INSTITUTE FOR RISK AND RELIABILITY

Leibniz Universität Hannover

Distribution-Free Bayesian Updating with Hybrid Uncertainties

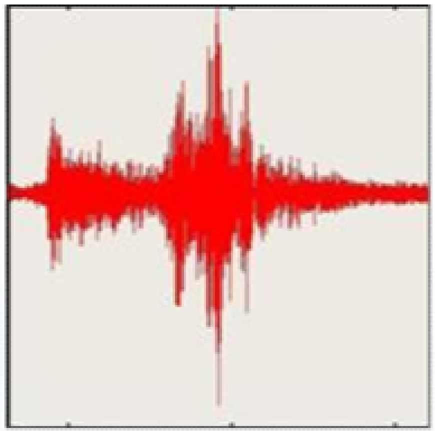
Matteo Broggi

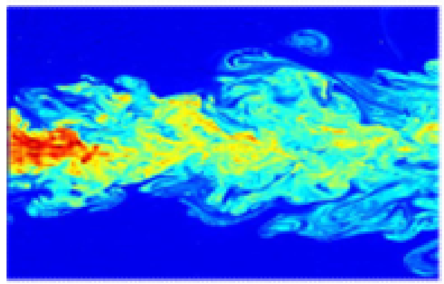
International Online Seminar on Interval Methods in Control Engineering

Spectrum of uncertainties

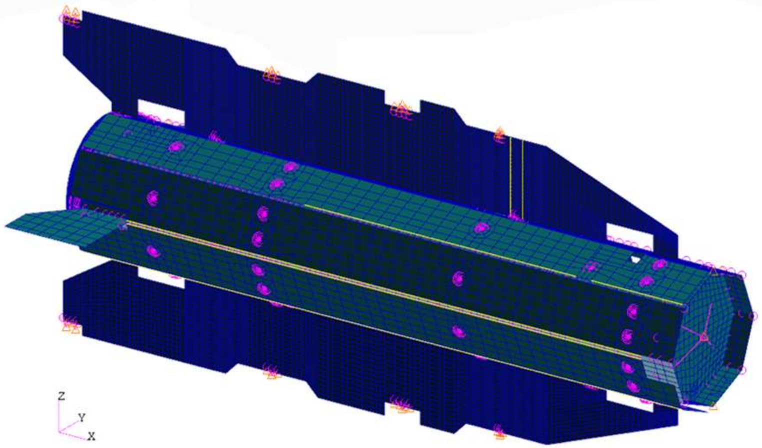
Physical

"Unavoidable" / Aleatory Uncertainties





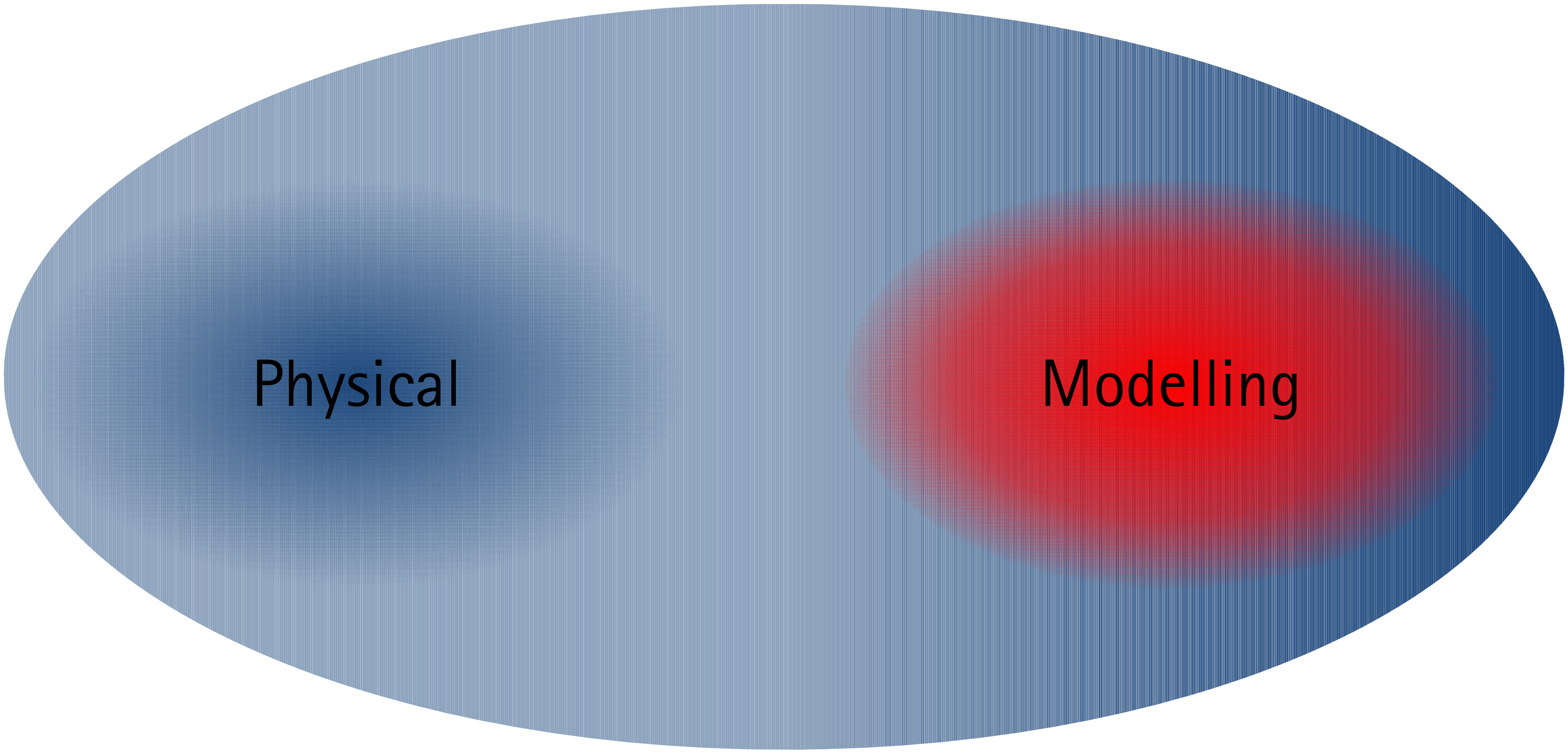
Spectrum of uncertainties



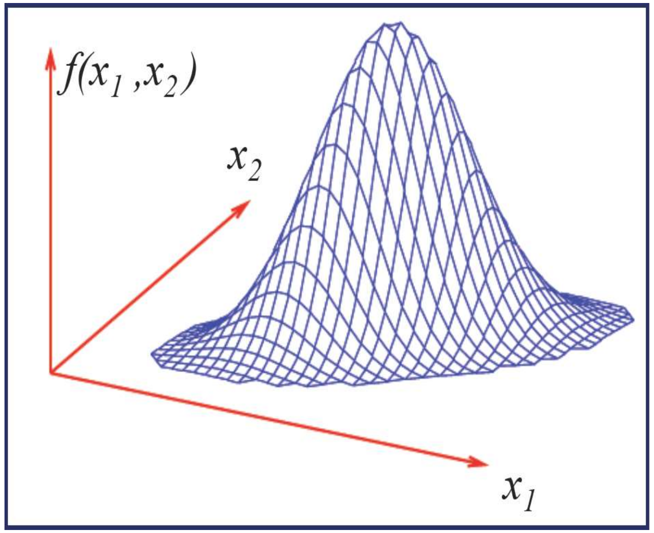
Modelling

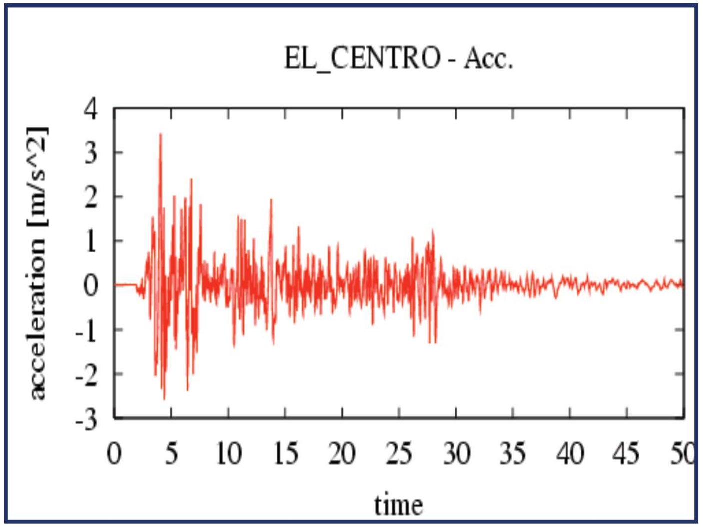
"Lack-of-knowledge" / Epistemic Uncertainties

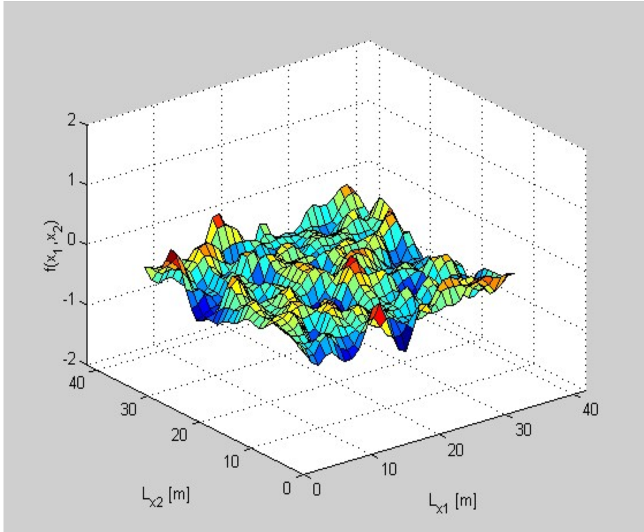
Spectrum of uncertainties



Aleatory uncertainties



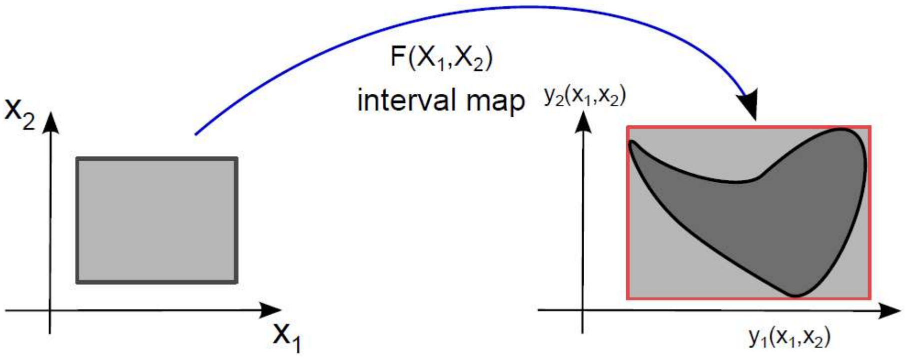


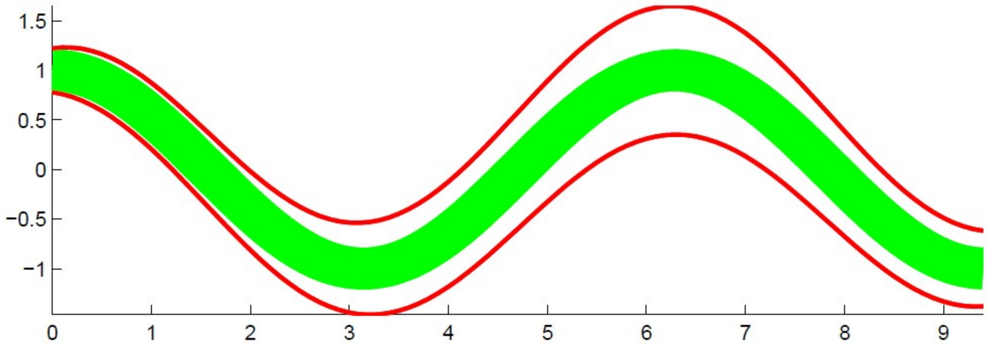


- Random variables
  - Random parameters, e.g., spring stiffness, dimension, static load
- Stochastic processes
  - Random functions of time e.g. Excitation time history, earthquake, dynamic load
- Random fields
  - Spatially fluctuating properties e.g. Young's modulus, Shell thickness

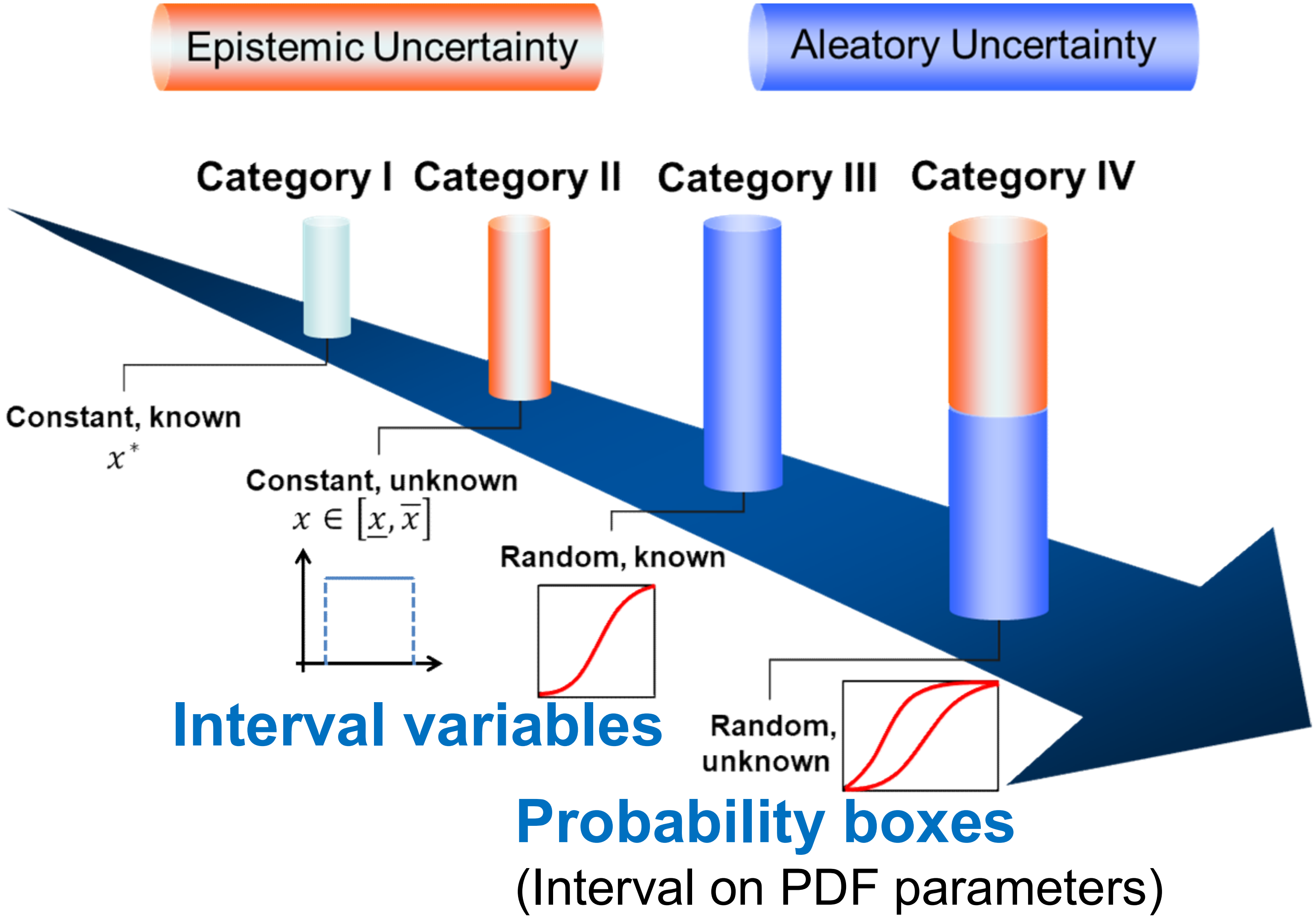
Epistemic uncertainties

- Statistical information often not available
  - e.g. unique structure
- Lack of knowledge
  - e.g. few or missing data
- Qualitative information
  - e.g. expert judgements





Parameter Categories

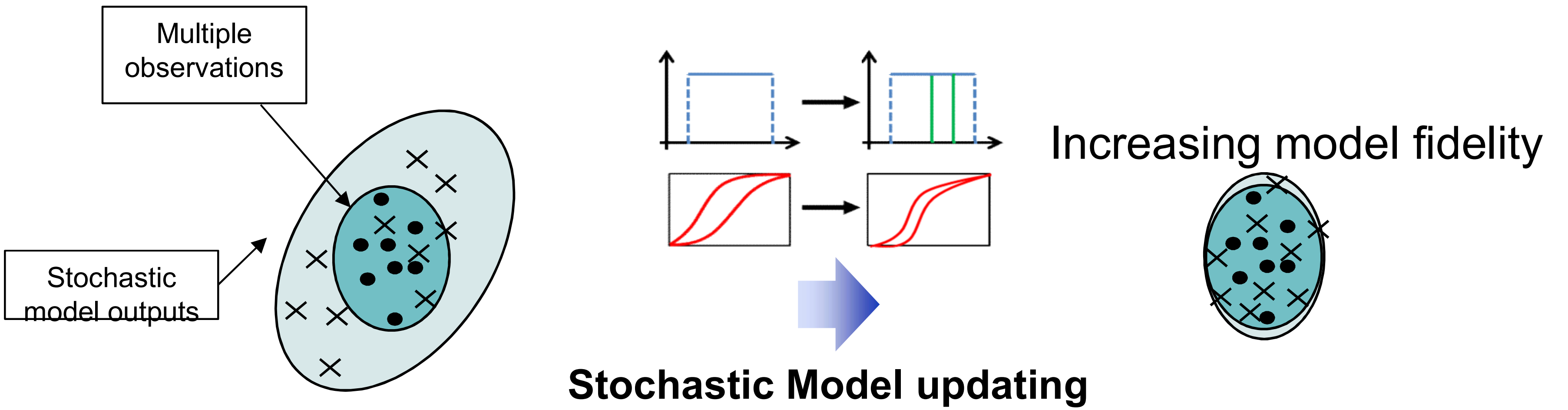


Outline

- Introduction
- Bayesian Updating
  - Likelihood with hybrid uncertainties
- Distribution-free model updating
- Numerical Examples
  - DLR AIRMOD
  - 2 DOF shear building
  - NASA Challenge Problem

Stochastic Model Updating

- Aleatory uncertainty makes the model output stochastic
  - Hypothesis: the scatter shown in experiment is purely aleatory
- Epistemic uncertainty of the model causes discrepancy between model outputs and observations
- How to increase the fidelity of the model?
  - Reduce epistemic uncertainty as much as possible based on the knowledge gained from observations



Bayesian Model Updating
Motivation

- Epistemic uncertainty as lack of knowledge in modelling physical quantities
  - Uncertain inputs
  - Approximate models
  - Limited information
- Real data (i.e., experiment) of the physical process available
  - "Indirect measure" of the uncertain parameters (i.e., compare experiments and model outputs)
- How to increase the fidelity of the model?
- How to include the knowledge gained from the experiment in the inputs and model?

Bayesian Model Updating

The goal is to update some prior information about the adjustable parameters Θ using the data D, obtaining a distribution for the optimal parameters:

Bayes' Theorem:

likelihood function prior distribution

posterior PDF

$$p(\theta \mid \mathcal{D}, \mathcal{M}) = \frac{p(\mathcal{D} \mid \theta, \mathcal{M}) p(\theta \mid \mathcal{M})}{p(\mathcal{D} \mid \mathcal{M})}$$

evidence (normalizing constant)

$\theta \ldots$ adjustable parameters

$\mathcal{D} \ldots$ experimental data

$\mathcal{M} \ldots$ model class

Prior PDF:

Expresses the initial knowledge about the ranges of the unknown epistemic parameters, e.g.:

- uniform distribution (e.g., interval) if only some engineering limits are known
- Gaussian distribution in case information about the mean value with uncertainty is available
- Other functions: triangular, trapezoidal, etc.

