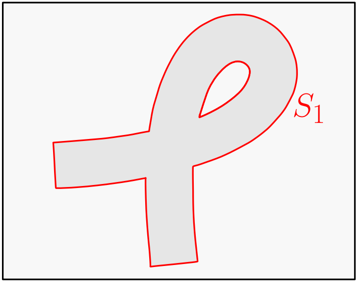
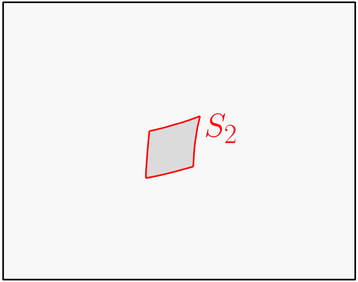


Computing the Winding Number

Let S_i be the closure of the union of the regions with a winding value greater or equal to i.

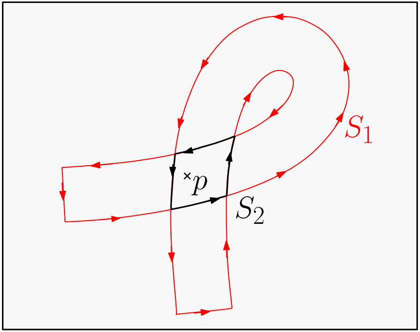




Computing the Winding Number

Theorem 1 in [Mcintyre 1993] [1] :

$$\eta(\gamma) = \sum_{i>0} \chi_{S_i} - \sum_{i<0} \chi_{S_i}$$



$$\eta(\gamma) = \chi_{S_1} + \chi_{S_2}$$

Where,

$$\chi_{S_i}(p) = \begin{cases} 0, & \text{if } p \notin S_i \\ 1, & \text{if } p \in S_i \end{cases}$$

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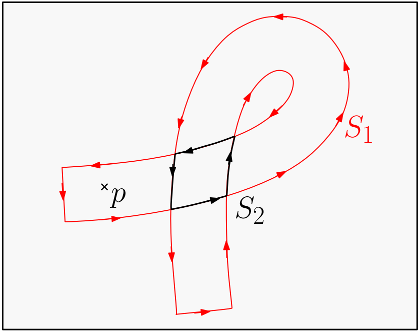

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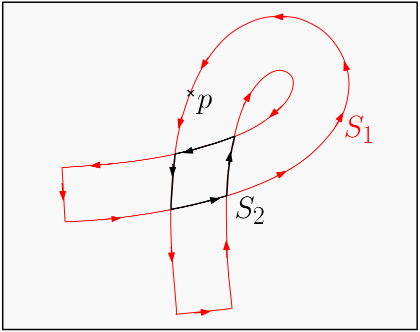

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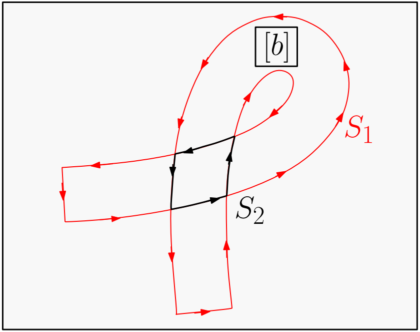

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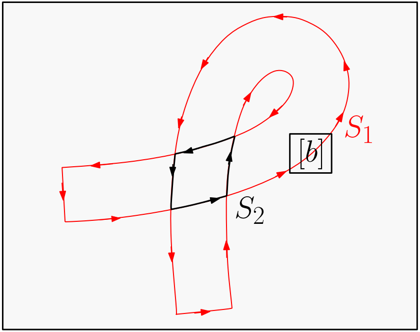

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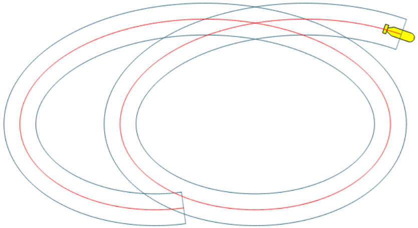

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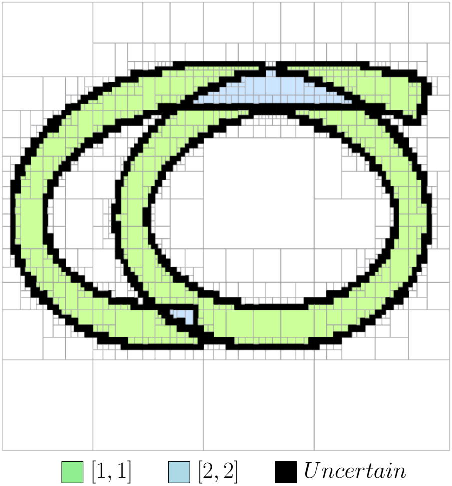

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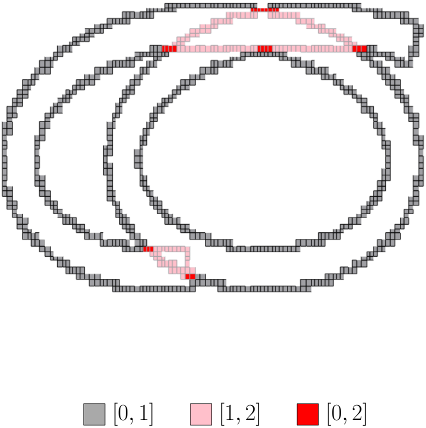

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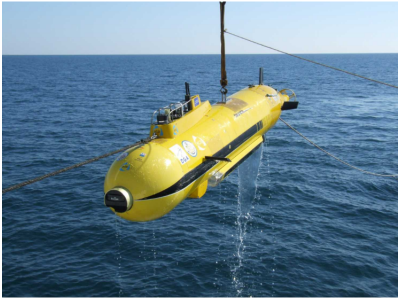


