Institut für Erdmessung

Research Training Group i.c.sens

Leibniz Universität Hannover

Intervals in Fault-free Error Modeling for GNSS Applications

- International Online Seminar on Interval Methods in Control Engineering -

Jingyao Su

Institut für Erdmessung

Gottfried Wilhelm Leibniz Universität Hannover

Overview

1. Background: GNSS positioning model
2. Interval for bounding observation uncertainty
3. Set representation for uncertainty in position domain
4. Conclusions and outlook

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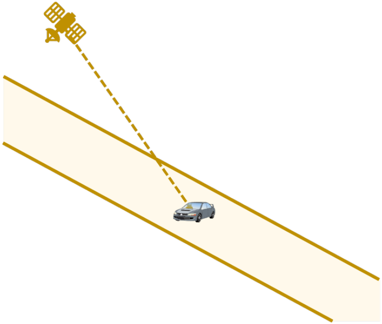
Background: GNSS range-based positioning

- Receiver receiving and tracking satellite signals
- Position is estimated using range measurements under certain satellite geometry
- Different error sources
- Techniques are developed to cancel, correct, or reduce the errors -> **remaining errors persist**
  - Imperfect correction models
  - Economical reasons
  - Special purposes

How the interval works:

Interval to represent uncertainty (bounded error) -> linear propagation -> bounding state error



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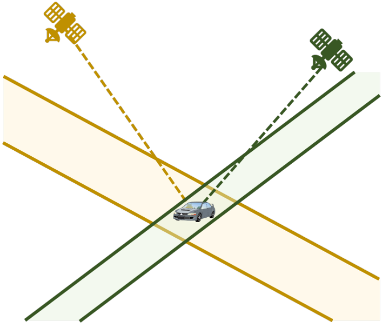

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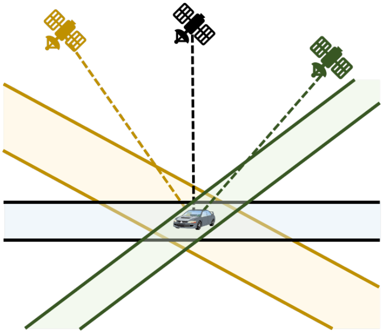

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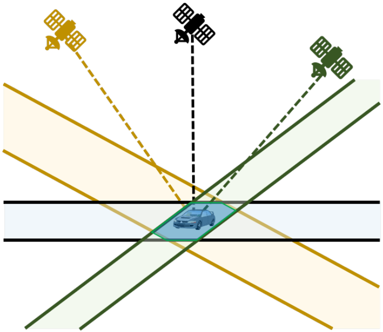

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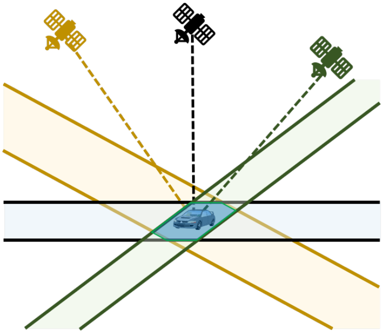

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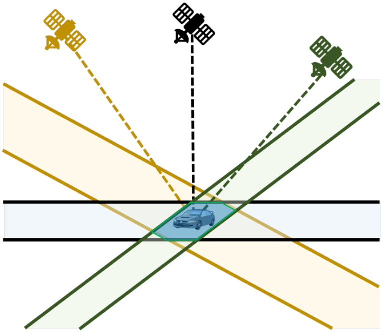
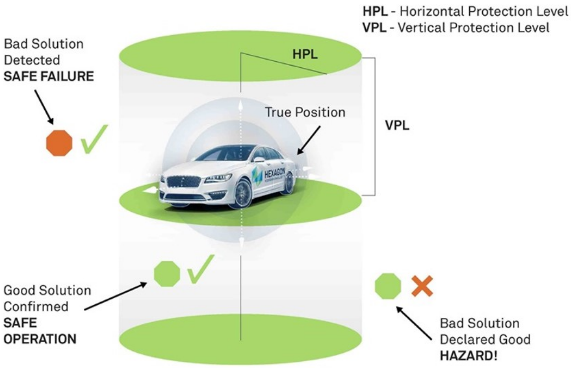


Critical question:

How to assess the uncertainty for remaining systematic errors as intervals?