Bayesian Model Updating

The goal is to update some prior information about the adjustable parameters Θ using the data D, obtaining a distribution for the optimal parameters:

Bayes' Theorem:

likelihood function

$$p(\theta \mid \mathcal{D}, \mathcal{M}) = \frac{p(\mathcal{D} \mid \theta, \mathcal{M})\, p(\theta \mid \mathcal{M})}{p(\mathcal{D} \mid \mathcal{M})}$$

$\theta \ldots$ adjustable parameters

$\mathcal{D} \ldots$ experimental data

$\mathcal{M} \ldots$ model class

Likelihood function (marginal distribution):

- Incorporates the information delivered by the data and updates the prior PDF.
- Needs to be adapted to deal with epistemic vs. hybrid uncertainties
- More details later in this talk

Bayesian Model Updating

The goal is to update some prior information about the adjustable parameters Θ using the data D, obtaining a distribution for the optimal parameters:

Bayes' Theorem:

posterior PDF

$$\boxed{p(\theta \mid \mathcal{D}, \mathcal{M})} = \frac{p(\mathcal{D} \mid \theta, \mathcal{M})\, p(\theta \mid \mathcal{M})}{p(\mathcal{D} \mid \mathcal{M})}$$

$\theta \ldots$ adjustable parameters

$\mathcal{D} \ldots$ experimental data

$\mathcal{M} \ldots$ model class

Posterior PDF:

- Combines prior knowledge and data in order to obtain the posterior PDF of the adjustable parameters.
- It provides the engineer with the information which parameter ranges are more probable than others.
- Generally impossible to determine closed form solution

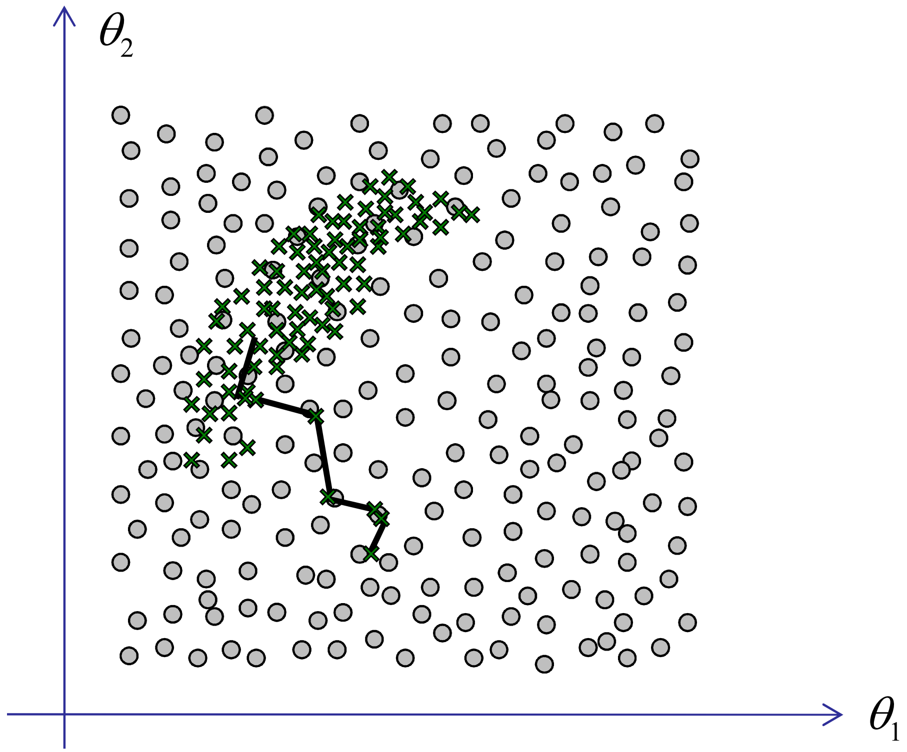
Generation of samples of posterior PDF

$$p(\theta \mid \mathcal{D}, \mathcal{M}) = \frac{p(\mathcal{D} \mid \theta, \mathcal{M})\, p(\theta \mid \mathcal{M})}{p(\mathcal{D} \mid \mathcal{M})}$$ ⇒ How can samples of the posterior PDF be generated?

Markov Chains can be used for the generation of samples of a complex distribution.

Possible shapes of posterior PDF:

1. Flat, widespread PDF:



- ○ Samples from prior distribution
- × Samples from posterior distribution

Generation of samples of posterior PDF

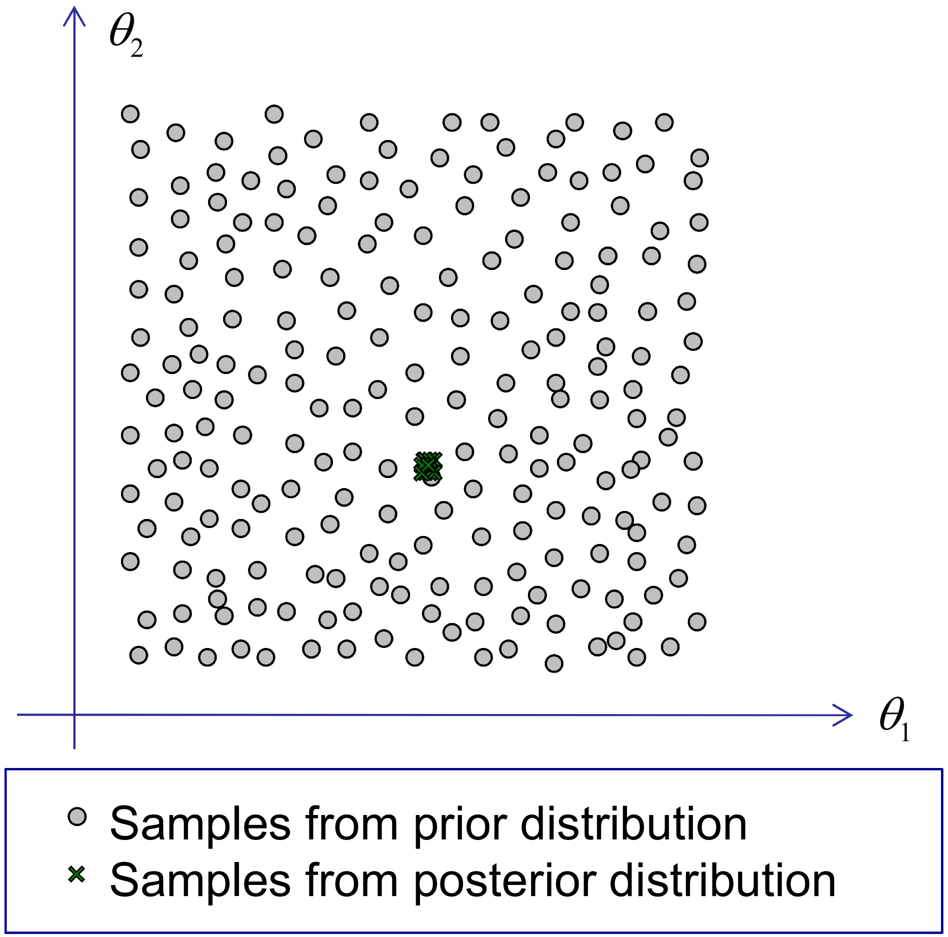
$$p(\theta \mid \mathcal{D},\mathcal{M}) = \frac{p(\mathcal{D}\mid\theta,\mathcal{M})\,p(\theta\mid\mathcal{M})}{p(\mathcal{D}\mid\mathcal{M})}$$

⇨ How can samples of the posterior PDF be generated?

Markov Chains can be used for the generation of samples of a complex distribution.

Possible shapes of posterior PDF:

1. Flat, widespread PDF:
2. Peaked PDF, concentrated over a small parameter range:
   Problems: starting point might be far away (inefficient), region with high probability mass might not be identified



Generation of samples of posterior PDF

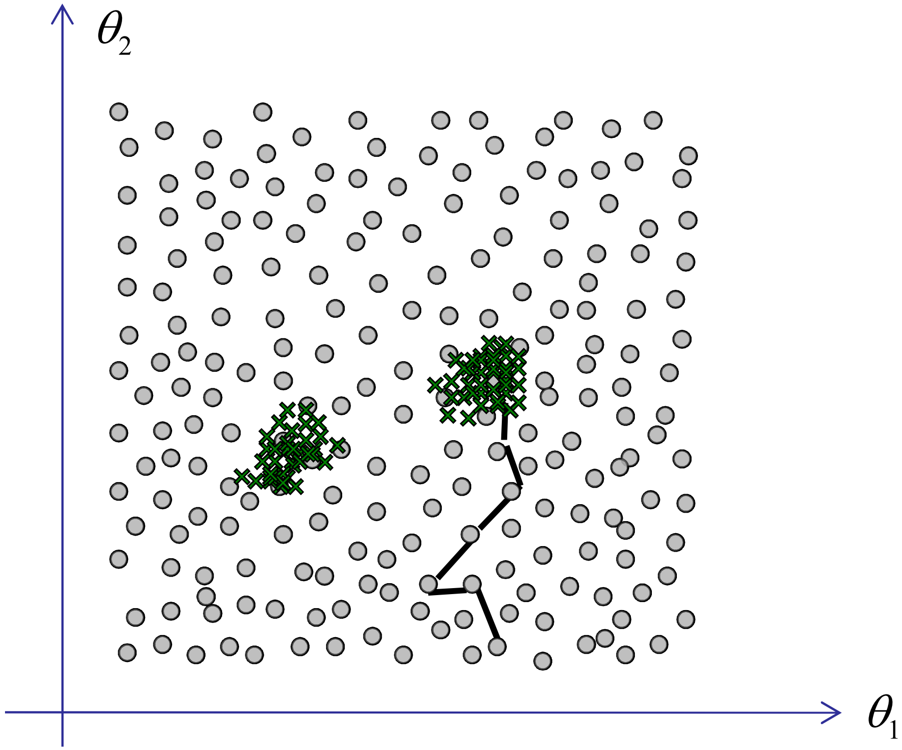
$$p(\theta \mid \mathcal{D}, \mathcal{M}) = \frac{p(\mathcal{D} \mid \theta, \mathcal{M})\, p(\theta \mid \mathcal{M})}{p(\mathcal{D} \mid \mathcal{M})}$$

⇒ How can samples of the posterior PDF be generated?

Markov Chains can be used for the generation of samples of a complex distribution.

Possible shapes of posterior PDF:

1. Flat, widespread PDF:
2. Peaked PDF, concentrated over a small parameter range:
3. Multi-modal PDF
   Problems: Low probability that algorithm identifies multiple regions with high probability mass.



○ Samples from prior distribution
× Samples from posterior distribution

Generation of samples of posterior PDF
Sampling based algorithms

- Transitional Markov Chain Monte Carlo[1]
  - Adaptively constructed Intermediate posteriors
- X-TMCMC[2]: TMCMC + adaptive meta-model
- Bayesian Updating using Structural Reliability[3,4]
  - Reformulation of the updating problem into a reliability problem (subset simulation)
- BUS with adaptive meta-model for improved numerical efficiency

1. Transitional Markov Chain Monte Carlo method for Bayesian updating, model class selection, and model averaging. J. Ching and Y.-C. Chen. *Journal of Engineering Mechanics*, 133:816–832, 2007.
2. X-TMCMC: Adaptive kriging for Bayesian inverse modelling, P. Angelikopoulos, C. Papadimitriou, P. Koumoutsakos, *CMAME*, 2015
3. Bayesian Updating with Structural Reliability Method, (D. Straub and I. Papaioannou, *Journal of Engineering Mechanics, Trans. ASCE*, 141(3), 2014.
4. Bayesian updating and model class selection with Subset Simulation, F.A. DiazDelaO, A. Garbuno-Inigo, S.K. Au, I. Yoshida, *CMAME*, 2017

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  - NASA Challenge Problem

Epistemic uncertainties only

- The model output is deterministic
- The likelihood incorporates the information delivered by the data and updates the prior PDF.
- How to evaluate the PDF of experiments when the model is deterministic?

→ Assumption on discrepancies between model output and experiments

- Source of discrepancies:
  - Measurement errors
  - Model errors

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Epistemic uncertainties only

- If the discrepancies between test and analysis are assumed to be normally distributed, the likelihood function is a Gaussian distribution.
  - Gaussian distribution gives the largest amount of uncertainty (maximum entropy)

$$p(\mathcal{D} \mid \theta, \mathcal{M}) = \prod_{i=1}^{N_{\mathcal{D}}} \frac{1}{(2\pi)^{Nm/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} \left(y(\theta) - \bar{y}_i \right)^T \Sigma^{-1} \left(y(\theta) - \bar{y}_i \right) \right)$$

Epistemic uncertainties only

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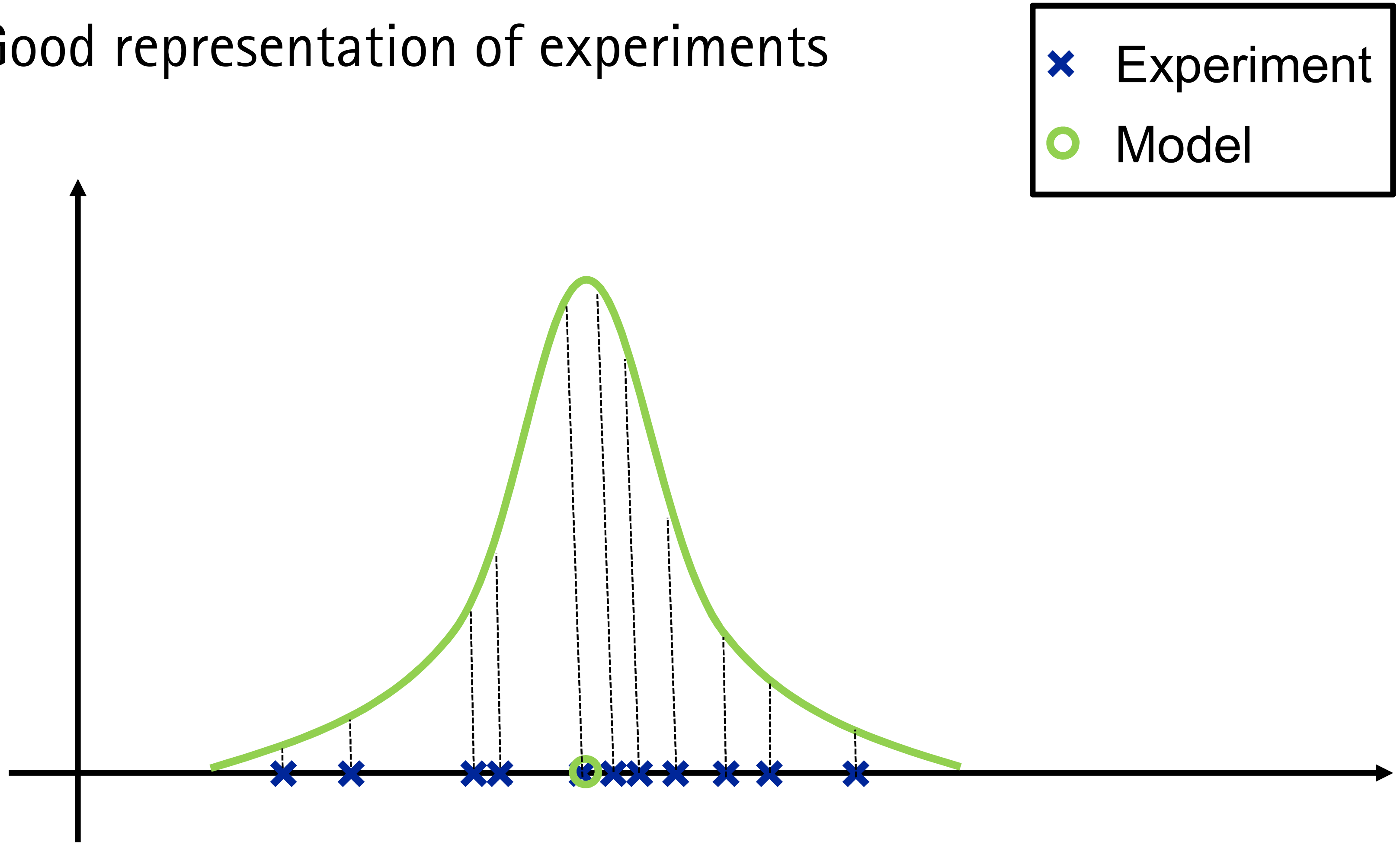
$$p(\mathcal{D} \mid \theta, \mathcal{M}) = \frac{1}{(2\pi\sigma^2)^{N_m/2}} \exp\left(-\frac{1}{2\sigma^2} \sum_{j=1}^{N_{\mathcal{D}}} \left(y(\theta) - \bar{y}_j \right)^2 \right)$$

No correlation between data

- Is Gaussian error a good assumption?
  - Likelihood can assume different function with different discrepancy models!

Epistemic uncertainties only

- Deterministic model output
- Good representation of experiments



Epistemic uncertainties only

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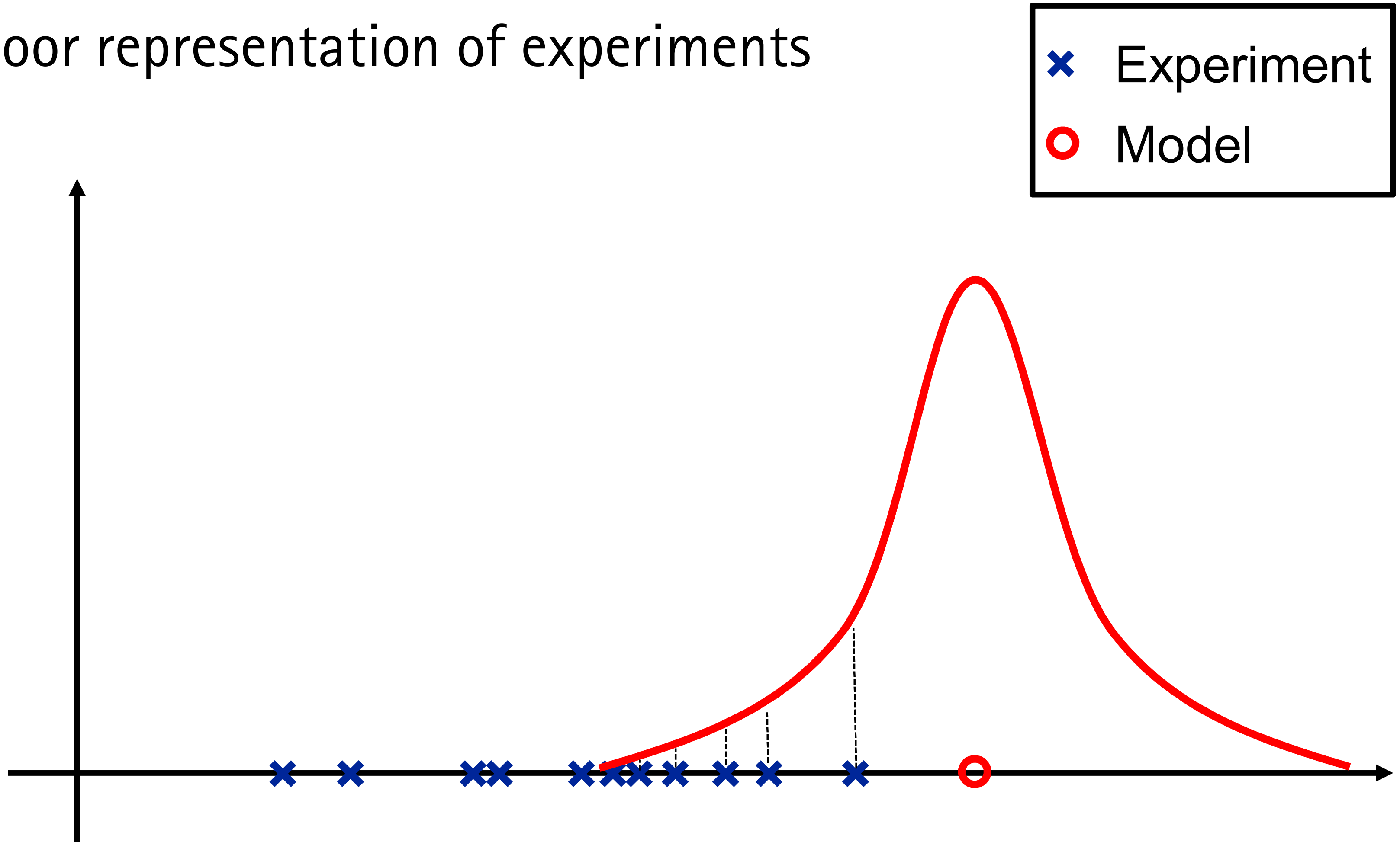


Epistemic and aleatory uncertainties

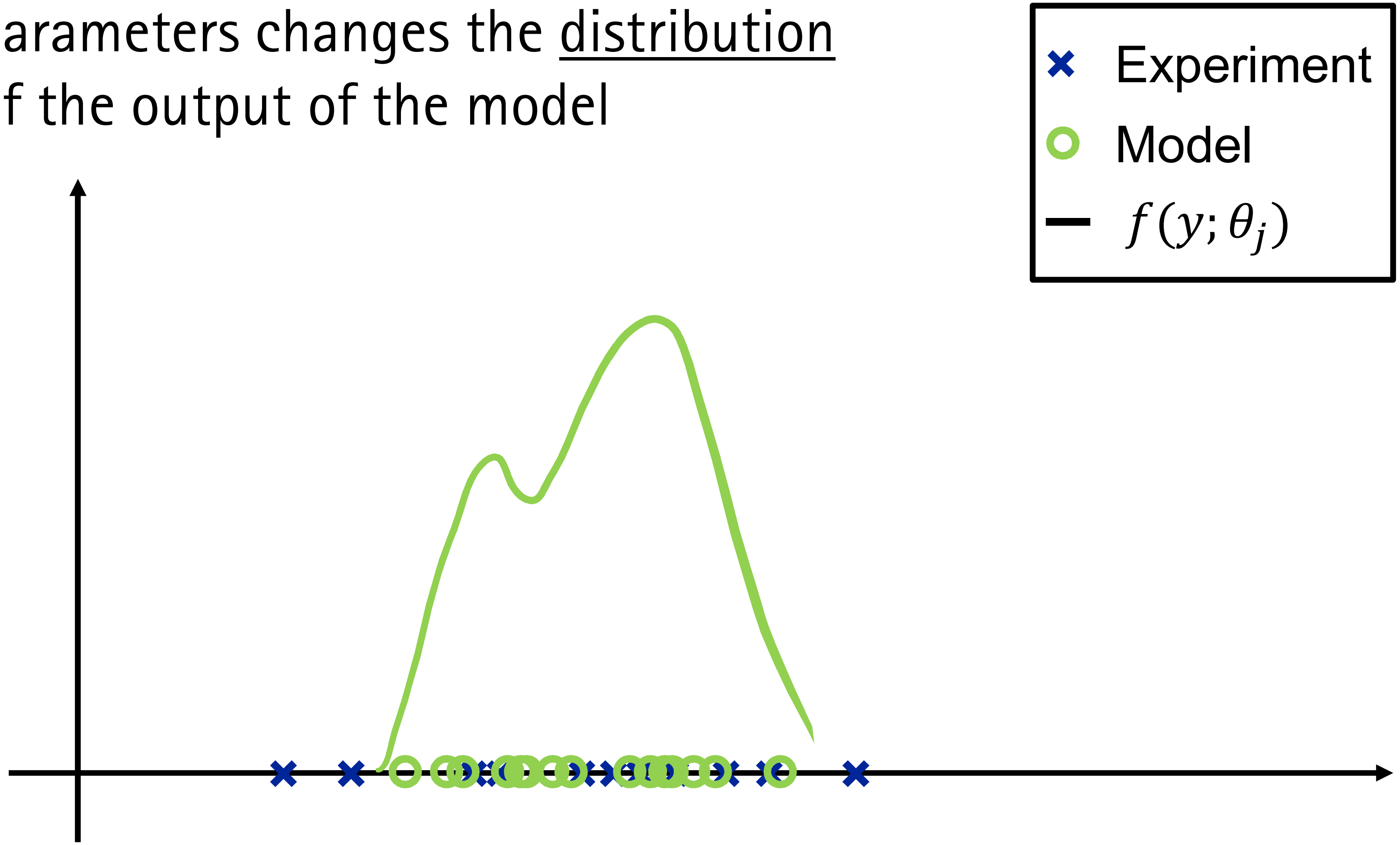
- The model output is <u>stochastic</u>
  - The likelihood can be exactly evaluated as

$$p(\mathfrak{D}|\theta, M) = \prod_{i=1}^{N_{\mathfrak{D}}} f^{M}_{\mathbf{Y}(\theta)}(\mathbf{Y}_i; \theta)$$