Data

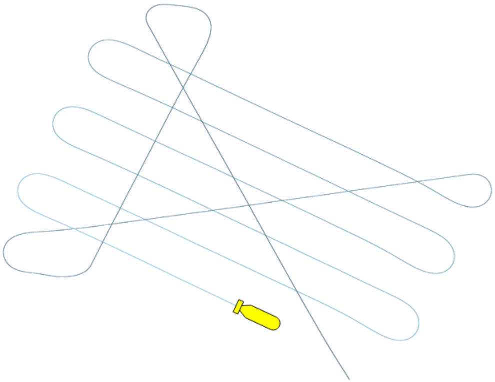
- DVL,
- IMU,
- Pressure.

Mission

- Classical survey path (law-mowing pattern),
- Roadstead of Brest (France, Brittany),
- 47 minutes.

Dataset and photo courtesy of DGA/GESMA.

Daurade



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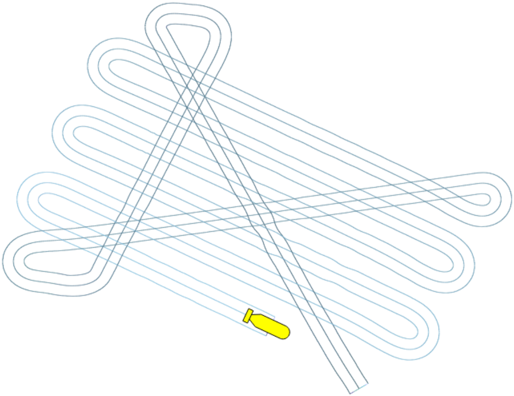

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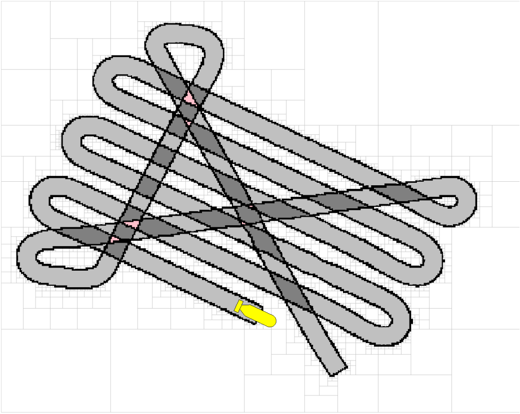

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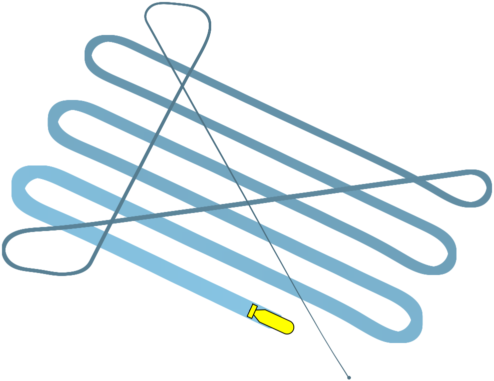

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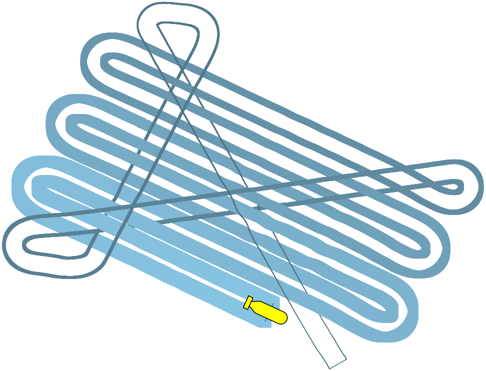

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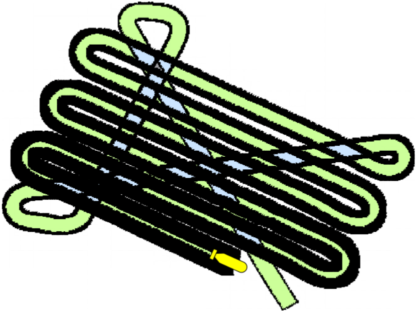

$[\eta](f(\partial W),.) = [1,1]$ $[\eta](f(\partial W),.)^- \neq [\eta](f(\partial W),.)^+$

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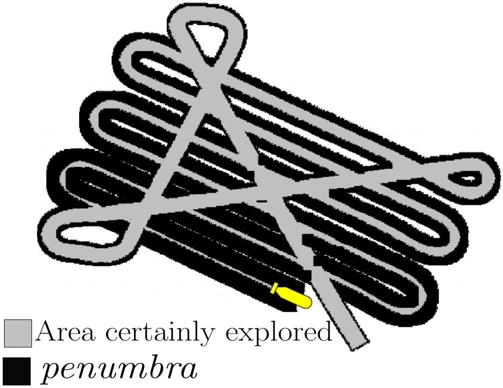

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Conclusions

Future Work:

- Relation between persistent homology and uncertain winding number values [Bhattacharya 2015] [1],
- coverage measures dealing with multiple robots [De Silva 2007] [2],
- uncertainty in the robot's trajectory using thick sets,
- contour reversing orientation,
- real-time computation,
- localization application using the exteroceptive data.

[1] S. Bhattacharya, R. Ghrist, and V. Kumar, "Persistent homology for path planning in uncertain environments,", 2015

[2] V. D. Silva and R. Ghrist, "Homological sensor networks,", 2007