Background: Error bounding for GNSS integrity monitoring



Source: hexagonpositioning.com/autonomous-x/our-approach/integrity

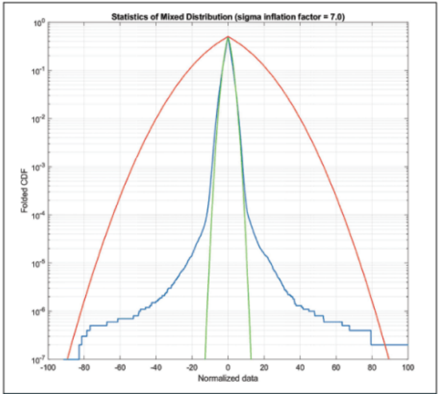


FIGURE 5 Tail Bounding of R with Sigma Inflation of 7.0.

InsideGNSS (2020). Why is bounding GNSS errors under rare or anomalous conditions important, and what makes it difficult?

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GNSS error sources and uncertainty budget

Satellite specific effects:

orbit errors, satellite clock error

Signal propagation specific effects:

- **ionospheric error:** forming differences, correction models
- **tropospheric error:** correction models
- **multipath effect:** code tracking error, difficult to model in a probabilistic manner
- **Non-line-of-sight (NLOS):** extra path delay due to indirect signal path

Receiver specific effects:

receiver clock error, hardware delays, antenna effects

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Method of sensitivity analysis for remaining systematic errors

Concept: A forward modeling

GNSS measurements → corrected by empirical models →

empirical correction models are built based on:
- assumptions deviating from reality
- approximations with uncertainty

↓

imprecisely known and associated with a range:
- constants
- parameters

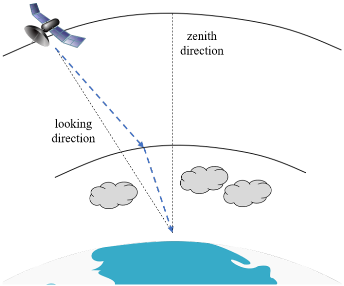
Influence factors:

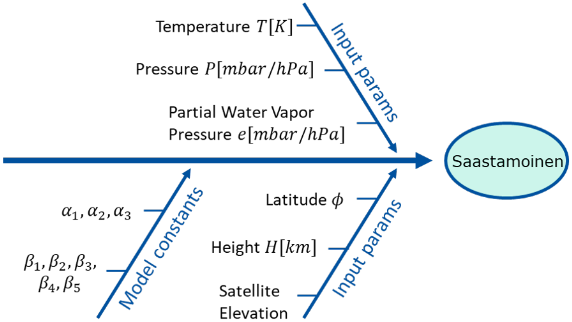
the overall uncertainty has contributions from all influence factors

Example of remaining tropospheric errors

Tropospheric delay and its correction model

- Signal delays caused by the neutral atmosphere
- The Saastamoinen correction model





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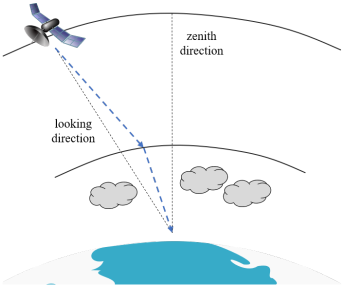

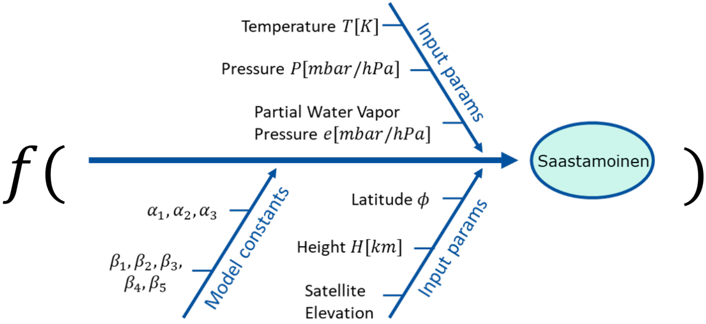

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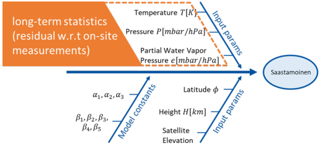

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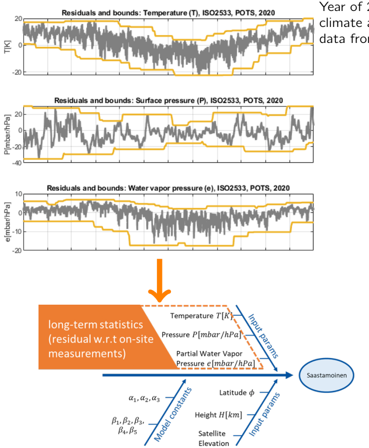