- Accurate knowledge of output PDF is required
  - E.g.: Kernel density PDF from MC simulation
  - Computationaly extermely intensive
    - High number of samples for each set of model parameters

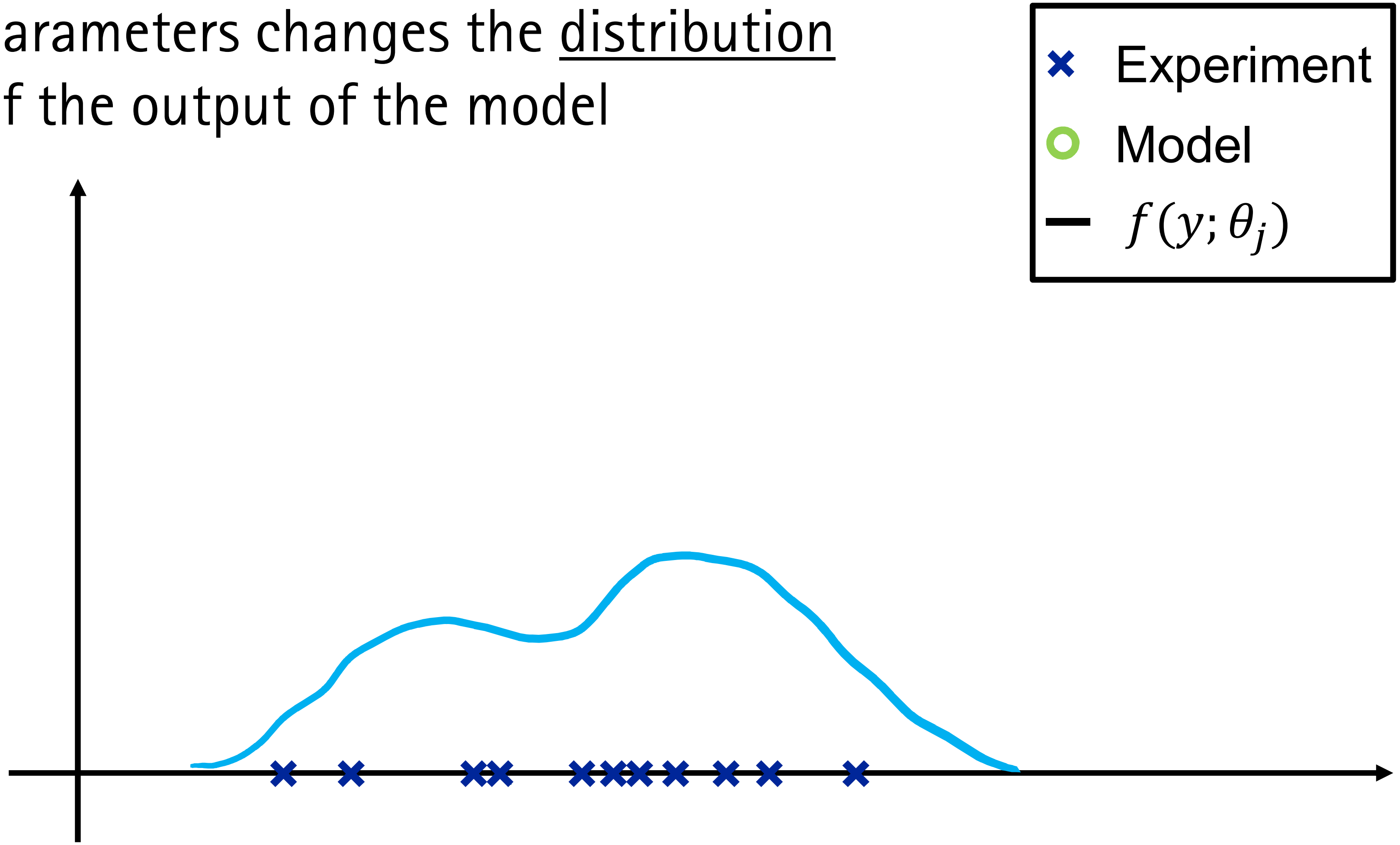
Epistemic and aleatory uncertainties

- Changing the value of the epistemic parameters changes the distribution of the output of the model



Epistemic and aleatory uncertainties

- Changing the value of the epistemic parameters changes the <u>distribution</u> of the output of the model



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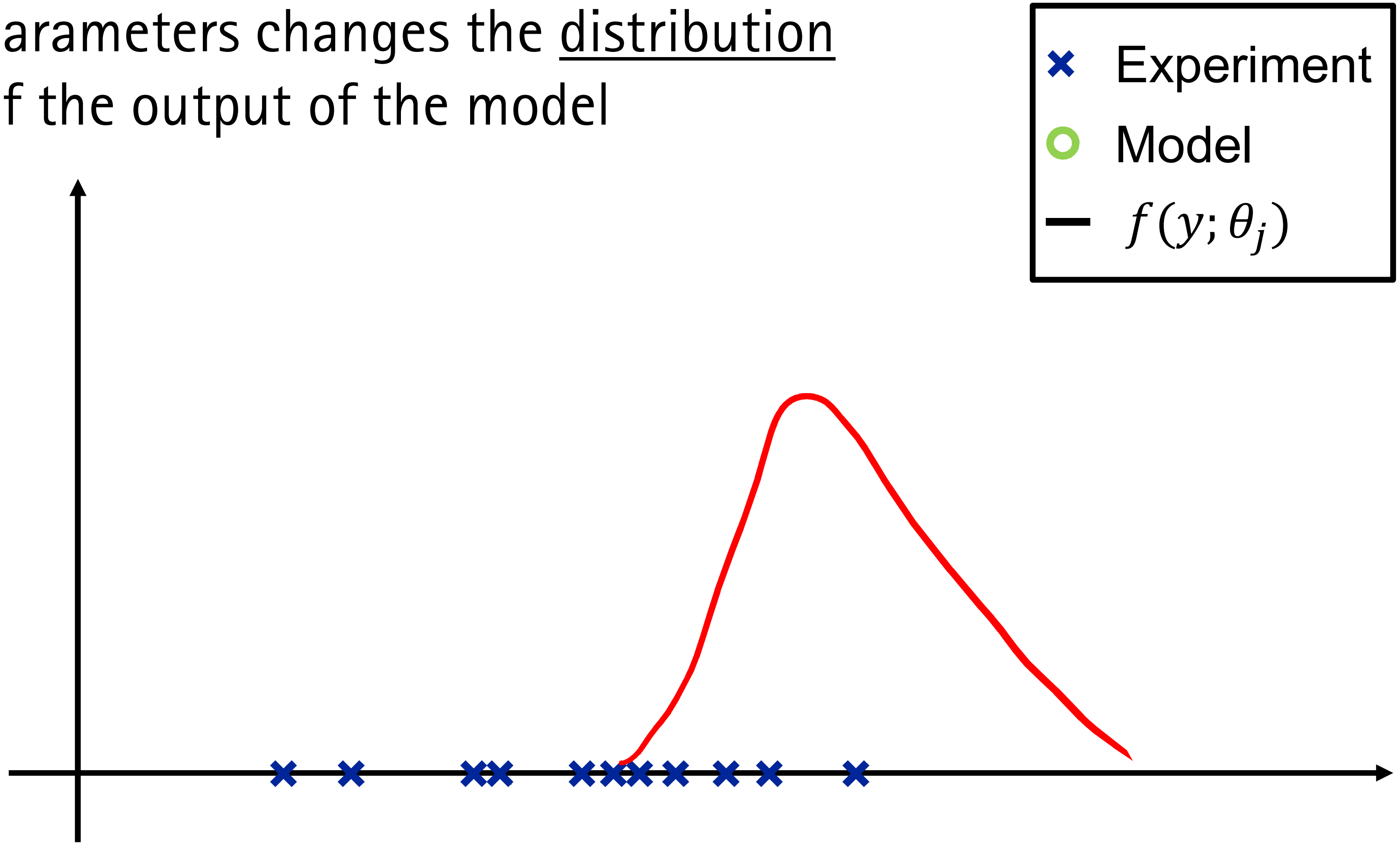

Epistemic and aleatory uncertainties

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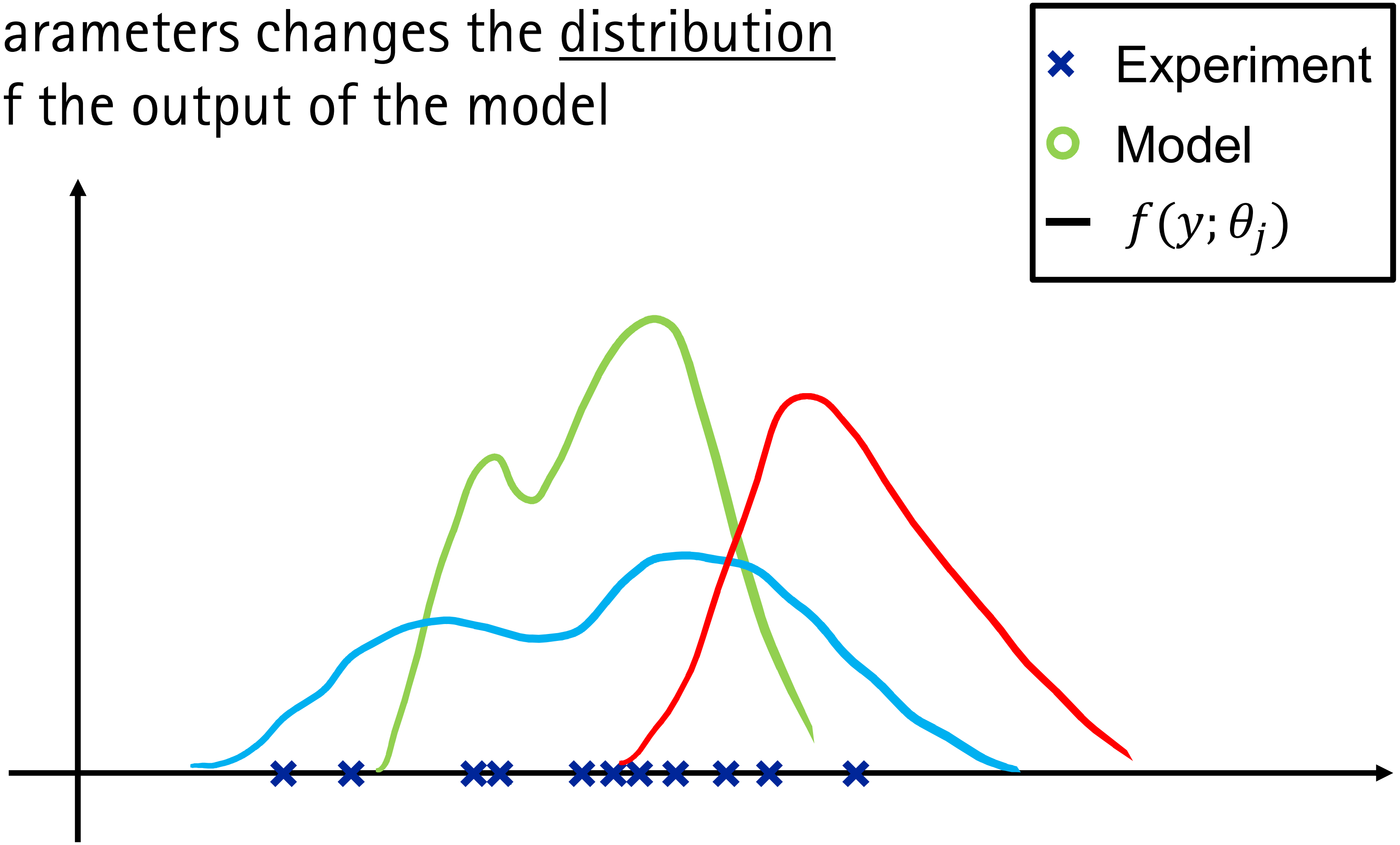


Approximate Bayesian Computation

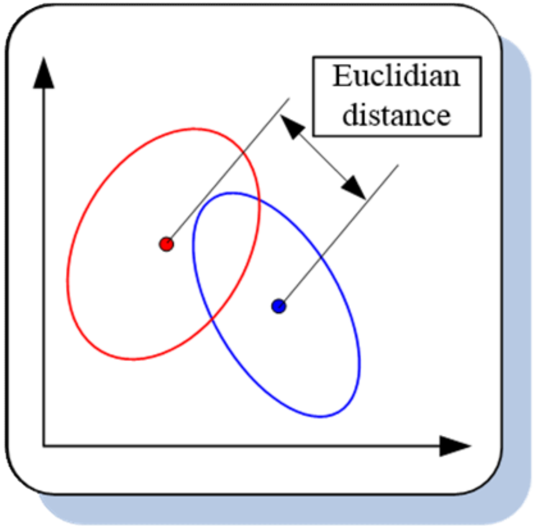
- Approximate likelihood
  - Synthetize information from both experimental observations and input
  - Avoid the full estimation of the model output PDF
- Based on combination of the Gaussian likelihood function:
  - $$P_L\left(\mathbf{Y}_{exp}\middle|\boldsymbol{\theta}\right) \propto \exp\left\{-\frac{d\left(\mathbf{Y}_i,\mathbf{Y}^M(\theta)\right)^2}{\varepsilon^2}\right\}$$

➢ d -- distance metric, i.e., distance of mean, quantiles, stochastic distance measures, etc.;

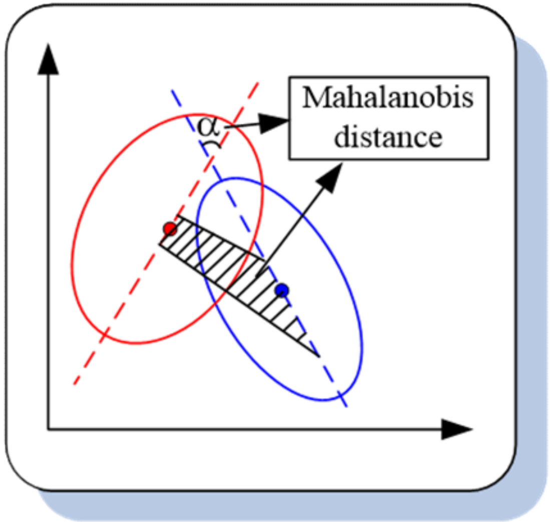
➢ ε -- width factor, controlling the centralization of the resulting posterior distributions of the inputs.

Approximate Bayesian Computation
Stochastic distance metrics

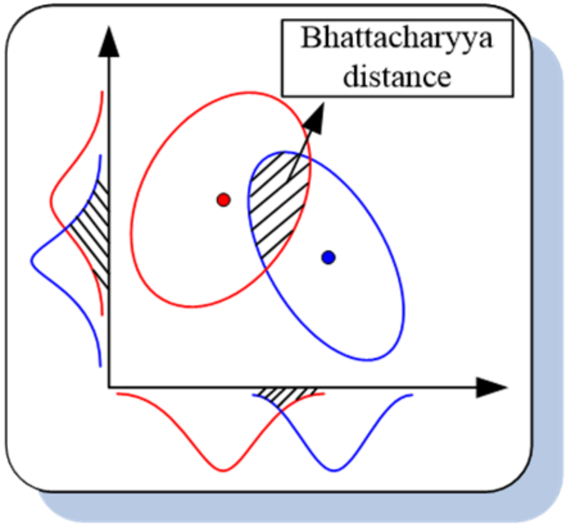
Capture increasing level of uncertainties



Geometrical distance



Variance-based distance



Distributional overlap distance

Approximate Bayesian Computation
Bhattacharyya distance metric

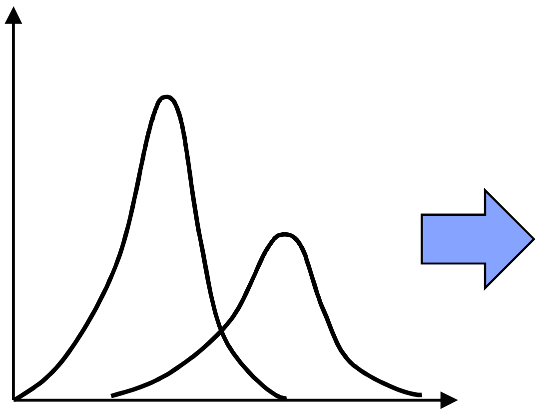
A quantitative and comprehensive comparison metric, not only focusing on single points but also capable for random samples, is required.

$$\mathbf{Y} = \begin{bmatrix} x_{11}, x_{12}, \dots, x_{1m} \\ x_{21}, x_{22}, \dots, x_{2m} \\ \vdots \quad \ddots \quad \vdots \\ x_{N1}, x_{N2}, \dots, x_{Nm} \end{bmatrix} \qquad \begin{matrix} \mathbf{Y}_{sim} \in \mathbb{R}^{N_{sim} \times m} \\ \\ \mathbf{Y}_{exp} \in \mathbb{R}^{N_{exp} \times m} \end{matrix}$$

Bhattacharyya distance:

$$d_B(\mathbf{Y}_{data}, \mathbf{Y}_{sim}) = -\log\left[\int_{\mathbb{y}} \sqrt{f_Y^{data}(y) f_Y^{sim}(y)} \mathrm{d}y\right]$$

Approximate Bayesian Computation
Bhattacharyya distance



Bhattacharyya distance

Overlap of the distributions/samples

- A binning algorithm[2] is utilized to evaluate the Bhattacharyya distance between two discrete distributions, using their Probability Mass Functions (PMFs).

$$d_B(\mathbf{Y}_{\text{data}}, \mathbf{Y}_{\text{sim}}) = -\log\left(\sum_{k=1}^{N_{bin}} \sqrt{\text{PMF}_{\text{data}}^{(k)} \text{PMF}_{\text{sim}}^{(k)}}\right)$$

  - Problem: when no overlap is present the distance is infinite

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Distribution-free model updating

- Model updating under hybrid uncertainties

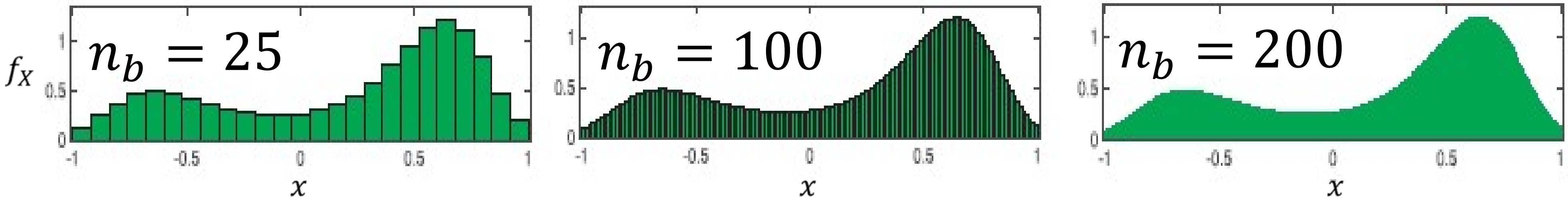
  Challenge

  - **Hypotheses on the distribution families of aleatory inputs;**
  - **Dependence structure among aleatory inputs;**
  - **Updating dynamic systems: very high dimensional observations.**

- Enhancement of subjective assumption-free framework
  - How to calibrate the PDF of aleatory inputs whose distribution families are unknown?
  - How to calibrate the correlated joint PDF?
  - How to incorporate the time dependent observations to update the dynamic systems?