Year of 2022,
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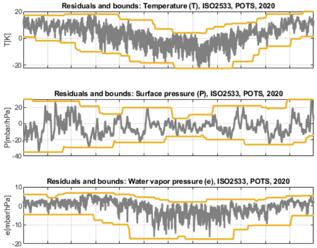

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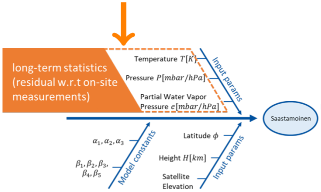

Sensitivity Analysis for
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$[ZPD]$
remaining
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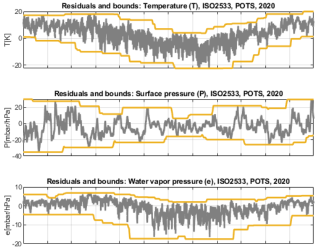

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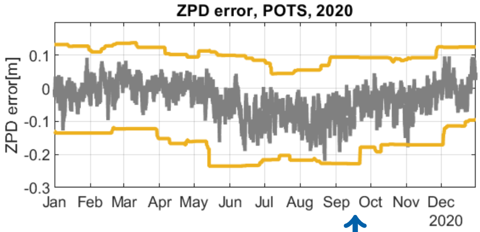

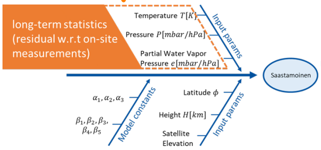

Sensitivity Analysis for
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Method of sensitivity analysis for remaining systematic errors

Influence factors:

- model constants
- model parameters
- auxiliary information

Sources of influence factors:

- construction process of the model
- expert knowledge or experience
- manufacturer‘s specification
- model‘s accuracy evaluations
- uncertainties assigned to reference data taken from handbooks

Implementation:

- Sensitivity w.r.t influence factors s by partial differentiation:

$$dL_i^k = \frac{\partial L_i^k}{\partial s} ds = \frac{\partial \rho_i^k}{\partial s} ds + \frac{\partial T_i^k}{\partial s} ds + \frac{\partial I_i^k}{\partial s} ds + \dots$$
$$= F ds$$
$$L_i^k = |F| \cdot s_r$$

- implement via interval arithmetic:

$$[f] \triangleq [\underline{\Delta}, \overline{\Delta}] = \sum_i^m [f_i] + f(x^*)$$
$$[f_i] = f([x_i]|x^*) - f(x^*)$$

with $f([x_i]|x^*)$ the resulting variation due to $[x_i]$ around $f(x^*)$

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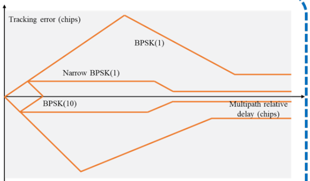
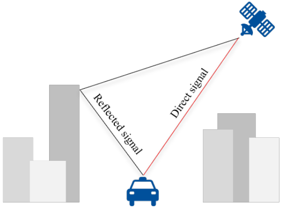
Envelope bounding models for GNSS multipath error

- Intervals sufficiently bound the multipath effect
- Upper and lower bounds derived from the multipath error envelope models
- Tracking errors oscillate between the two curves due to changes in the phase



GPS multipath error envelopes



[1] Icking et al (2022). Multipath characterization using ray-tracing in urban trenches

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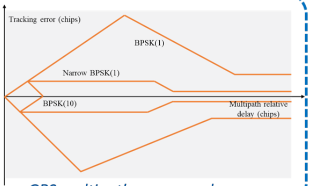
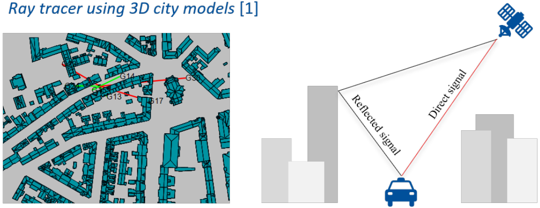


GPS multipath error envelopes

Ray tracer using 3D city models [1]



Envelope model

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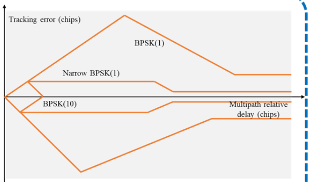

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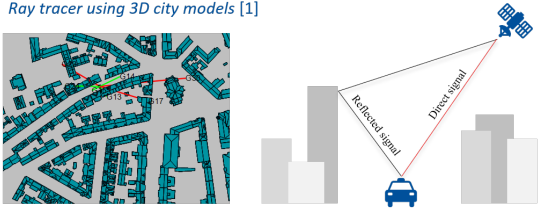