Staircase Random Variable (SRV)

- Univariate random variable x, having:
  - Bounded support set: $\Omega_x = [\underline{x}, \overline{x}]$;
  - First four moments: $\boldsymbol{\theta} = [\mu, m_2, \tilde{m}_3, \tilde{m}_4]$.
    - $\tilde{m}_3$ – skewness $(= m_3/m_2^{3/2})$, $\tilde{m}_4$ – kurtosis $(= m_4/m_2^2)$
- Staircase density function
  - Piecewise constant function: $f_X(x) = \begin{cases} l_i & \forall x \in (x_i, x_{i+1}], \text{for } 1 \le i \le n_b \\ 0 & \text{otherwise} \end{cases}$
    - n_b – number of bins
  - e.g., $\Omega_x = [-1, 1]$, $\boldsymbol{\theta} = [0.2, 0.3, -0.61, 2.0]$



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Staircase Density Function
Optimization problem

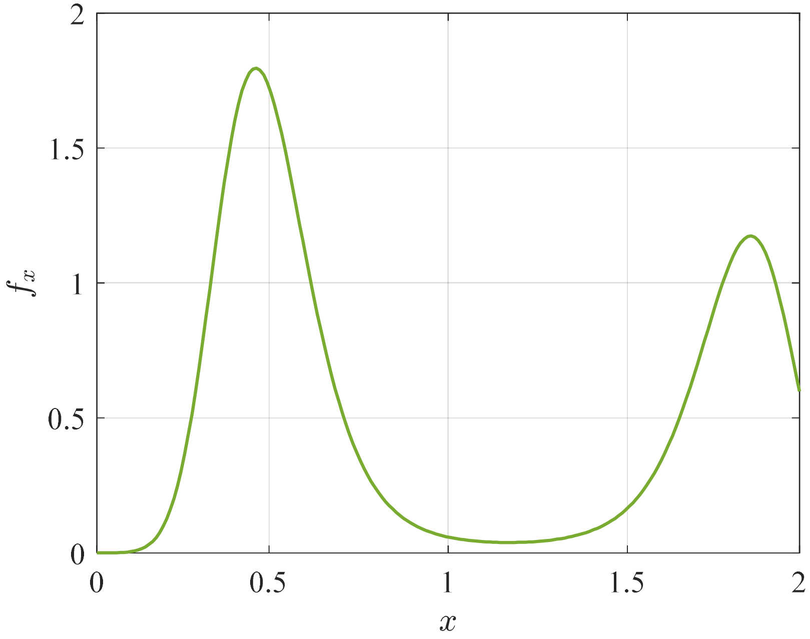
- Optimization problem on moment matching constraints:

$$\hat{\boldsymbol{l}} = \underset{\boldsymbol{l}\geq 0}{\operatorname{argmin}}\{J(\boldsymbol{l}):\ \mathbf{A}(\boldsymbol{\theta}, n_b)\boldsymbol{l} = \mathbf{b}(\boldsymbol{\theta}), \boldsymbol{\theta} \in \Theta\}$$

  - $J(\boldsymbol{l})$ – cost function, e.g., maximum entropy;
  - Θ – $\boldsymbol{\theta}$ feasible domain, $g_i(\boldsymbol{\theta}) \leq 0, i = 1, \cdots, 14.$

	Moment constraints		Moment constraints
Mean μ_i	$g_1 = \underline{x}_i - \mu_i$	Kurtosis $\tilde{m}_{4i}$	$g_{10} = -\tilde{m}_{4i} m_{2i}^2$
	$g_2 = \mu_i - \overline{x}_i$		$g_{11} = 12\tilde{m}_{4i} m_{2i}^2 - (\overline{x}_i - \underline{x}_i)^4$
Variance m_{2i}	$g_3 = -m_{2i}$		$g_{12} = \left(\tilde{m}_{4i} m_{2i}^2 - v_i m_{2i} - u_i \tilde{m}_{3i} m_{2i}^{3/2}\right)(v_i - m_{2i}) + \left(\tilde{m}_{3i} m_{2i}^{3/2} - \mu_i m_{2i}\right)^2$
	$g_4 = m_{2i} - v_i$		
Skewness $\tilde{m}_{3i}$	$g_5 = m_{2i}^2 - m_{2i}(\mu_i - \underline{x}_i)^2 - \tilde{m}_{3i} m_{2i}^{3/2}(\mu_i - \underline{x}_i)$		$g_{13} = \tilde{m}_{3i}^2 m_{2i}^3 + m_{2i}^3 - \tilde{m}_{4i} m_{2i}^3$
	$g_6 = \tilde{m}_{3i} m_{2i}^{3/2}(\overline{x}_i - \mu_i) - m_{2i}(\overline{x}_i - \mu_i)^2 + m_{2i}^2$		
	$g_7 = 4m_{2i}^2 + \tilde{m}_{3i}^2 m_{2i}^3 - m_{2i}^2(\overline{x}_i - \underline{x}_i)^2$		
	$g_8 = 6\sqrt{3}\tilde{m}_{3i} m_{2i}^{3/2} - (\overline{x}_i - \underline{x}_i)^3$		
	$g_9 = -6\sqrt{3}\tilde{m}_{3i} m_{2i}^{3/2} - (\overline{x}_i - \underline{x}_i)^3$		

Staircase Density Function
Illustrations



- $\Omega_x = [0,2]$, $\boldsymbol{\theta} = [1.0, 0.33, 0, 1.8]$
  - ➢ Uniform distribution
- $\Omega_x = [0,2]$, $\boldsymbol{\theta} = [0.57, 0.10, 0.59, 2.86]$
  - ➢ Beta distribution (left skewed)
- $\Omega_x = [0,2]$, $\boldsymbol{\theta} = [1.0, 0.42\ 0.42, 1.37]$
  - ➢ Bi-modal distribution

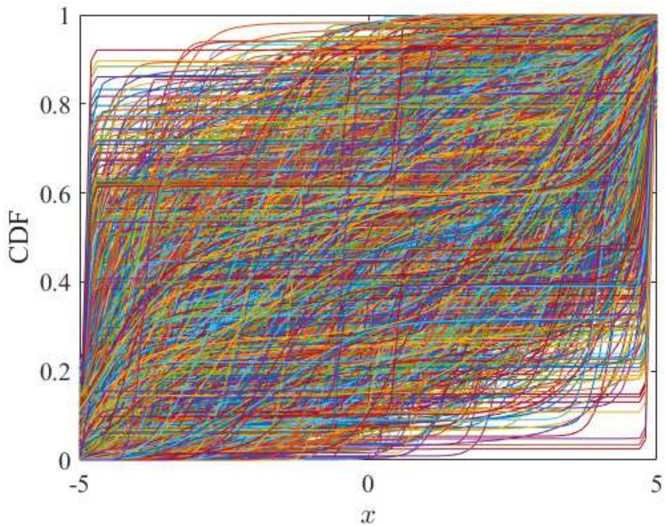
- Approximates a wide range of density shapes
  - e.g., very skewed or/and multi-modal distributions
- Allows to characterize aleatory inputs whose distribution families are unknown

Distribution-free Model Updating

- Given a support set Ω_x, the initial knowledge of $\boldsymbol{\theta}$ is expressed by intervals satisfying the moment constraints:

$$\mu \in [\underline{x}, \overline{x}],\ m_2 \in \left[0, \frac{(\overline{x}-\underline{x})^2}{4}\right],\ m_3 \in \left[-\frac{(\overline{x}-\underline{x})^3}{6}, \frac{(\overline{x}-\underline{x})^3}{6}\right],\ m_4 \in \left[0, \frac{(\overline{x}-\underline{x})^4}{12}\right]$$

- e.g., $\Omega_x = [-5, 5]$



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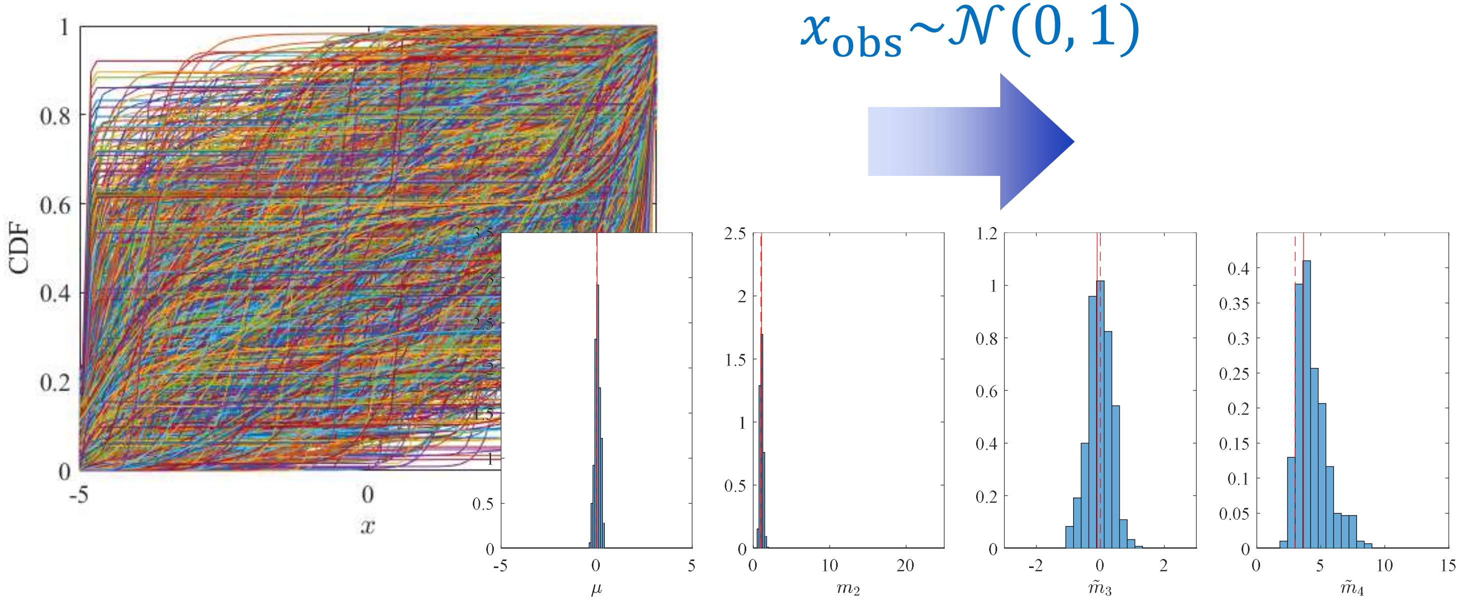
- e.g., $\Omega_x = [-5, 5]$

Bayesian updating

$x_{\text{obs}} \sim \mathcal{N}(0, 1)$



Distribution-free Model Updating

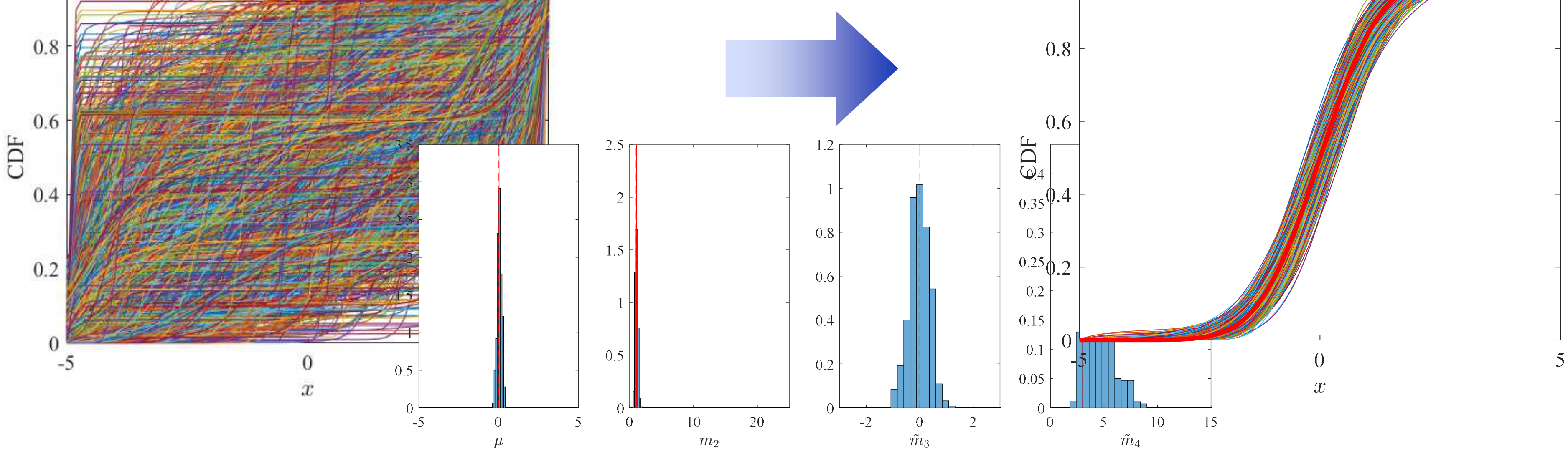
- Given a support set Ω_x, the initial knowledge of $\boldsymbol{\theta}$ is expressed by intervals satisfying the moment constraints:

$$\mu \in [\underline{x}, \overline{x}],\ m_2 \in \left[0, \frac{(\overline{x}-\underline{x})^2}{4}\right],\ m_3 \in \left[-\frac{(\overline{x}-\underline{x})^3}{6}, \frac{(\overline{x}-\underline{x})^3}{6}\right],\ m_4 \in \left[0, \frac{(\overline{x}-\underline{x})^4}{12}\right]$$

- e.g., $\Omega_x = [-5, 5]$

Bayesian updating

$x_{\text{obs}} \sim \mathcal{N}(0, 1)$



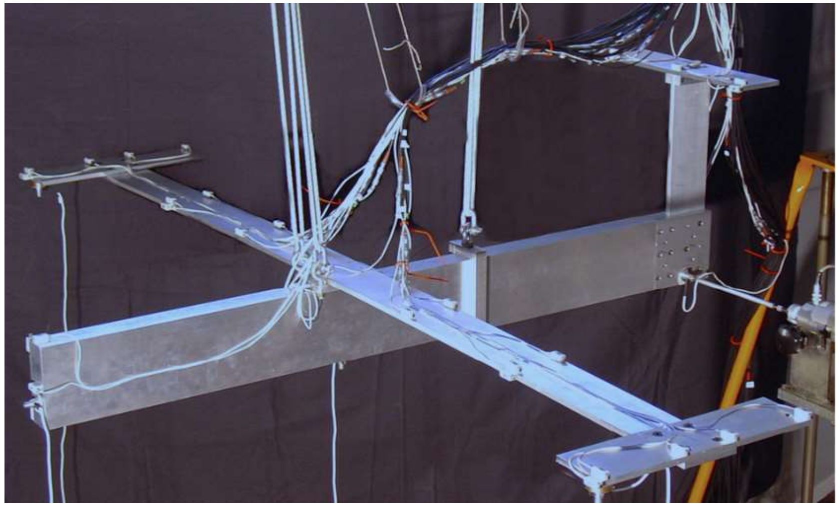
Outline

- Introduction
- Bayesian Updating
  - Likelihood with hybrid uncertainties
- Distribution-free model updating
- Numerical Examples
  - DLR AIRMOD
  - 2 DOF shear building
  - NASA Challenge Problem

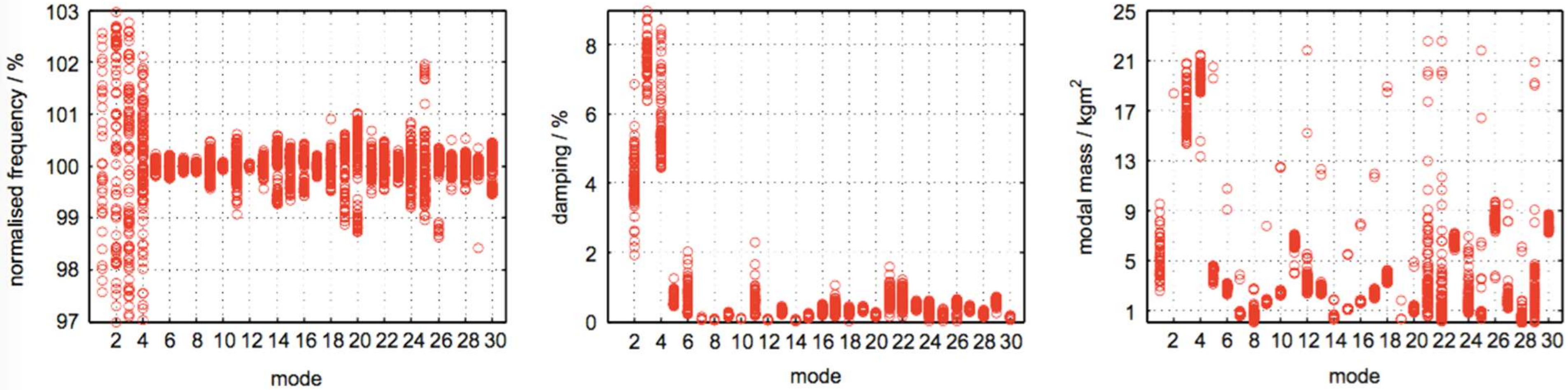
Numerical Example

DLR-AIRMOD

- Replica of GARTEUR SM-AG19 benchmark structure
  - 2m wingspan, 1.5m length, 0.46m height, 44kg weight
  - Disassembled and reassembled 130 times (86 tests usable)
- Excited with random signal in the frequency 0-400 Hz



DLR-AIRMOD vibration testing

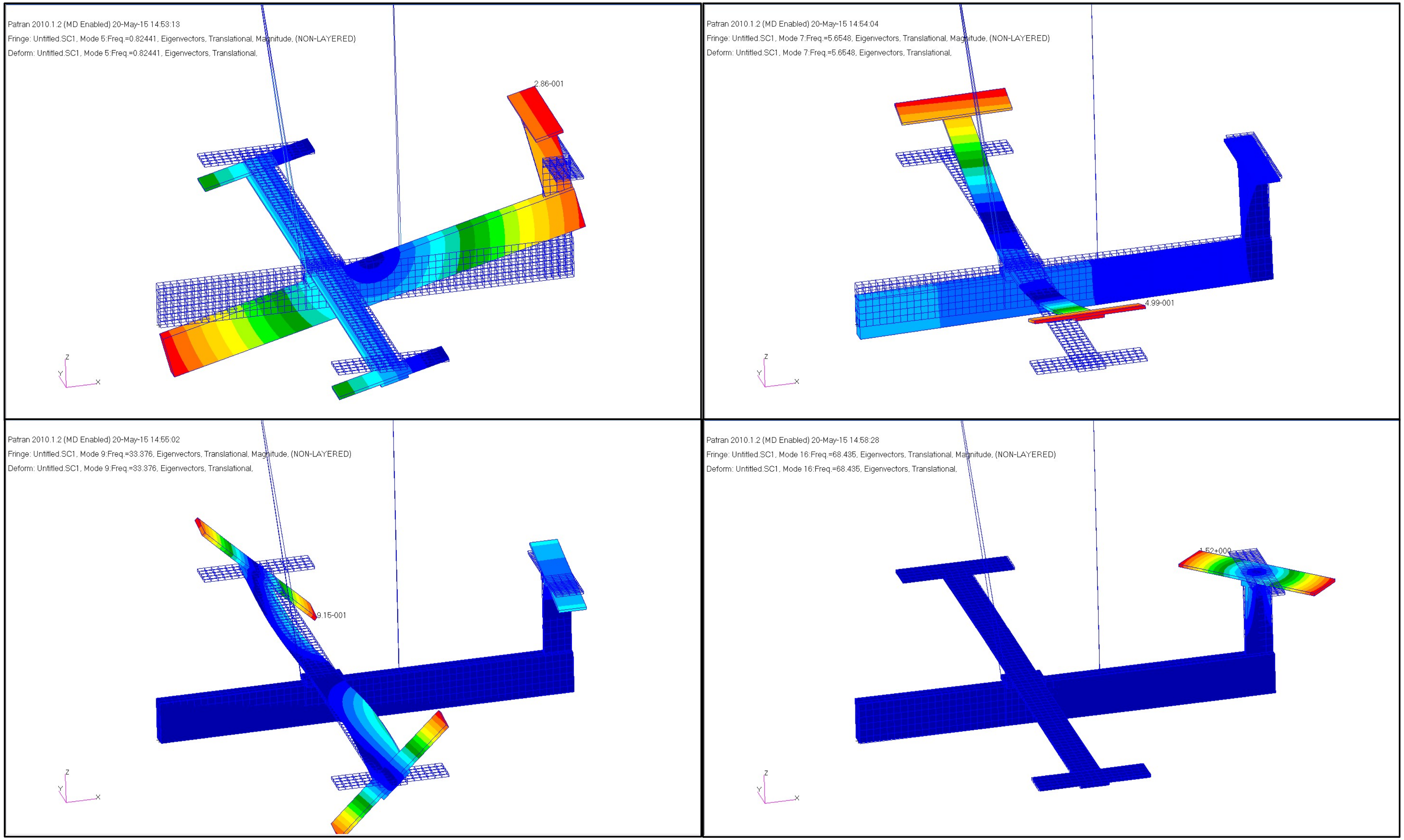


Numerical Example

DLR-AIRMOD

- Numerical model
    - Modeled in MSC.NASTRAN
    - 3136 grid points
    - 1446 solid elements (CHEXA, CPENTA) for the main aluminium structure
    - 561 CELAS1 for joints modeling
    - 73 concentrated mass for cables, instrumentation

Numerical Example



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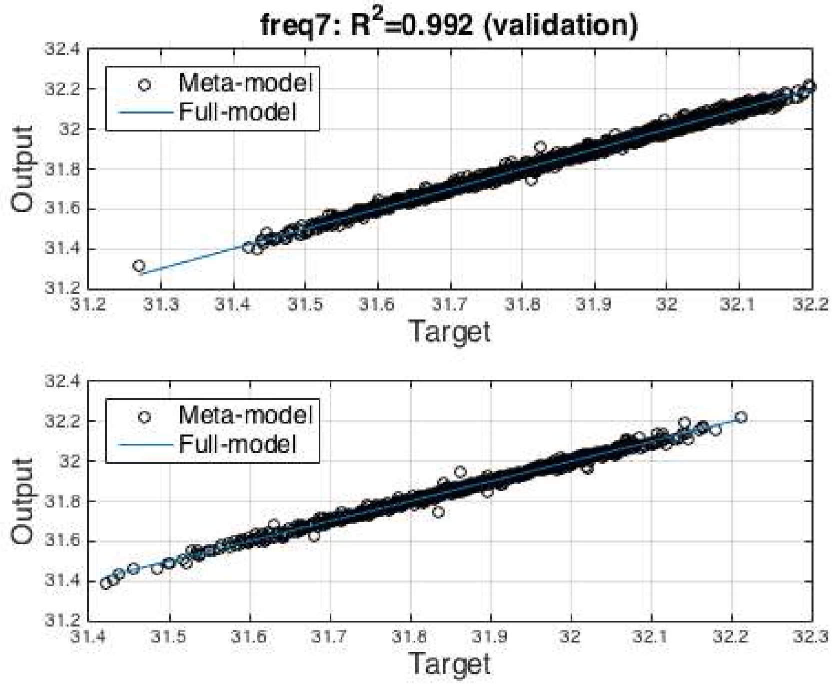
Numerical Example
Uncertain inputs