Envelope model

- δ (extra path delay from ray tracer)
- $\Gamma, \Gamma_{min}, \Gamma_{max}$ (autocorrelation sidelobe levels)
- d (correlator spacing)
- $\alpha = f(\theta, G, R)$ (M/D ratio, incidence angle, antenna gains, reflection coefficients)

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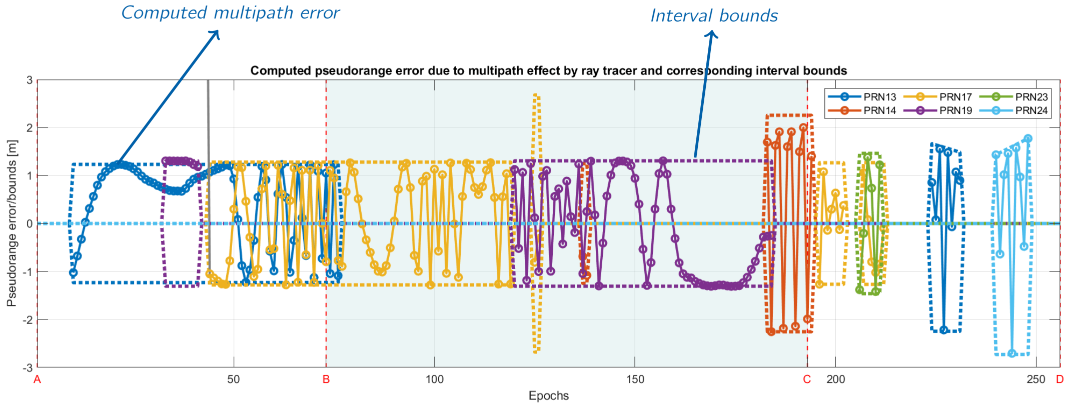
Example result for multipath bounding



Su & Schön, Advances in deterministic approaches for bounding uncertainty and integrity monitoring of autonomous navigation, ION GNSS+ 2022

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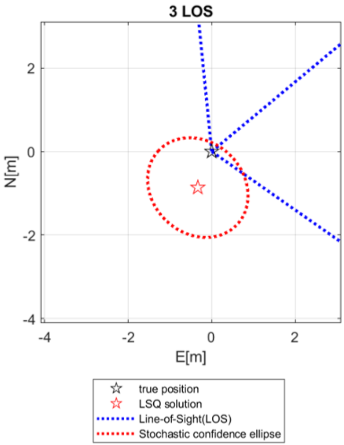
Error modeling via interval extension of the least-squares estimator

Classical least-squares estimator:

- Solving the linearized GNSS observation equation: $y = A\Delta x + e$, with y being the observation residual, Δx the state of interest (difference to an initial point x_0)
- the least-squares estimator: $\Delta\hat{x} = (A^\top PA)^{-1}A^\top Py$
- the final estimate: $\hat{x} = x_0 + \Delta\hat{x}$



- The assumption of normal distribution is violated -> Remaining systematics exist
- The confidence interval / confidence ellipse does not reflect the realistic uncertainty.

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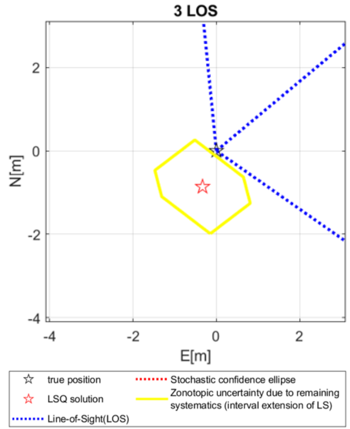
- the least-squares estimator: $\Delta\hat{x} = (A^T PA)^{-1} A^T Py$
- the final estimate: $\hat{x} = x_0 + \Delta\hat{x}$

- Intervals representing remaining systematic uncertainty [s]
- Interval extension of least-squares estimator [1]:

$$\mathcal{X} \triangleq \{\Delta\hat{x}_E \in \mathbb{R}^n | \Delta\hat{x}_E = (A^T PA)^{-1} A^T P(y - [s])\}$$
$$= \{\Delta\hat{x}_E \in \mathbb{R}^n | \Delta\hat{x}_E = Ky + K\Delta_s \cdot [-1, 1]\}$$

with $K = (A^T PA)^{-1} A^T P$

- Zonotope $\mathcal{Z}(\Delta\hat{x}, K diag(s))$ with center being the classical LS estimator $\Delta\hat{x}$ and generators $K diag(s)$



[1] Kutterer, H. (1994).Intervallmathematische Behandlung endlicher Unschärfen linearer Ausgleichungsmodelle. Beck.