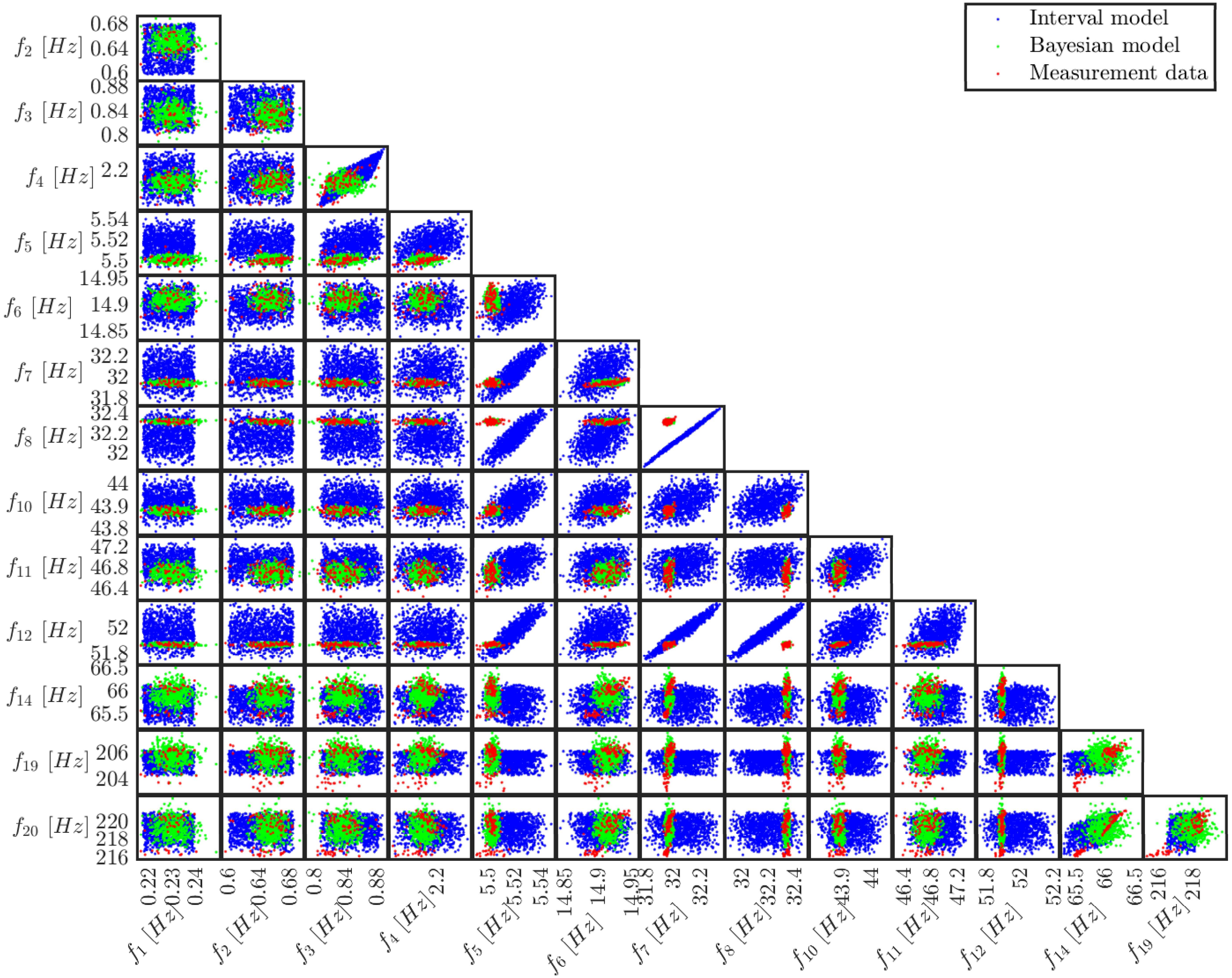
	Type	Location	Description	Init. val.	Unit
θ_{01}	Stiffness	Front bungee cord	Support stiffness	1.80E+03	N/m^2
θ_{02}	Stiffness	Rear bungee cord	Support stiffness	7.50E+03	N/m^2
θ_{03}	Stiffness	VTP/HTP joint	Sensor cable – y dir^n	1.30E+02	N/m
θ_{04}	Stiffness	Wing/fuselage joint	Sensor cable – y dir^n (top)	7.00E+01	N/m
θ_{05}	Stiffness	Wing/fuselage joint	Sensor cable – y dir^n (bott^m)	7.00E+01	N/m
θ_{06}	Stiffness	VTP/HTP joint	Joint stiffness – x, y dir^{ns}	1.00E+07	N/m
θ_{07}	Stiffness	VTP/HTP joint	Joint stiffness – z dir^n	1.00E+09	N/m
θ_{08}	Mass	VTP/HTP joint	Sensor cables	2.00E-01	kg
θ_{09}	Mass	Wingtip right wing	Screws and glue	1.86E-01	kg
θ_{10}	Mass	Wingtip left wing	Screws and glue	1.86E-01	kg
θ_{11}	Mass	Wingtip left/right	Sensor cables on wings	1.50E-02	kg
θ_{12}	Mass	Out^r wing left/right	Sensor cables on wings	1.50E-02	kg
θ_{13}	Mass	Inn^r wing left/right	Sensor cables on wings	1.50E-02	kg
θ_{14}	Stiffness	Wing/fuselage joint	Joint stiffness – x dir^n	2.00E+07	N/m
θ_{15}	Stiffness	Wing/fuselage joint	Joint stiffness – y dir^n	2.00E+07	N/m
θ_{16}	Stiffness	Wing/fuselage joint	Joint stiffness – z dir^n	7.00E+06	N/m
θ_{17}	Stiffness	VTP/fuselage joint	Joint stiffness – x dir^n	5.00E+07	N/m
θ_{18}	Stiffness	VTP/fuselage joint	Joint stiffness – y dir^n	1.00E+07	N/m

Numerical Example
Computational aspects

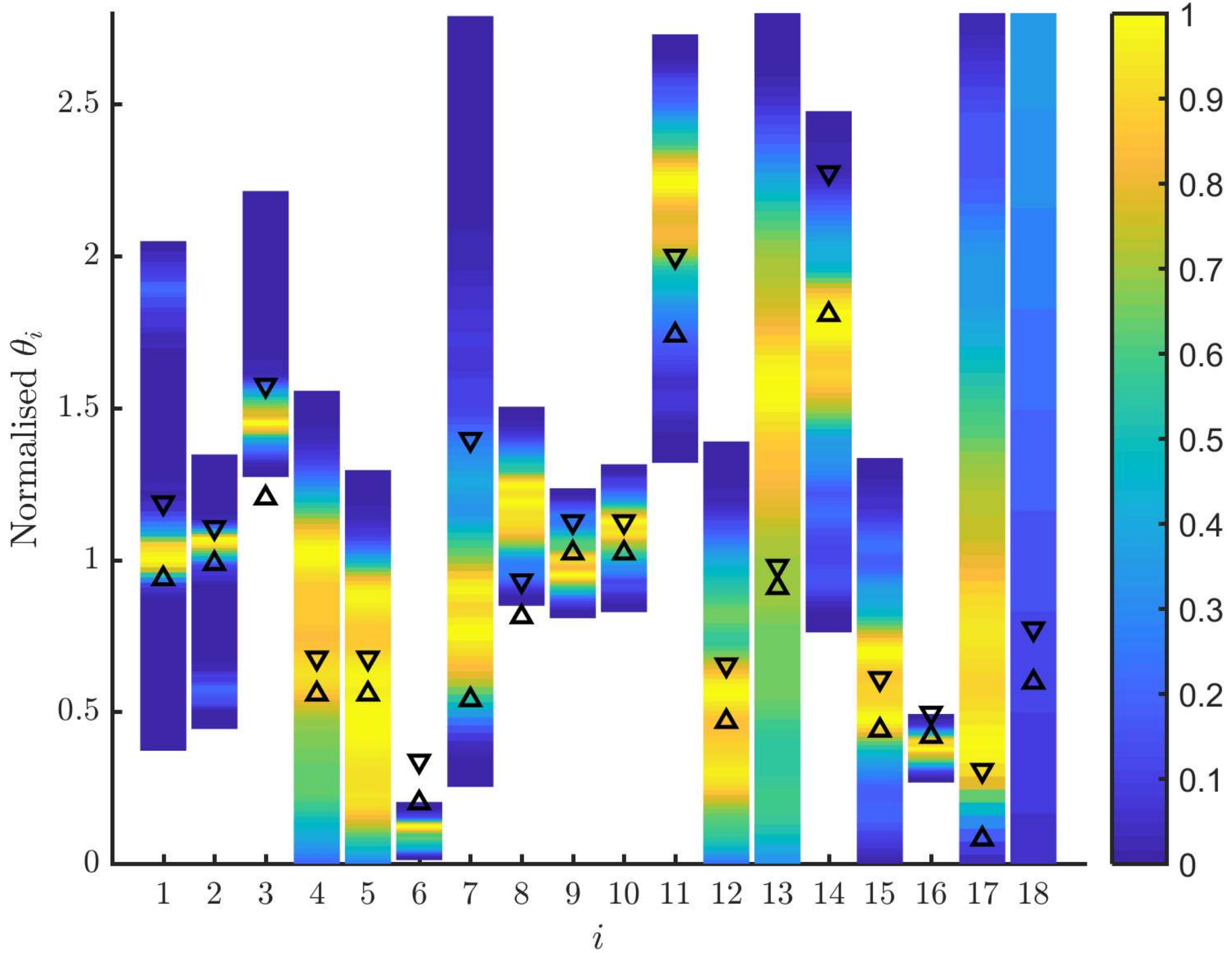
- Bayesian updating compared to interval field updating
- Full finite element analysis ~10s
- Optimization algorithm + NN for Interval Field
  - Particle swarm optimization in 32-dimensional space
- TMCMC + NN for Bayesian Updating
  - TMCMC with 500 samples, 29 steps to convergence
- Two analyses:
  - Full experimental data set – 86 experiments
  - Reduced experimental data set – 5 experiments



Numerical Example
Results – Full experimental data set

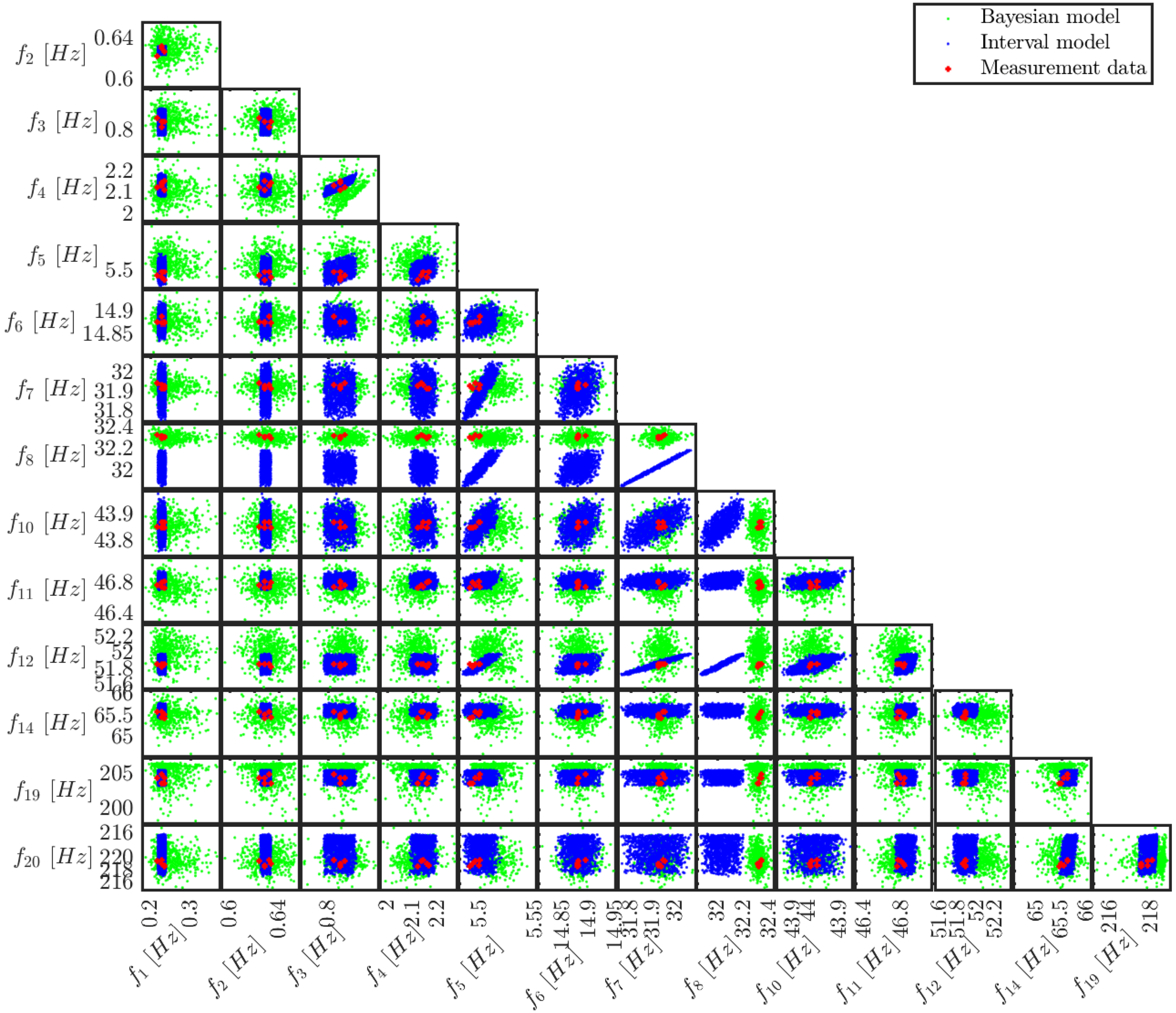


Numerical Example
Results – Full experimental data set



Numerical Example

Results – Reduced data set



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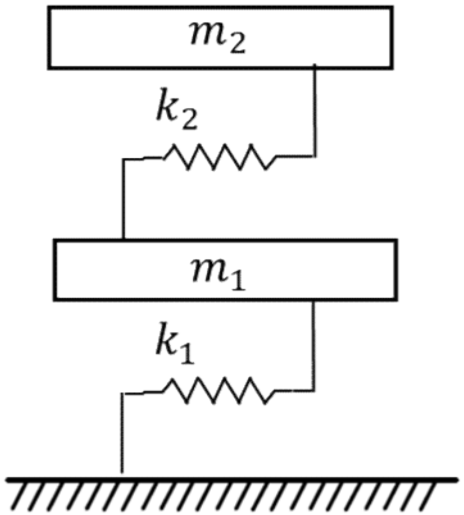
2-DOF Shear Building Example

- A simple engineering application originally introduced by Beck and Au (2002)

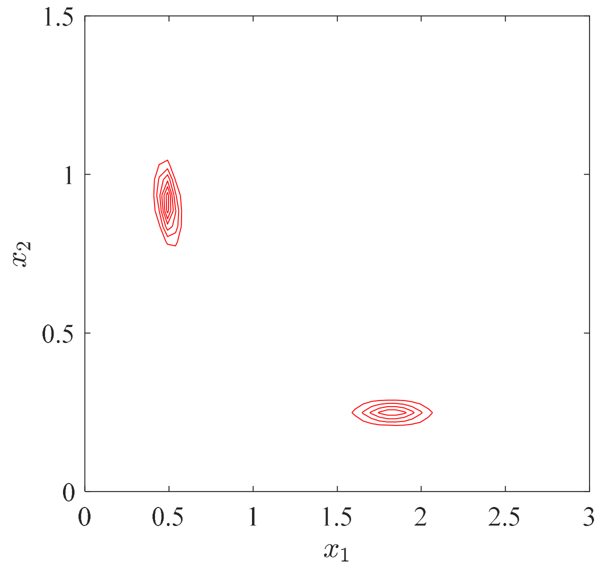


- $m_1 = 16.531 \times 10^3$ kg, $m_2 = 16.131 \times 10^3$ kg
- $k_1 = \bar{k}x_1$, $k_2 = \bar{k}x_2$
  - $\bar{k}$ – nominal value ($= 29.7 \times 10^6$ N/m)
  - x_1, x_2 – epistemic inputs to be updated

- First two natural frequencies f_1, f_2 are the observed features used in Bayesian updating.

2-DOF Shear Building Example
Problem setting

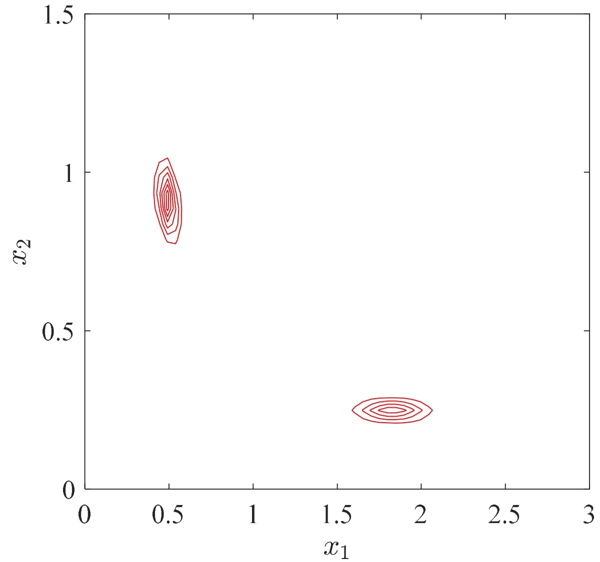
Beck and Au (2002)	Epistemic inputs: x_1,x_2	Prior PDF: Lognormal MPVs: [1.3, 0.8], Unit SDs	Observed data: $\hat{f}_1 = 3.13$ Hz, $\hat{f}_2 = 9.83$ Hz

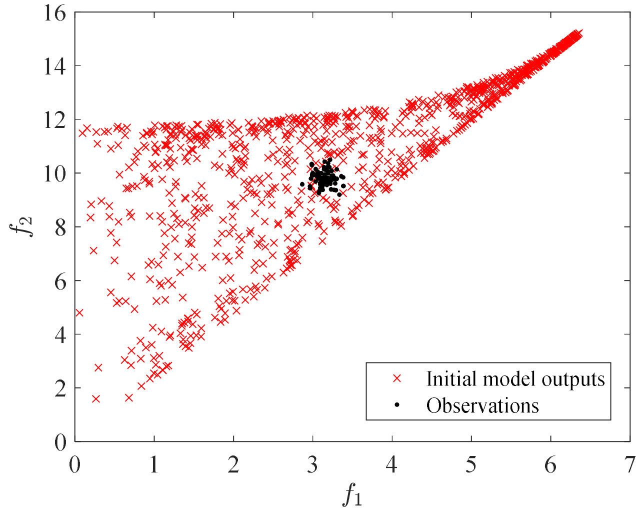


2-DOF Shear Building Example
Problem setting

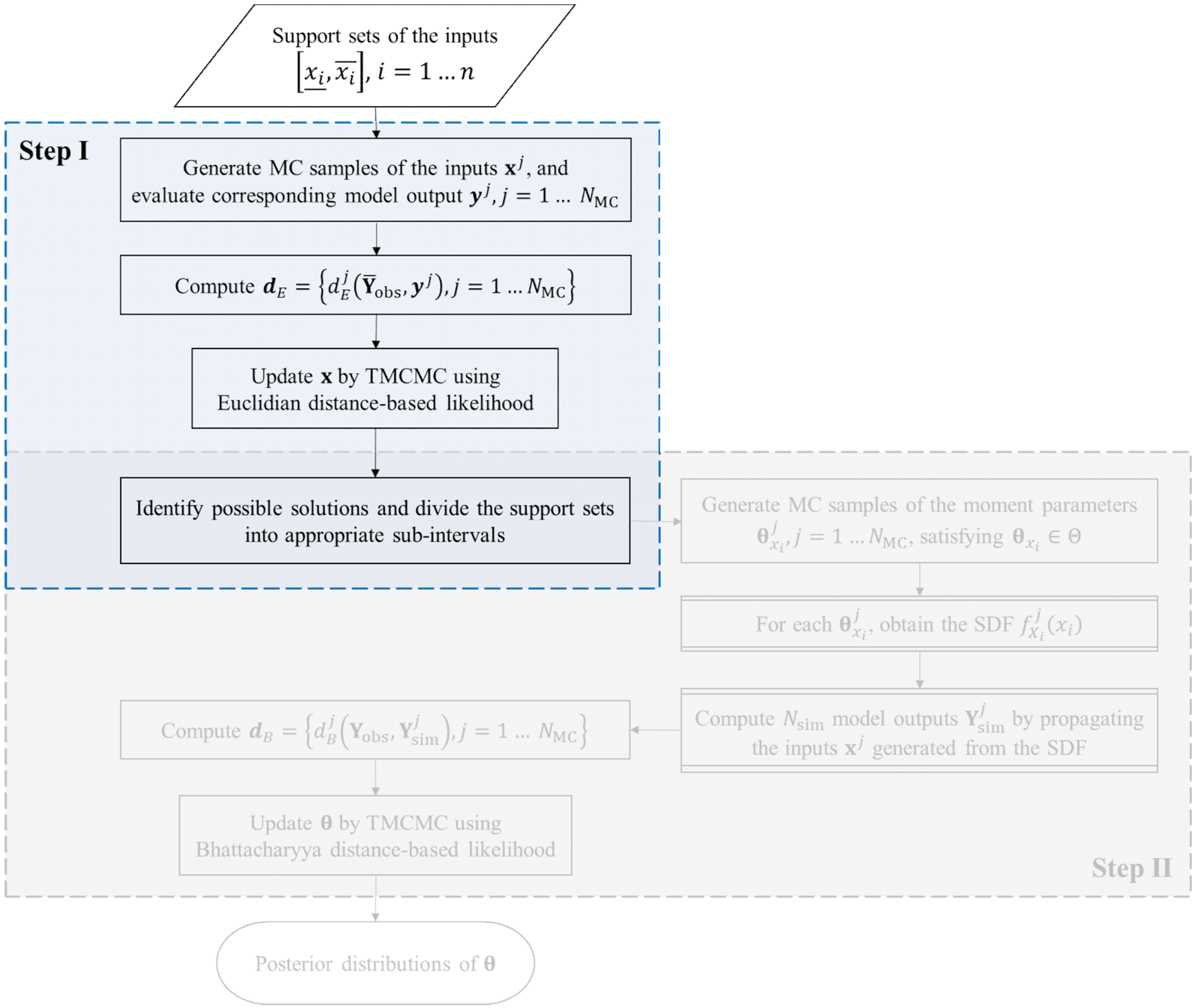
Non-unique solutions

Beck and Au (2002)	Epistemic inputs: x_1,x_2	Prior PDF: Lognormal MPVs: [1.3, 0.8], Unit SDs	Observed data: $\hat{f}_1 = 3.13$ Hz, $\hat{f}_2 = 9.83$ Hz
Proposed	Aleatory inputs: x_1,x_2 Epistemic inputs: $\left\{\begin{matrix}\mu_i, m_{2i}, \\ \widetilde{m}_{3i}, \widetilde{m}_{4i}\end{matrix}\right\}$, $i = 1, 2$	Prior PDF: $x_1 \in [0, 3.0]$, $x_2 \in [0, 1.5]$	Observed data: 100 pairs of $\langle f_1, f_2 \rangle$ obtained by assigning the posterior PDF in Beck and Au (2002) to x_1, x_2