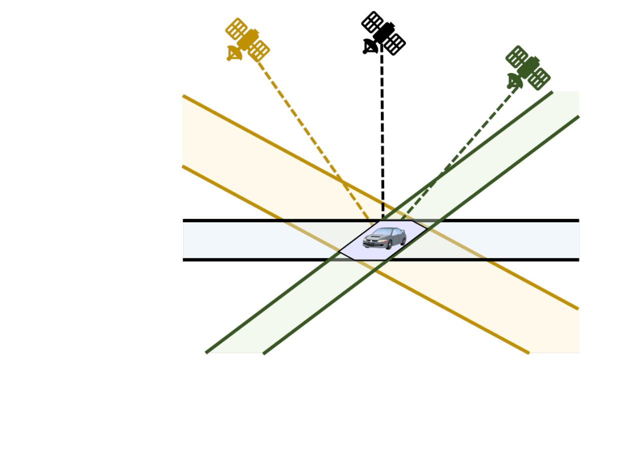
Satellite positioning as constraints satisfaction

Positioning as constraint satisfaction:

GNSS positioning problem is reformulated as $y = A\Delta x + [e]$

$$\begin{cases} A\Delta x \leq y - \underline{e} \\ -A\Delta x \leq -y + \overline{e} \end{cases}$$

Polytope $P_{\Delta x}$ as the set solution



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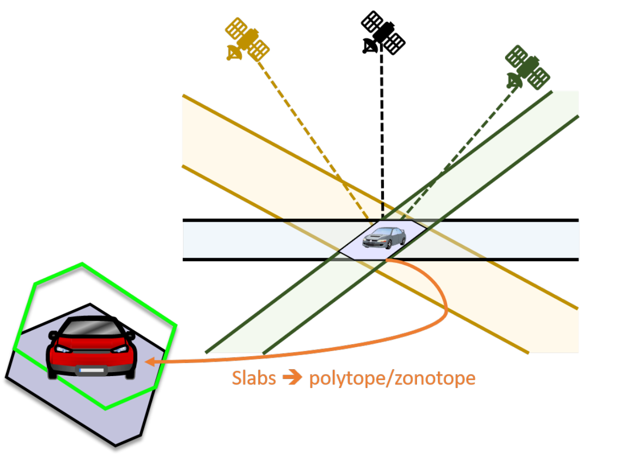
Polytope $P_{\Delta x}$ as the set solution

- Polytopic set solution as feasible solution set? inconsistency area?
- Ideal situation (noise-free) -> a zonotope (conditioning symmetric intervals)
- Zonotopic nominal solution as geometrical confidence.

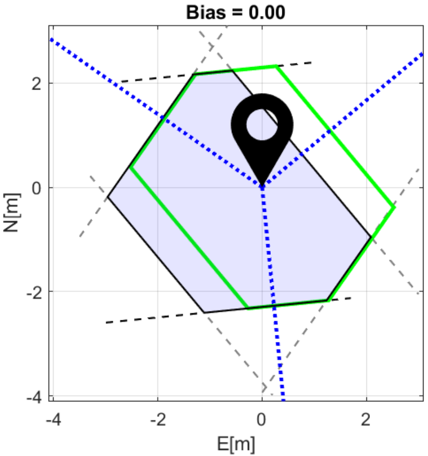


Fault detection via constraints satisfaction

Indicator of fault:

- Existence of faults result in inconsistency of observations
- Inconsistency of intervals (slabs) leads to no intersection (empty set):

$$\bigcup (y_i - [e_i]) = \emptyset$$



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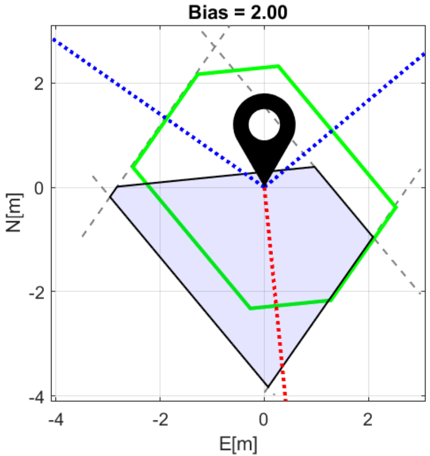

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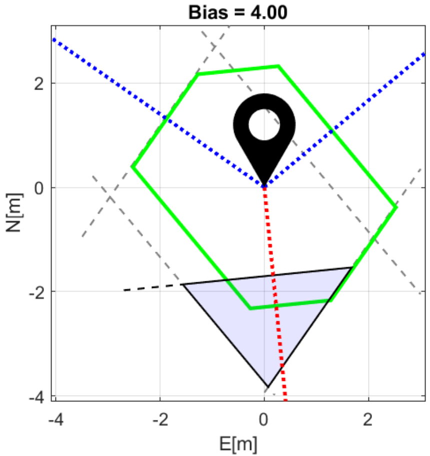

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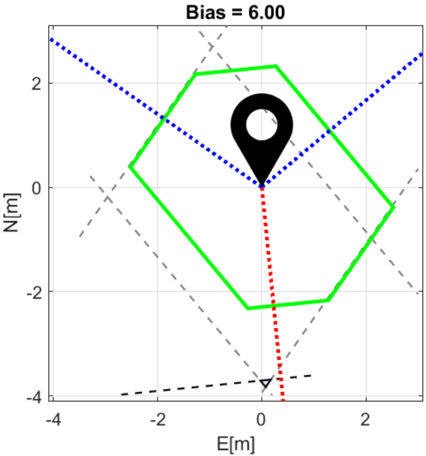

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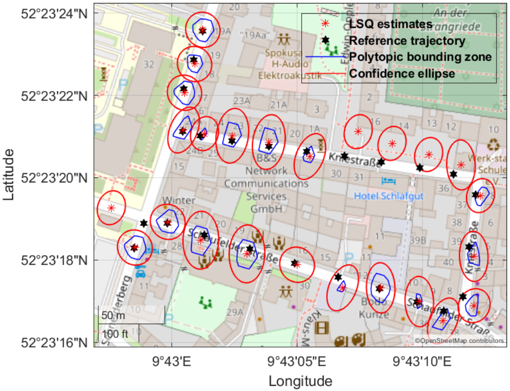
Examples in real world



Schön et al (2022). Towards Integrity for GNSS-based urban navigation-challenges and lessons learned

Open questions: sensitivity of detection

How to define the sensitivity of detection?

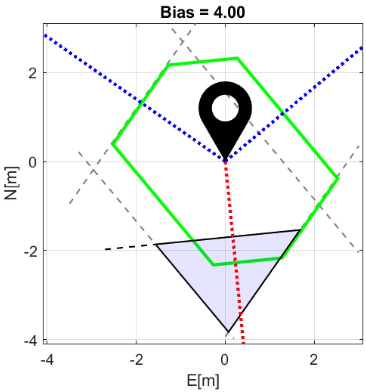
- Detection is established (empty set achieved) only when the bias is sufficiently large
- Minimal detectable bias indicates the threshold of achieving empty set

How to evaluate the sensitivity of detection?

- Equivalent to: how to **determine** Minimal detectable bias?
- Achievement of empty set is highly dependent on geometry
- Achievement of empty set is associated with individual signals

How to enhance the sensitivity of detection?

- Equivalent to: how to **reduce** Minimal detectable bias?
- By adjusting interval size? polytope reshaped
- Modeling based on physical feature? weighting model established



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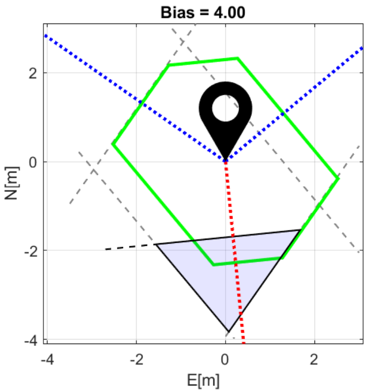

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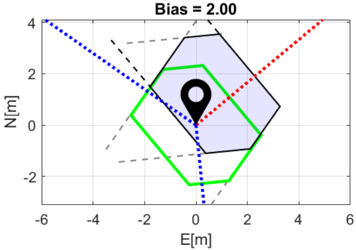

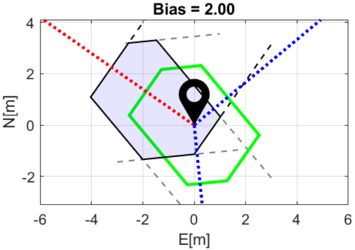

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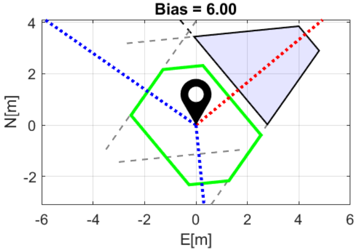
- Minimal detectable bias indicates the threshold of achieving empty set

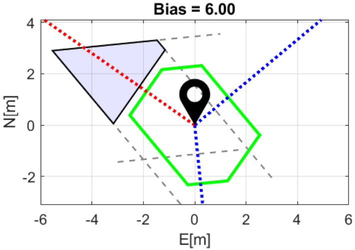
How to evaluate the sensitivity of detection?