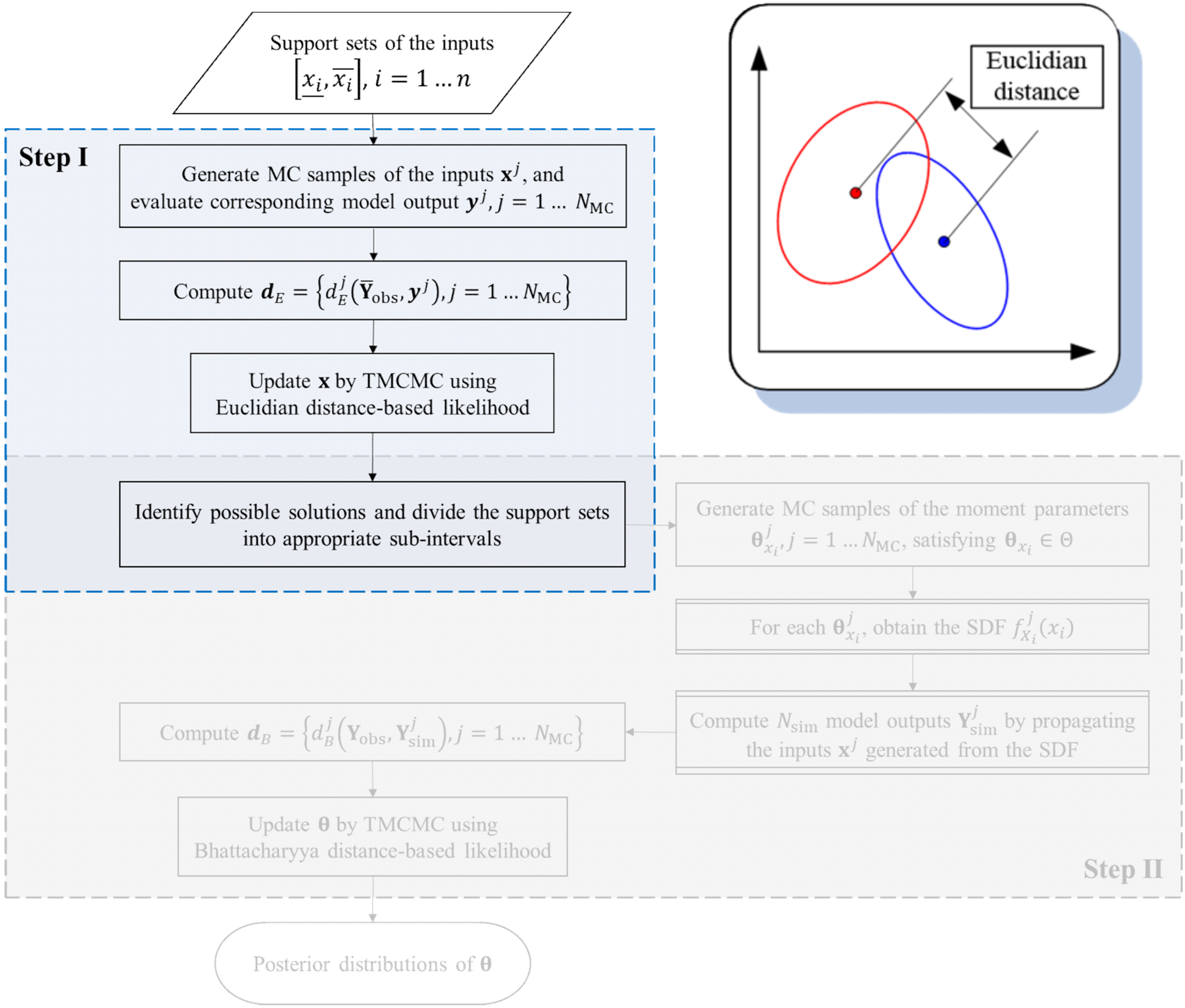




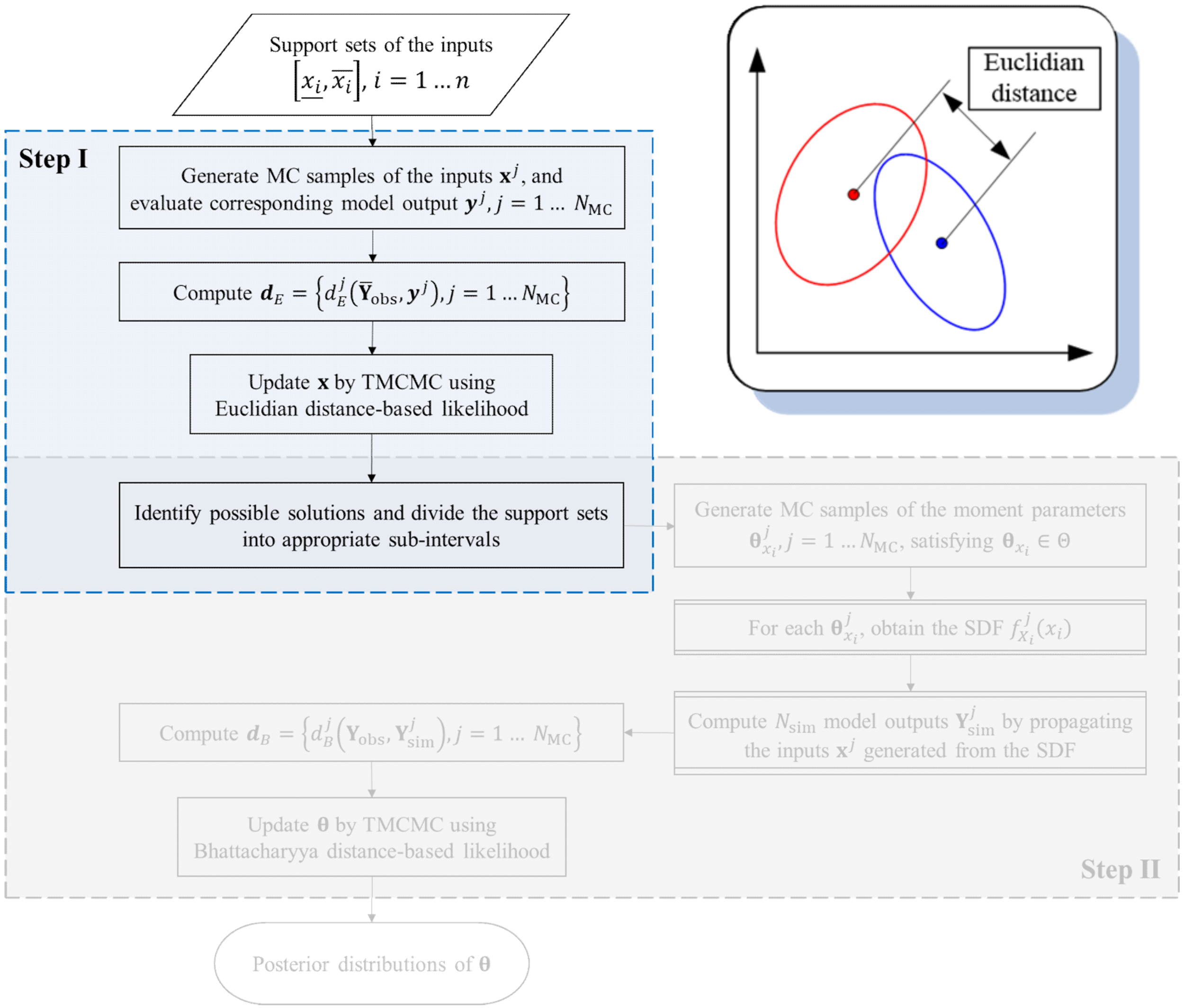
2-DOF Shear Building Example
Two-step deterministic/stochastic approach

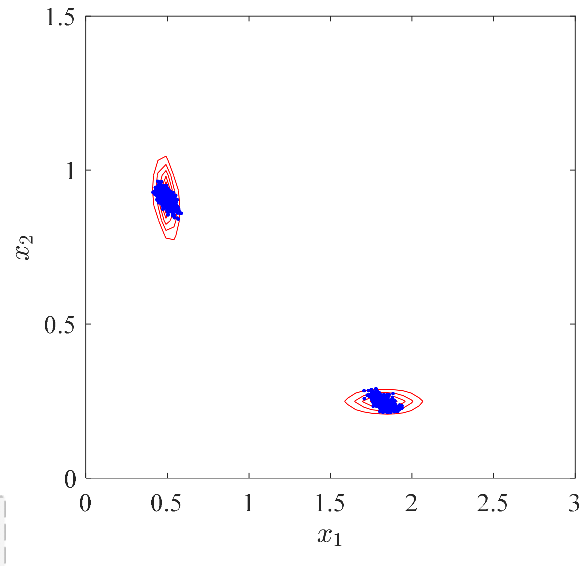


2-DOF Shear Building Example
Two-step deterministic/stochastic approach



2-DOF Shear Building Example
Two-step deterministic/stochastic approach





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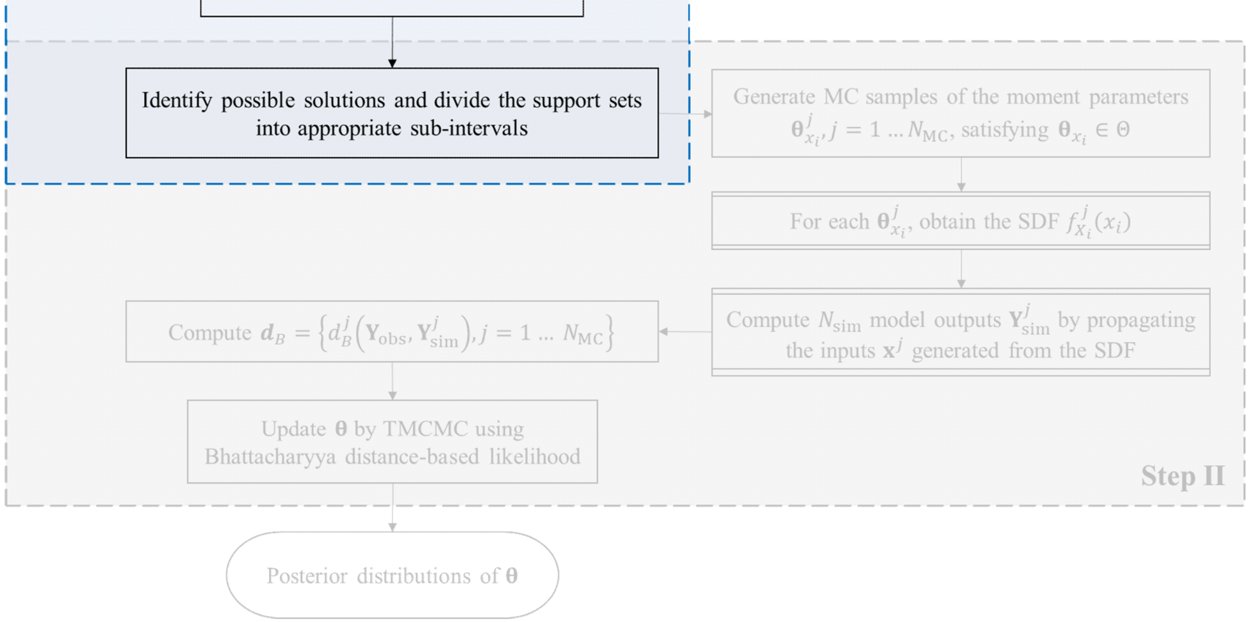

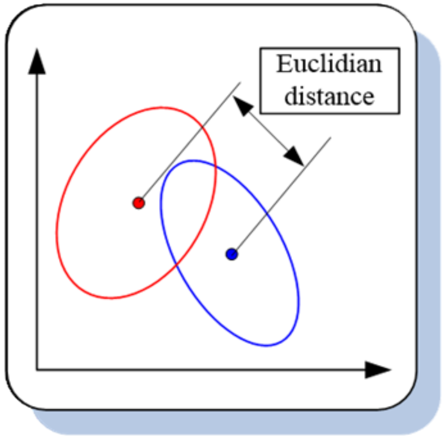

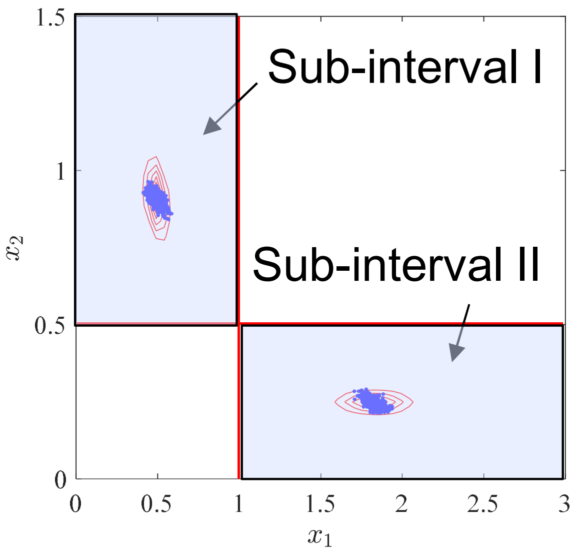

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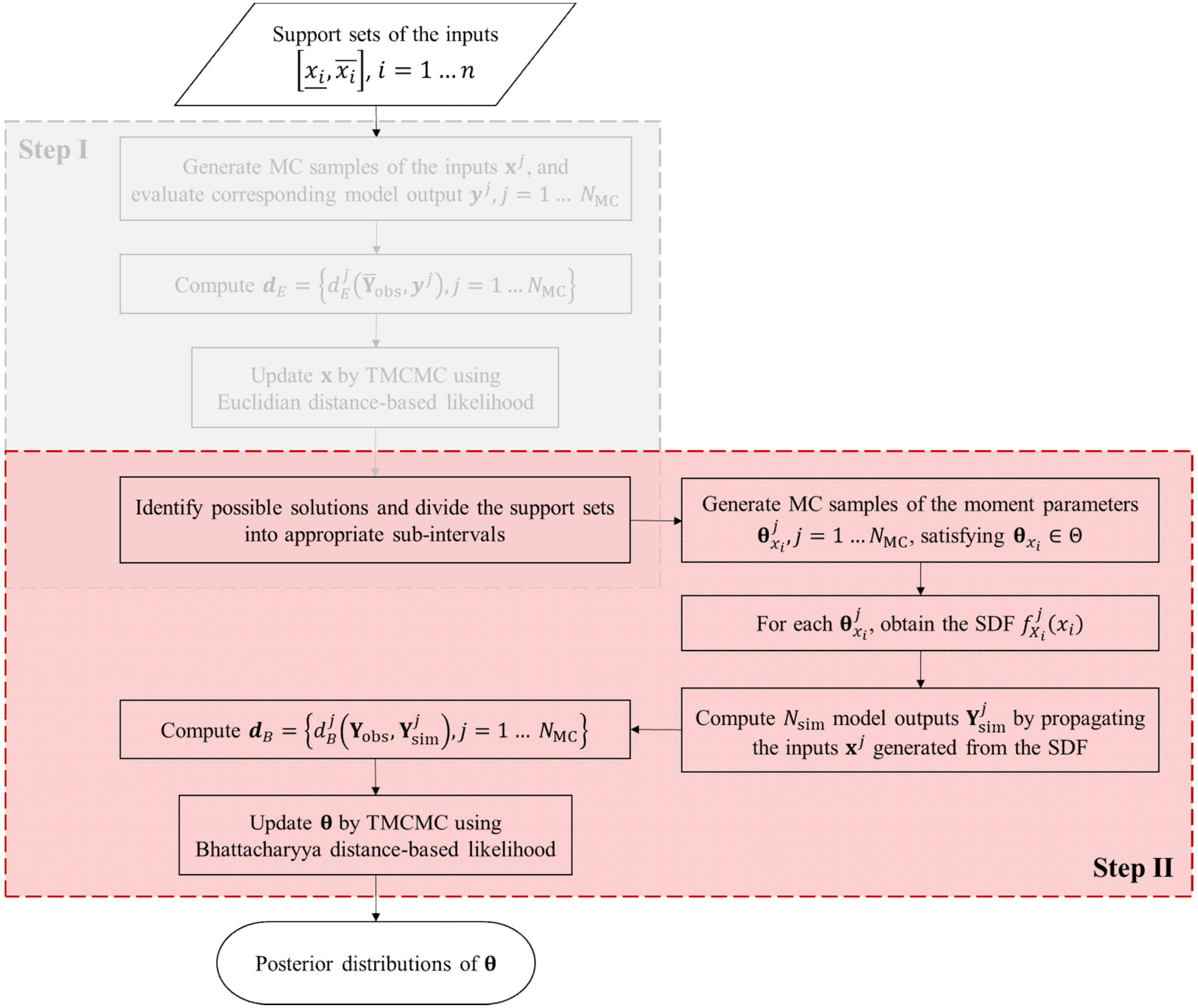
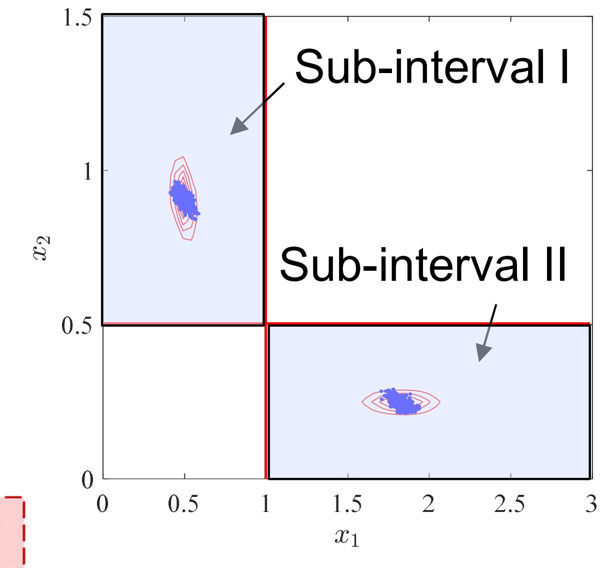
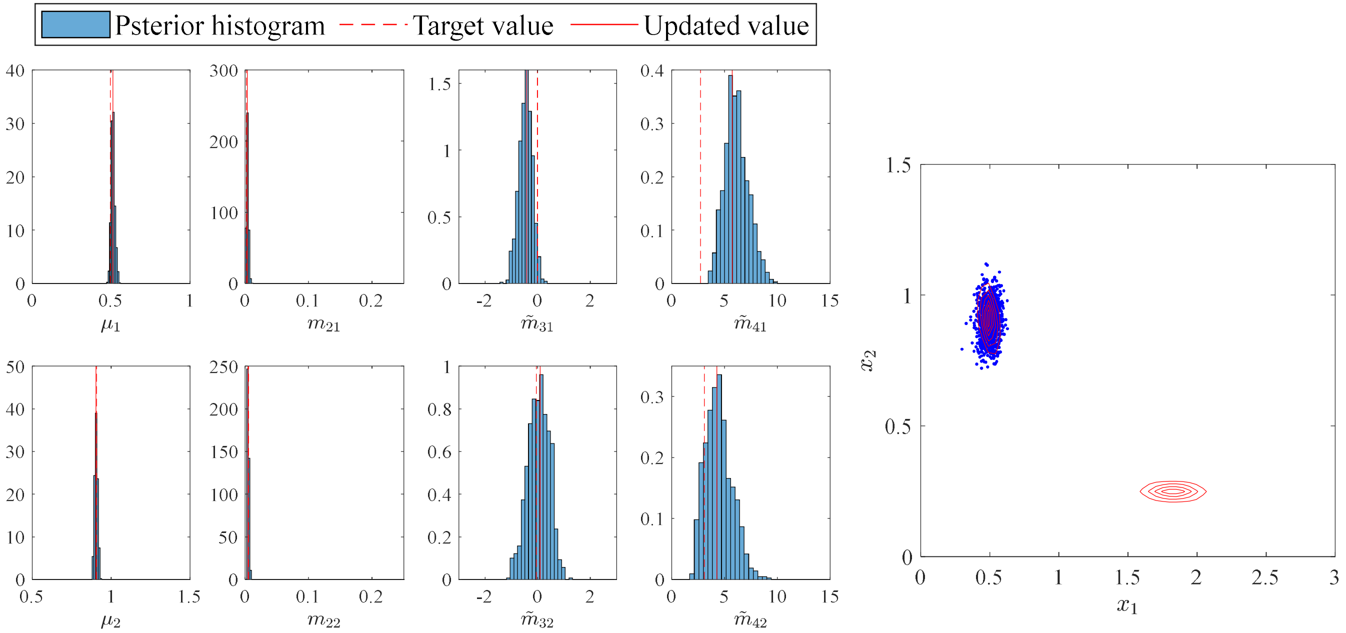
Two-step deterministic/stochastic approach