- Equivalent to: how to **determine** Minimal detectable bias?
- Achievement of empty set is highly dependent on geometry
- Achievement of empty set is associated with individual signals

How to enhance the sensitivity of detection?

- Equivalent to: how to **reduce** Minimal detectable bias?
- By adjusting interval size? polytope reshaped
- Modeling based on physical feature? weighting model established





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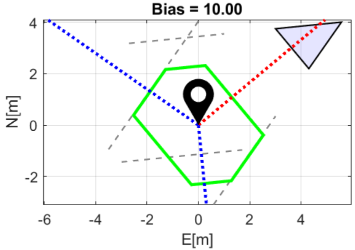

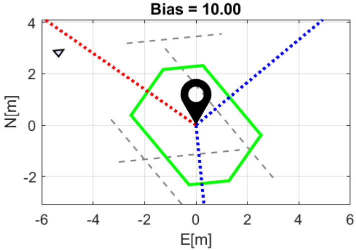

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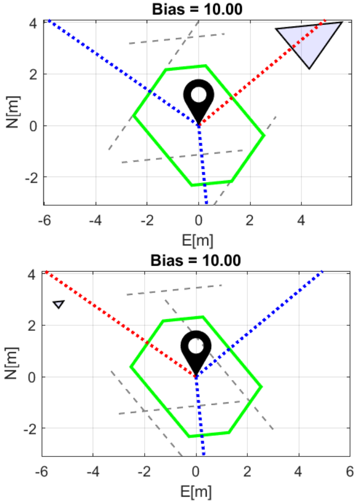

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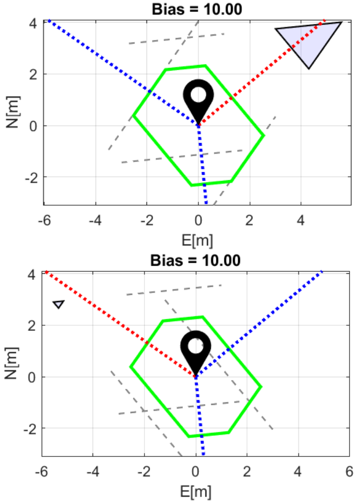


Examples in real world: Enhancing sensitivity of detection

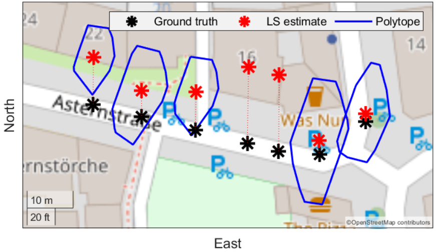
Examples of weighting models

- IDEN (identical weighting)
- EDM (Elevation-dependent model)
- SDM (Signal-strength-dependent model)

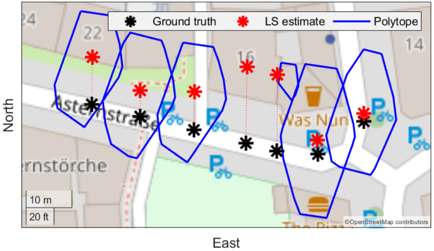
To observe: *Can we detect all potential faults?*

Su & Schön, Advances in deterministic approaches for bounding uncertainty and integrity monitoring of autonomous navigation, ION GNSS+ 2022

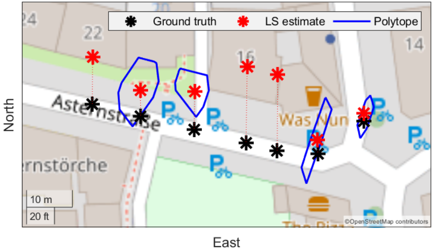
EDM:



IDEN:



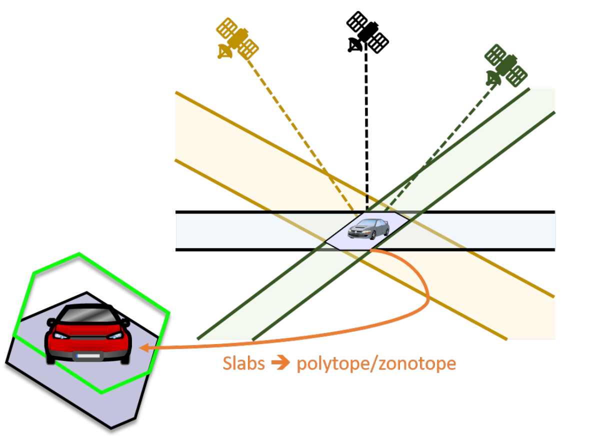
SDM:



Open questions: feasible solution set

feasible solution set

- A set consisting of all the feasible solutions of the system of linear inequalities.
- Obvious with fault-free assumptions.



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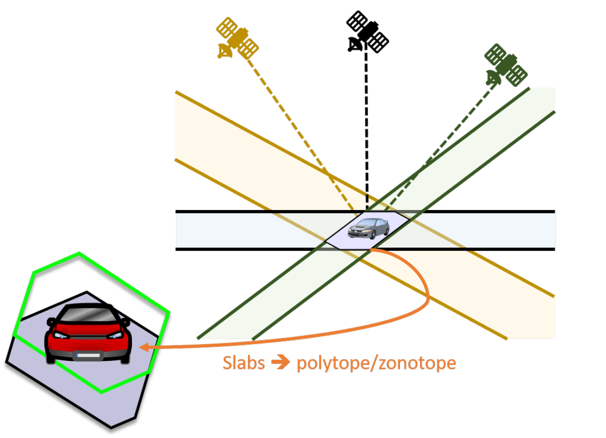

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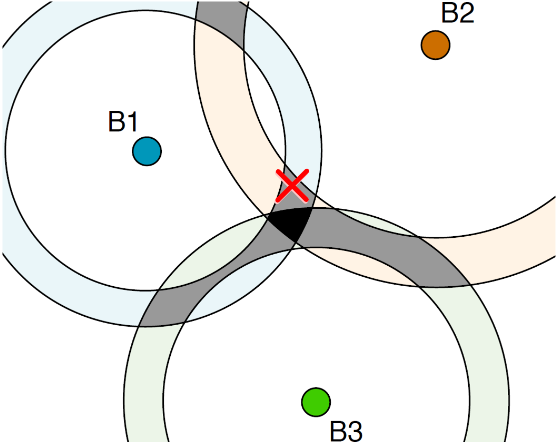

Drevelle & Bonnifait (2009). High integrity GNSS location zone characterization using interval analysis.

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- Impact of Minimal Detectable Bias?

Drevelle & Bonnifait (2009). High integrity GNSS location zone characterization using interval analysis.

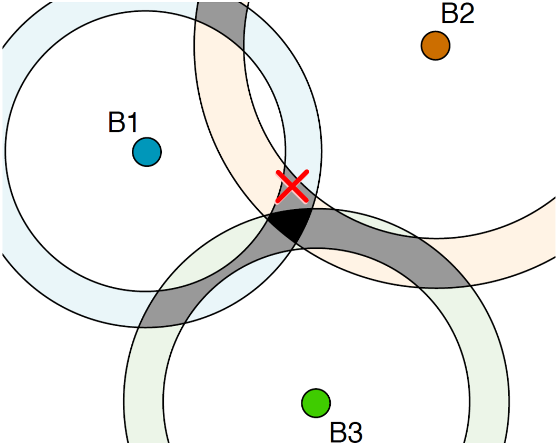

Table of Contents

Conclusions and outlook

Conclusions

- Interval used to adequately bound the remaining systematic uncertainty
  - Sensitivity analysis: remaining tropospheric error
  - Envelope bounding: GNSS multipath effect
- Set representations for GNSS integrity applications
  - Zonotope by interval extension of the least squares estimator for representing uncertainty in the position domain
  - Polytope by constraint satisfaction for fault detection

Future work & Questions to be answered

- Definition and determination of the minimal detectable bias
- Enhancing detection capability of set-based detector
- Possibility of dynamic state estimation

Related publications

- Su, J., & Schön, S. (2021, September). Improved Observation Interval Bounding for Multi-GNSS Integrity Monitoring in Urban Navigation. In *Proceedings of the 34th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2021)* (pp. 4141-4156). doi: 10.33012/2021.18078
- Su, J., & Schön, S. (2022, April). Deterministic approaches for bounding GNSS uncertainty: A comparative analysis. In *2022 10th Workshop on Satellite Navigation Technology (NAVITEC)* (pp. 1-8). IEEE. doi: 10.1109/NAVITEC53682.2022.9847545
- Su, J., Jiang, Y., Schön, S. & Wagner, B. (2022, July). How to determine uncertainty interval: Practice in GNSS and LiDAR localization. In *Summer Workshop on Interval Methods (SWIM) 2022*.
- Su, J., & Schön, S. (2022, September). Advances in Deterministic Approaches for Bounding Uncertainty and Integrity Monitoring of Autonomous Navigation. In *Proceedings of the 35th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2022)* (pp. 1442-1454). doi: 10.33012/2022.18418

Thank you very much for your attention!
Questions?

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