2-DOF Shear Building Example
Results – main step



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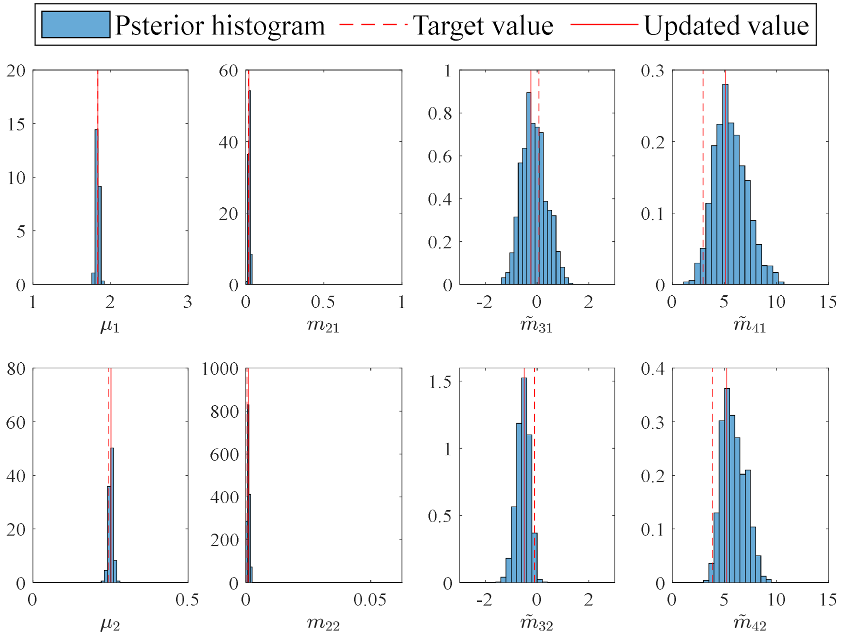
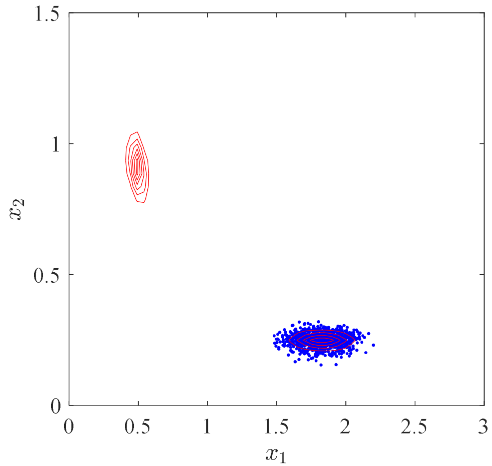
Results – main step





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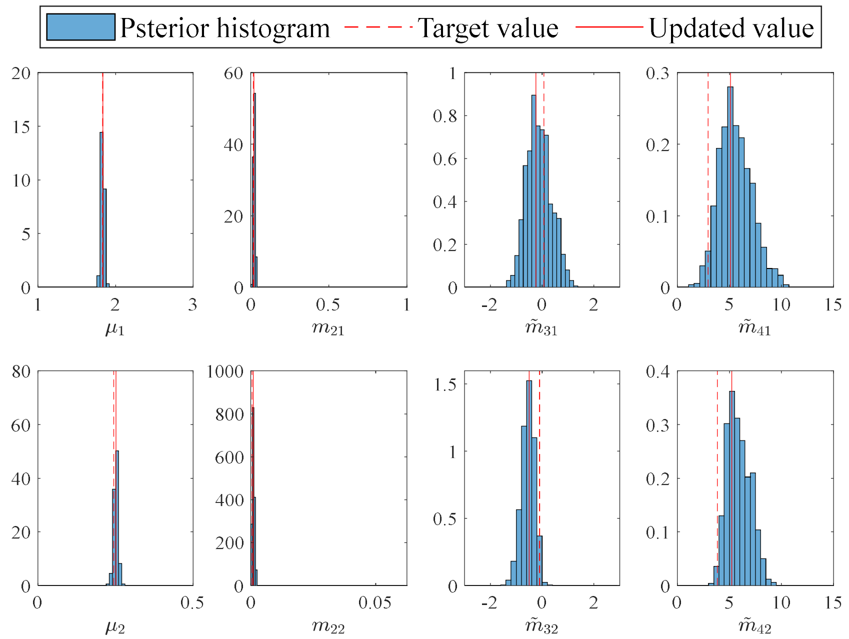

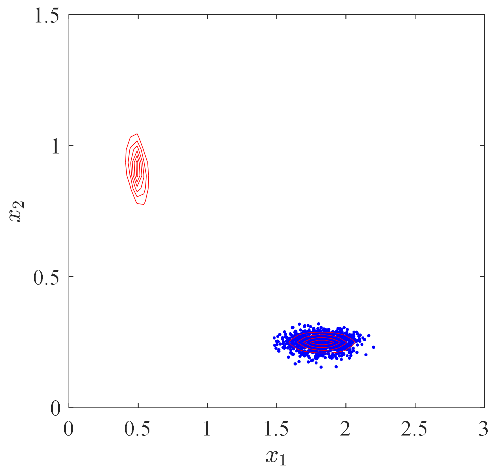

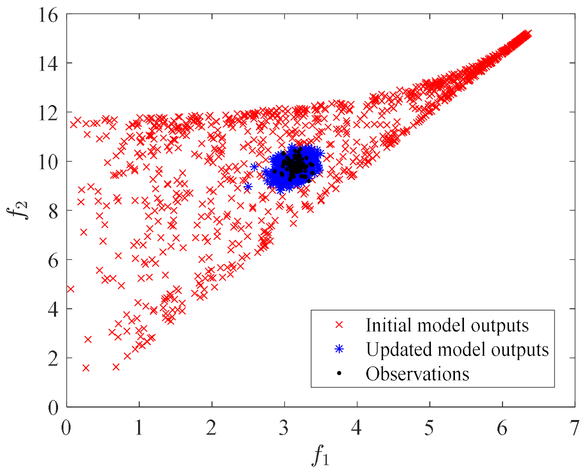

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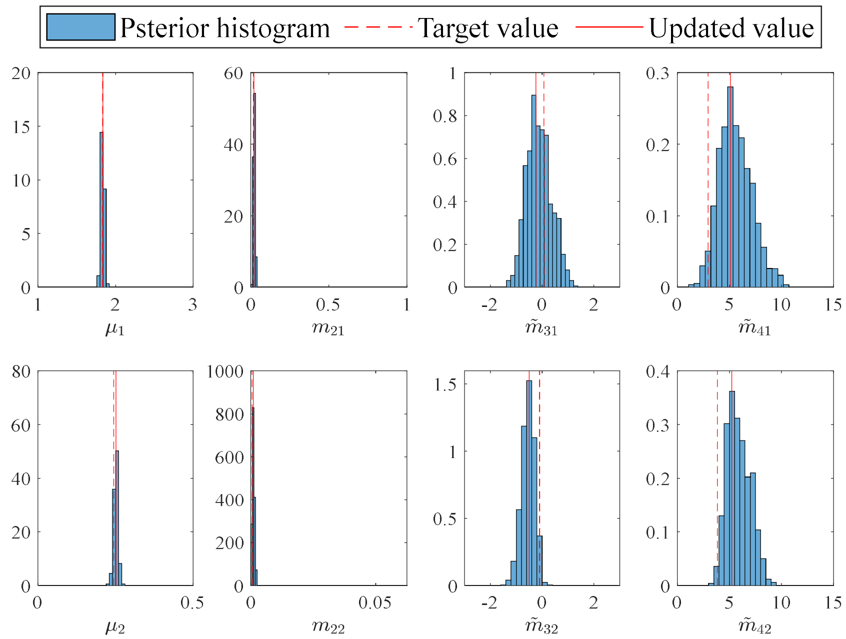
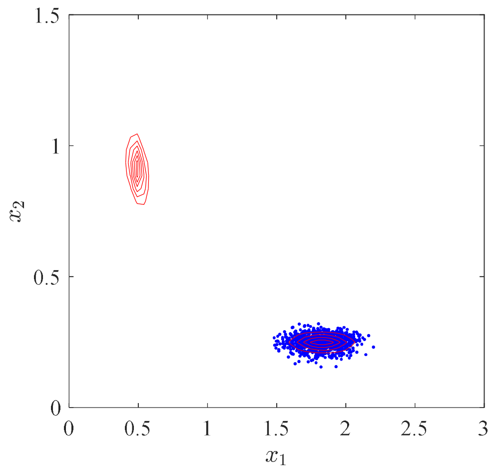
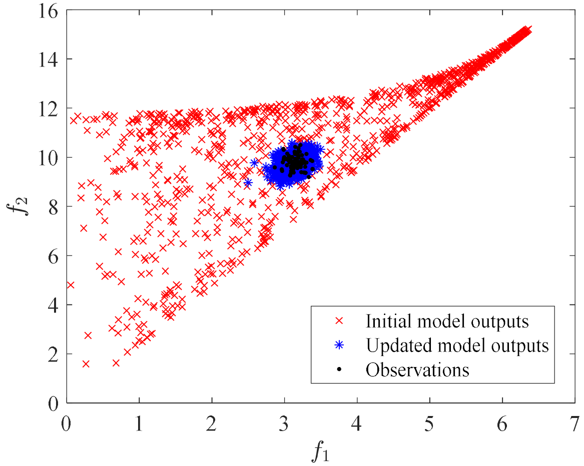


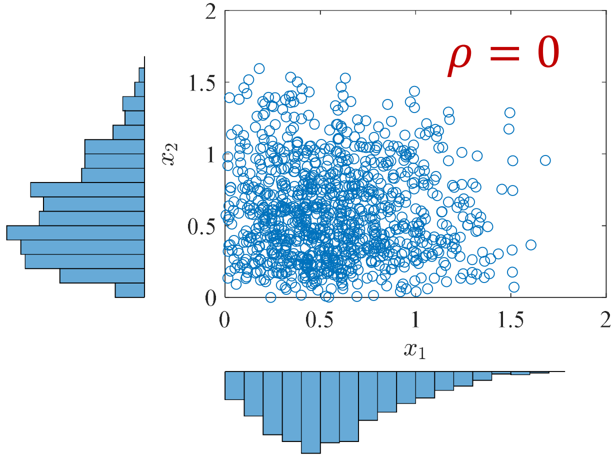


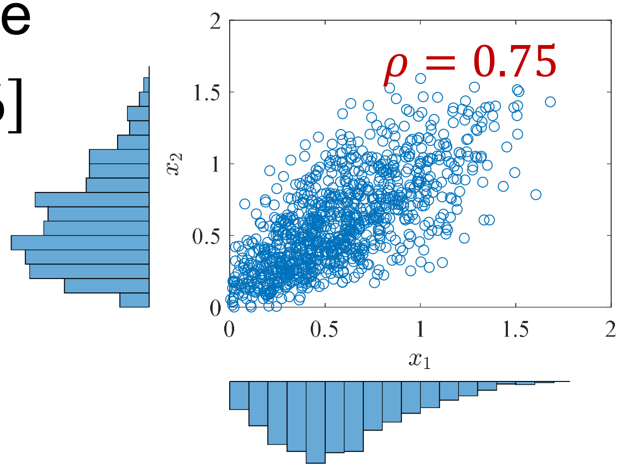


- Proposed updating framework combining the Bhattacharyya distance and staircase density functions is capable of calibrating aleatory inputs without any assumptions on their distribution families.
- Non-unique solutions can be avoid by the two-step Euclidian/Bhattacharyya distances-based updating procedure.

Gaussian Copula Function

- Joint CDF can be uniquely given as: $F_X(\boldsymbol{x}) = C\left(F_{X_1}(x_1), F_{X_2}(x_2)\right)$
  - C – copula function
  - $F_{X_i}(x_i)$ – marginal distribution
- Gaussian copula
  - Only needs the correlation matrix to determine the correlation structure
  - Easily generalized to the multi-variate case
  - e.g., $\Omega_x = [0, 2]$, $\boldsymbol{\theta} = [0.57, 0.10, 0.59, 2.86]$
- Bayesian updating formulation
  - Prior PDF: $\rho \in [-1, 1]$
  - Constraint condition: $\mathcal{P} = \{\boldsymbol{\rho}: \mathrm{chol}(\boldsymbol{\rho}) \neq \emptyset\}$





2-DOF Shear Building Example
Problem setting

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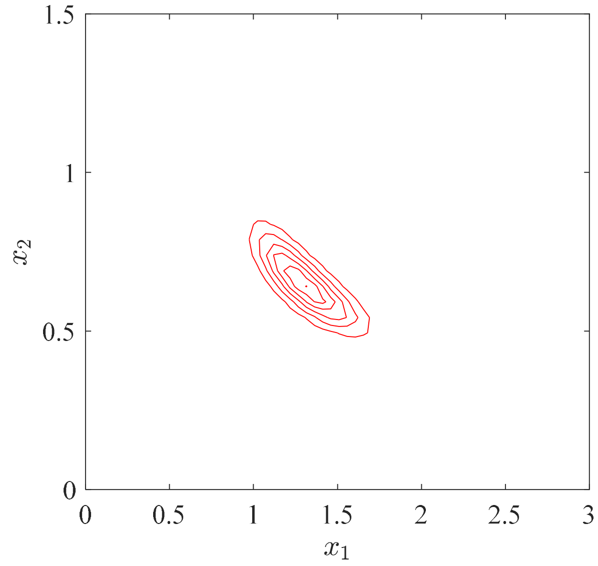
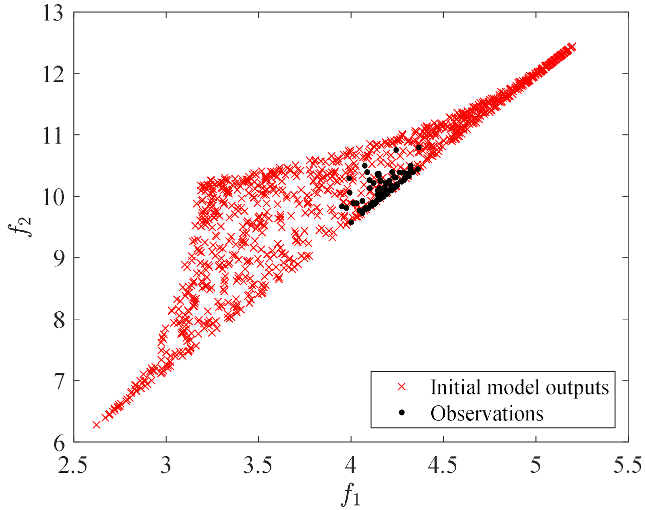
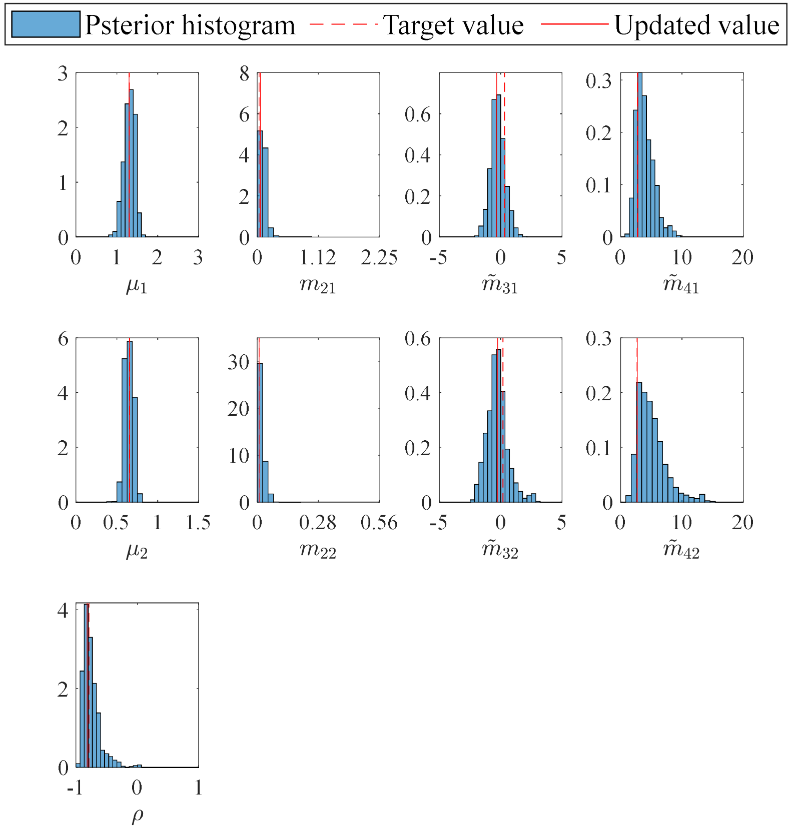




2-DOF Shear Building Example
Results



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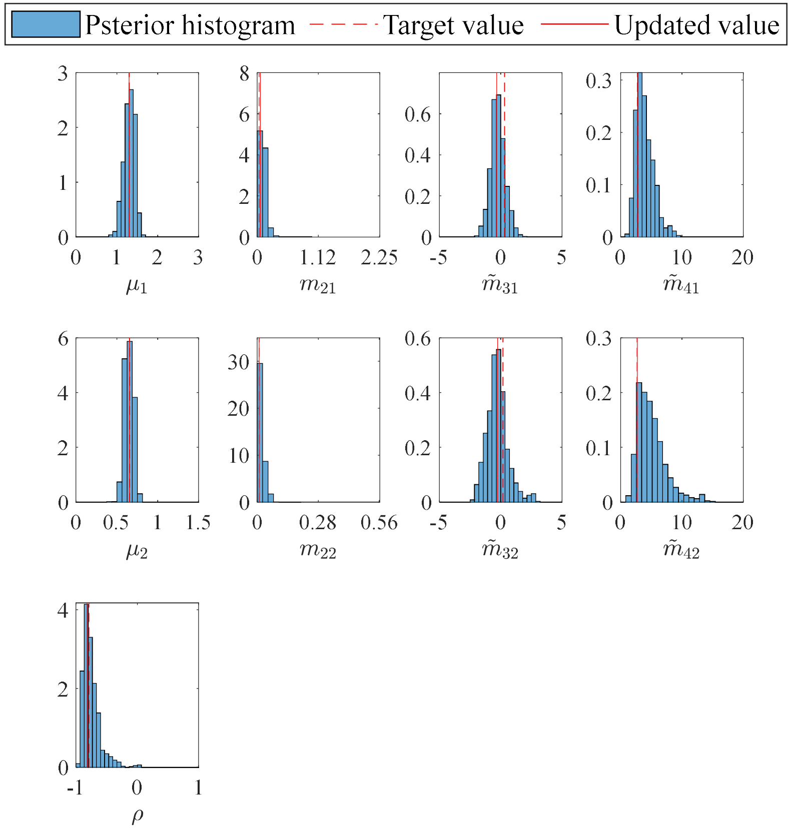

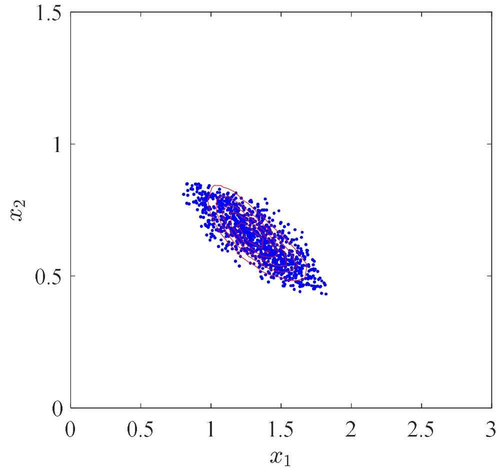

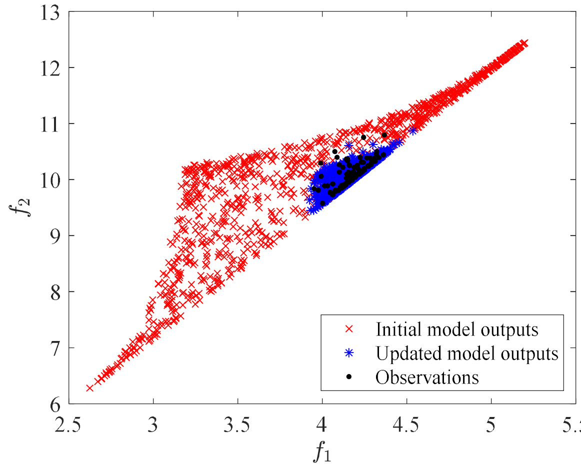

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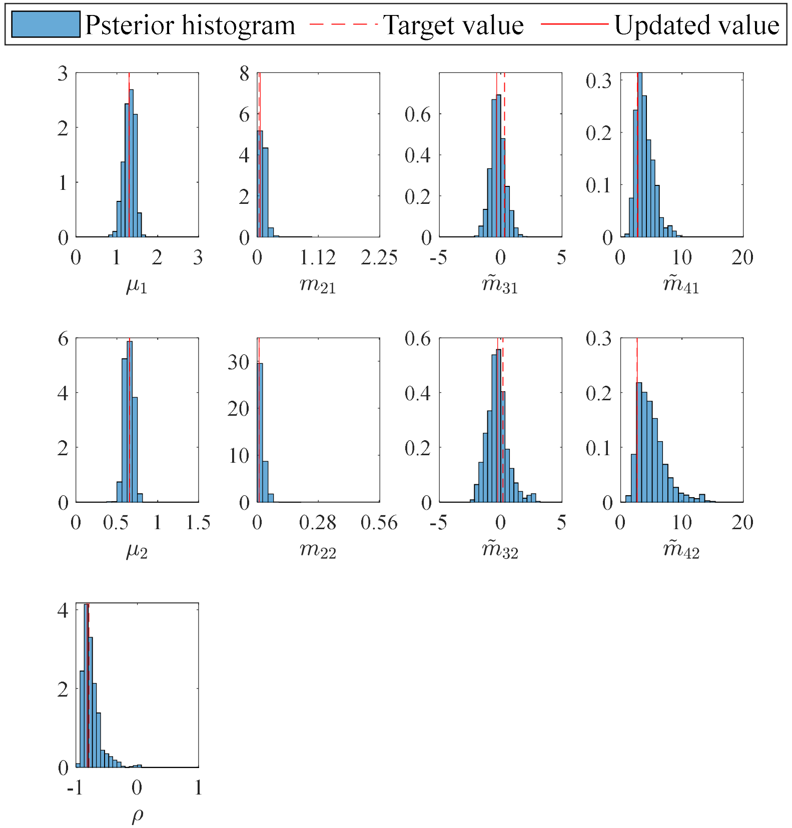

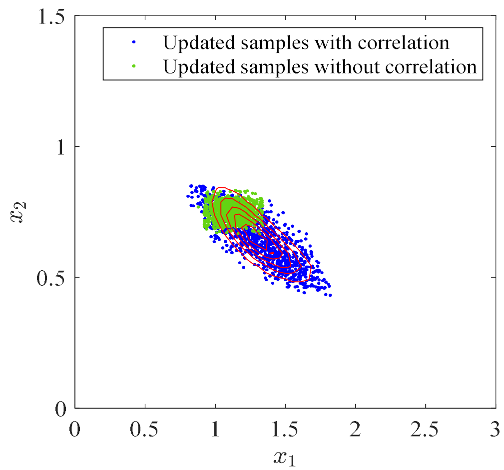

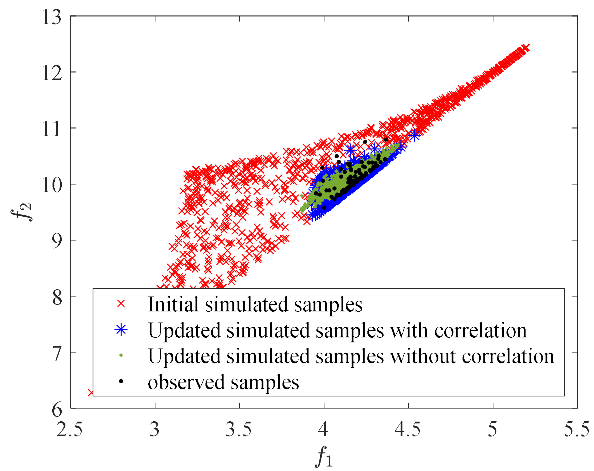

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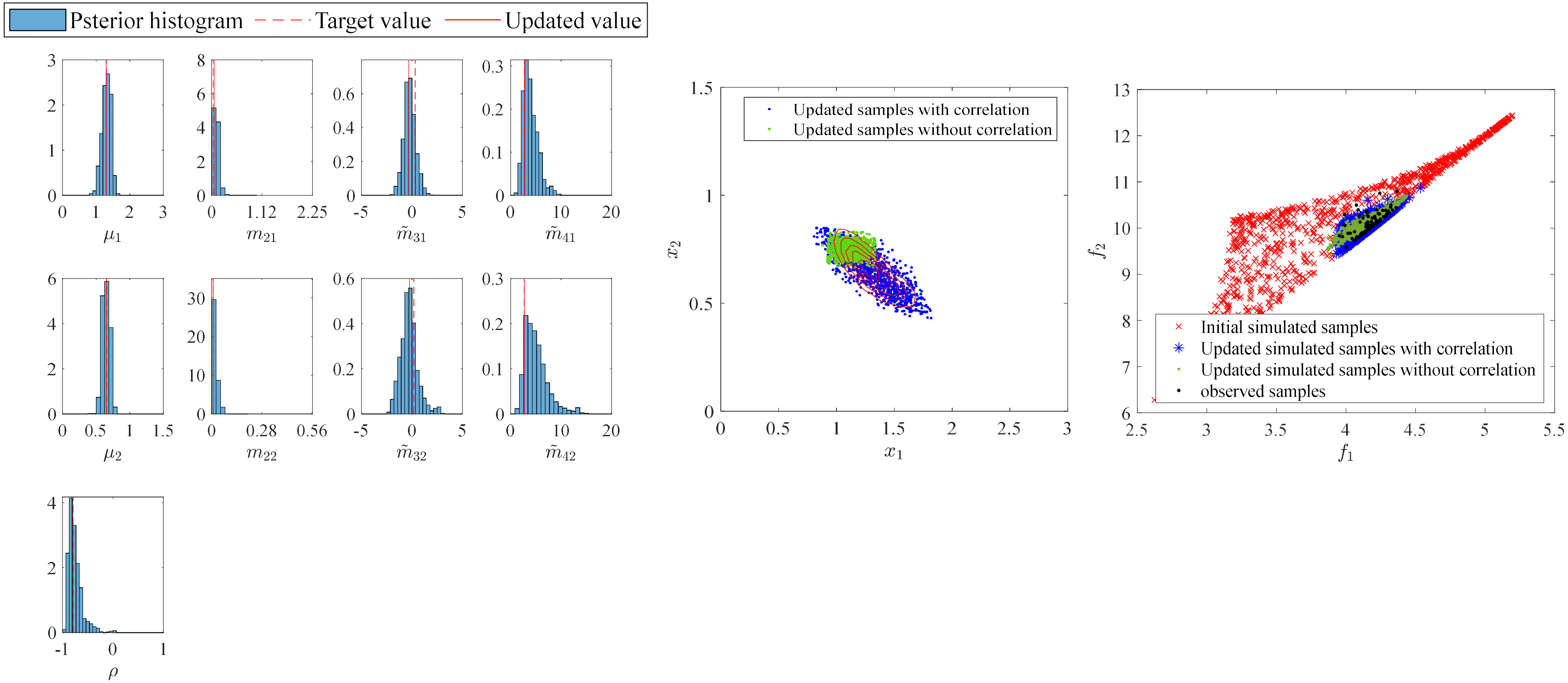

- Proposed updating framework combining the Gaussian copula function and staircase density functions is capable of calibrating the joint distribution of correlated aleatory inputs.

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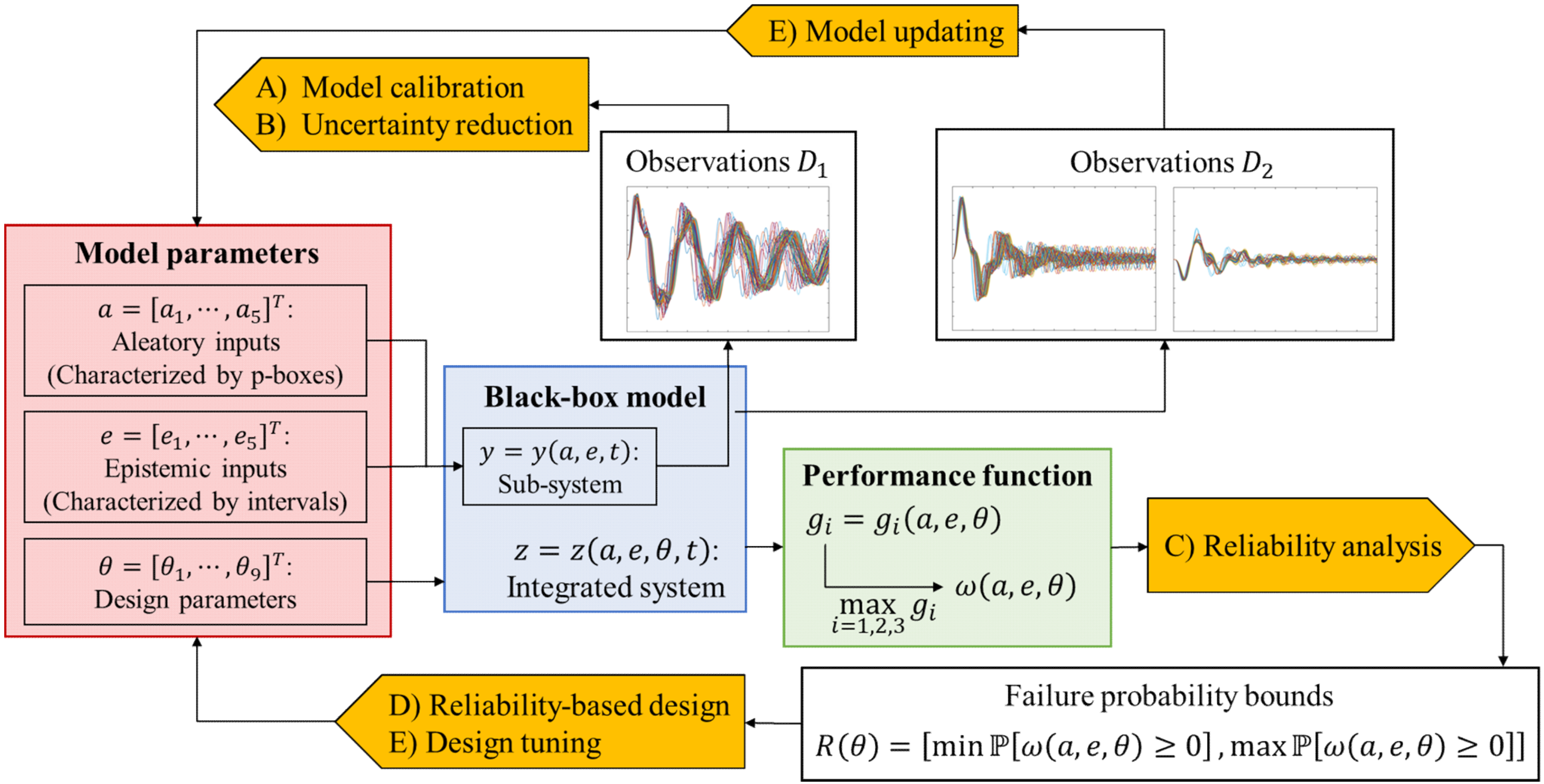
NASA UQ challenge 2019
Problem statement



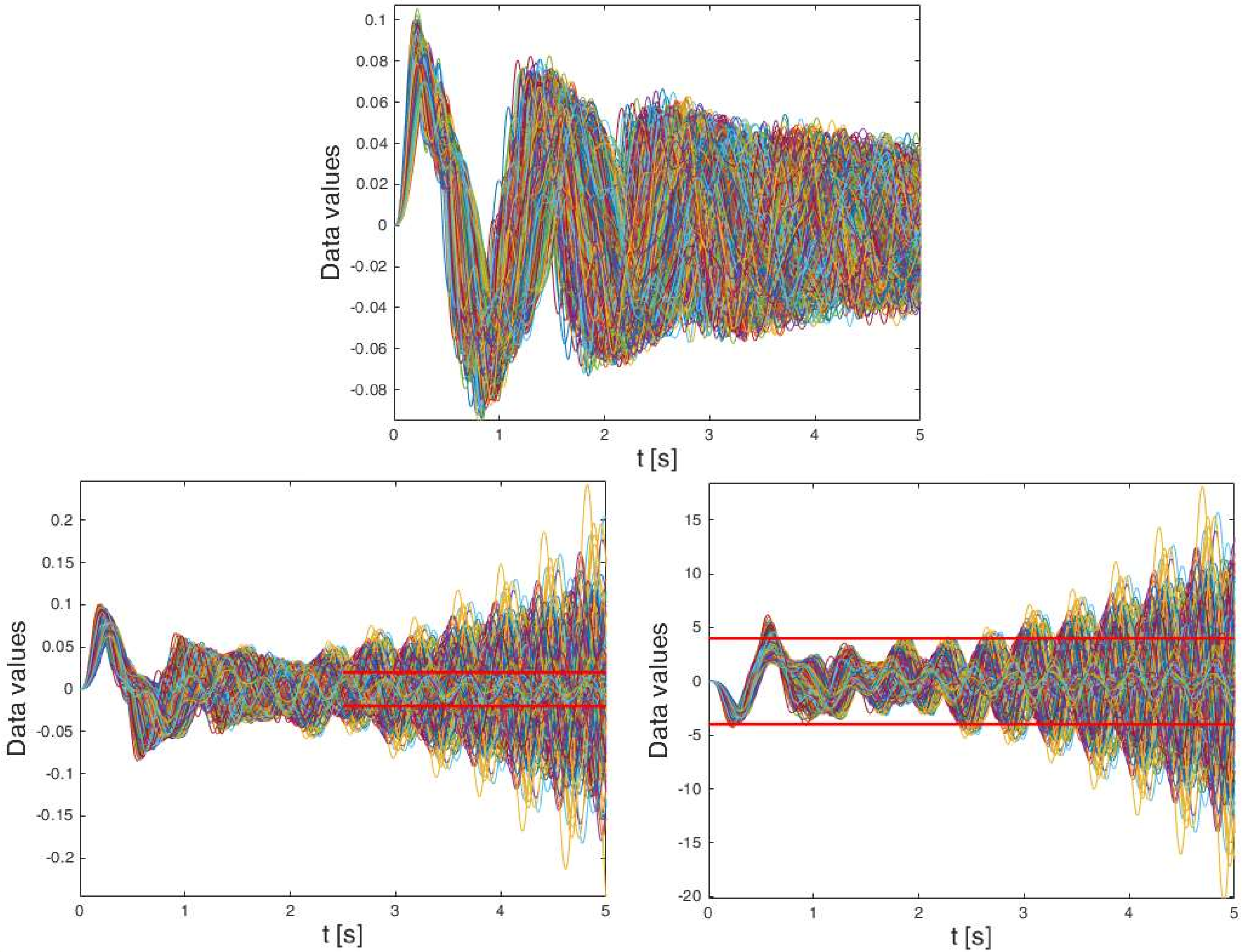
NASA UQ challenge 2019
Model updating subproblems

- 5 aleatory inputs $a \sim f_a$
  - Distribution family is unknown a priori
  - Support domain is given: $a \in [0,2]^5$
- 4 epistemic inputs $e \sim E$
  - Support domain is given: $e \in [0,2]^4$
- 100 sets of observations $y = y(a,e,t)$, $z = z(a,e,\theta,t)$

Our solution:

- f_a is assumed to be staircase density functions.
- Totally $4 + 5 \times 4 = 24$ epistemic parameters are updated by the Bhattacharyya distance-based ABC, comparing the time series through a moving window procedure.

Model Calibration (Subproblem A)
Prior model outputs



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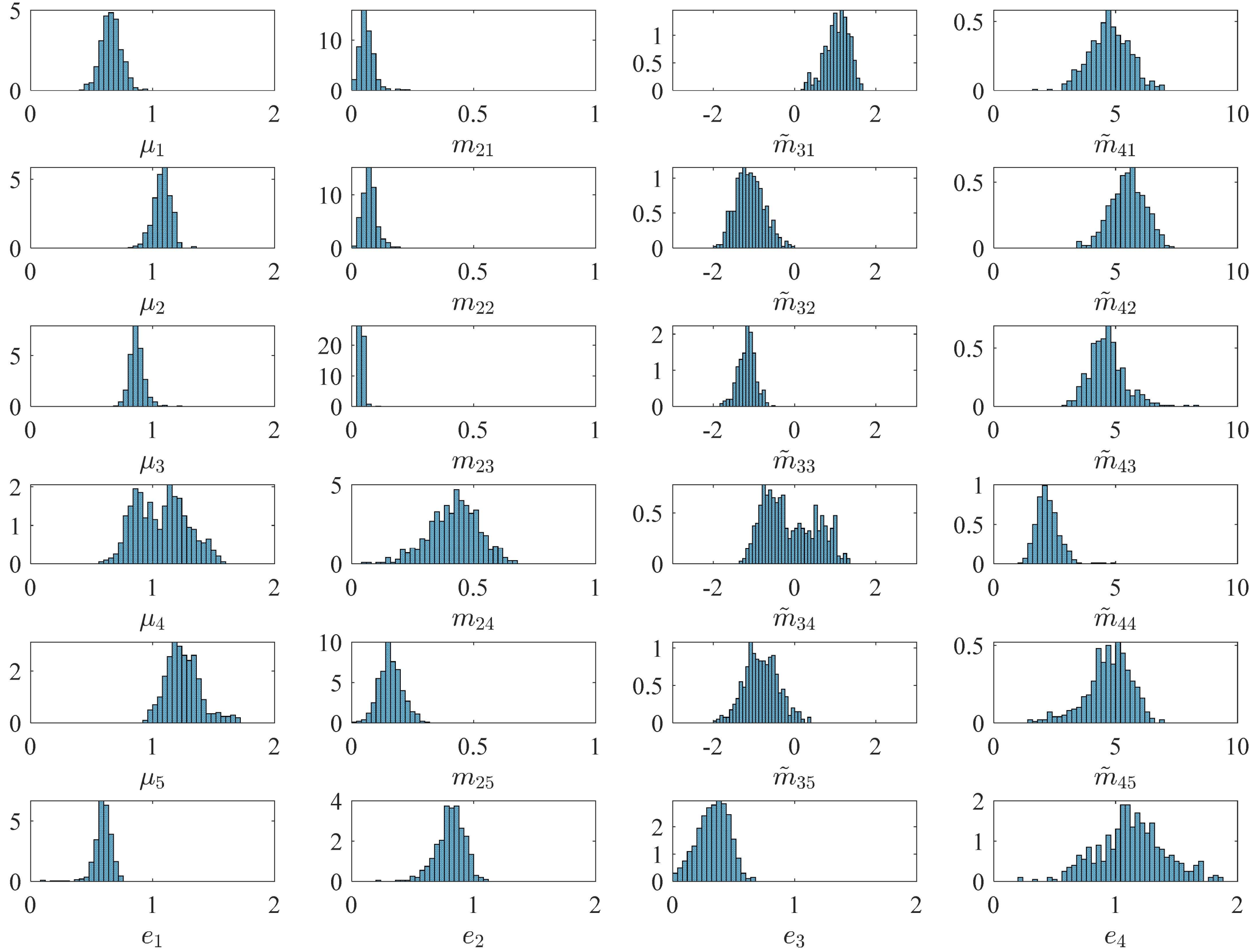
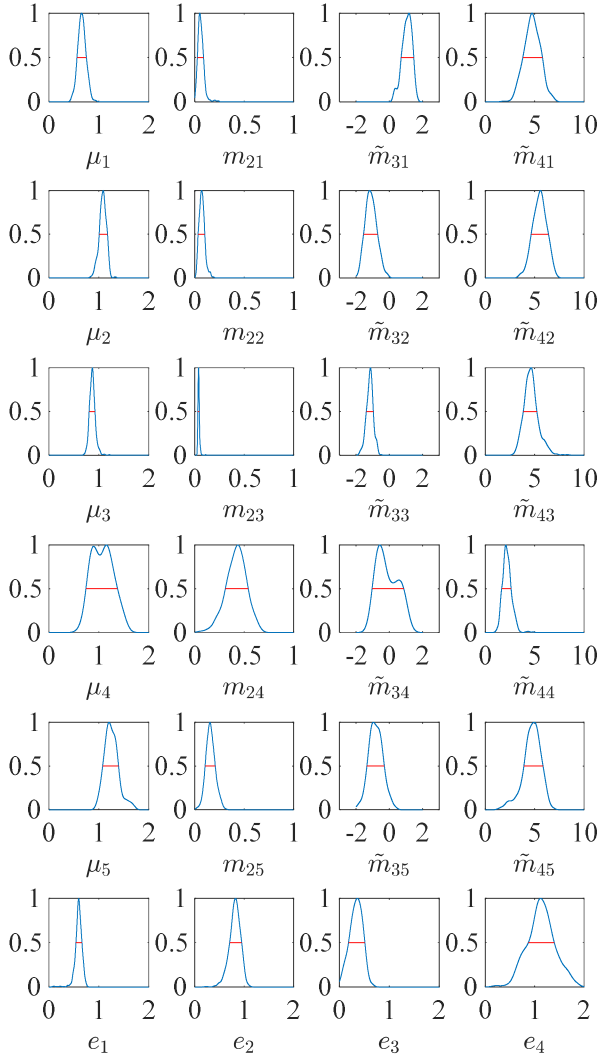
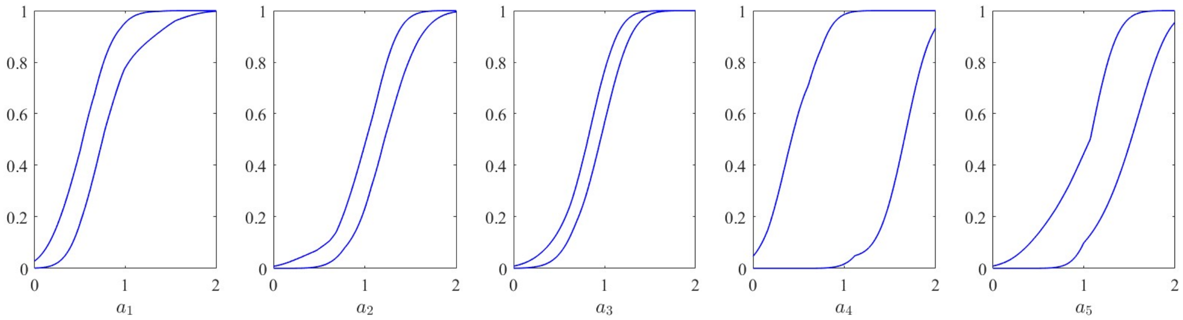
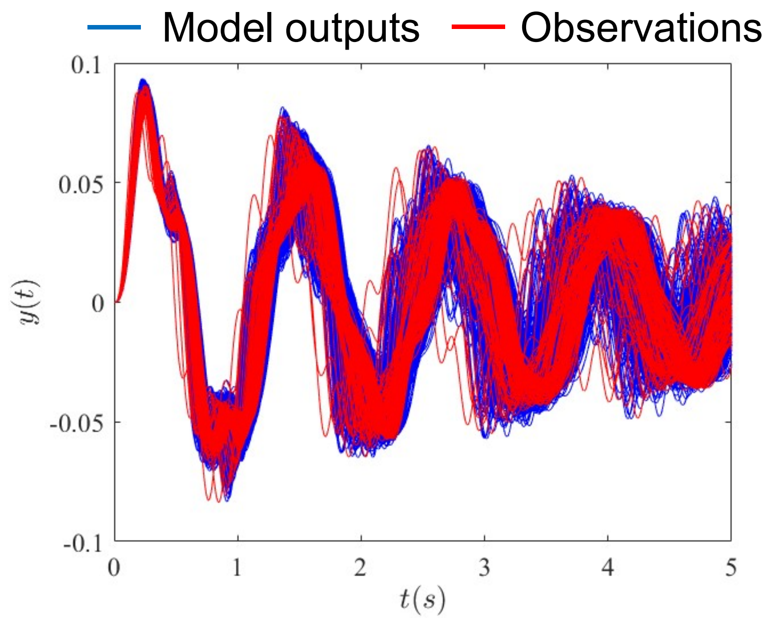
Model Calibration (Subproblem A)
Posterior distributions



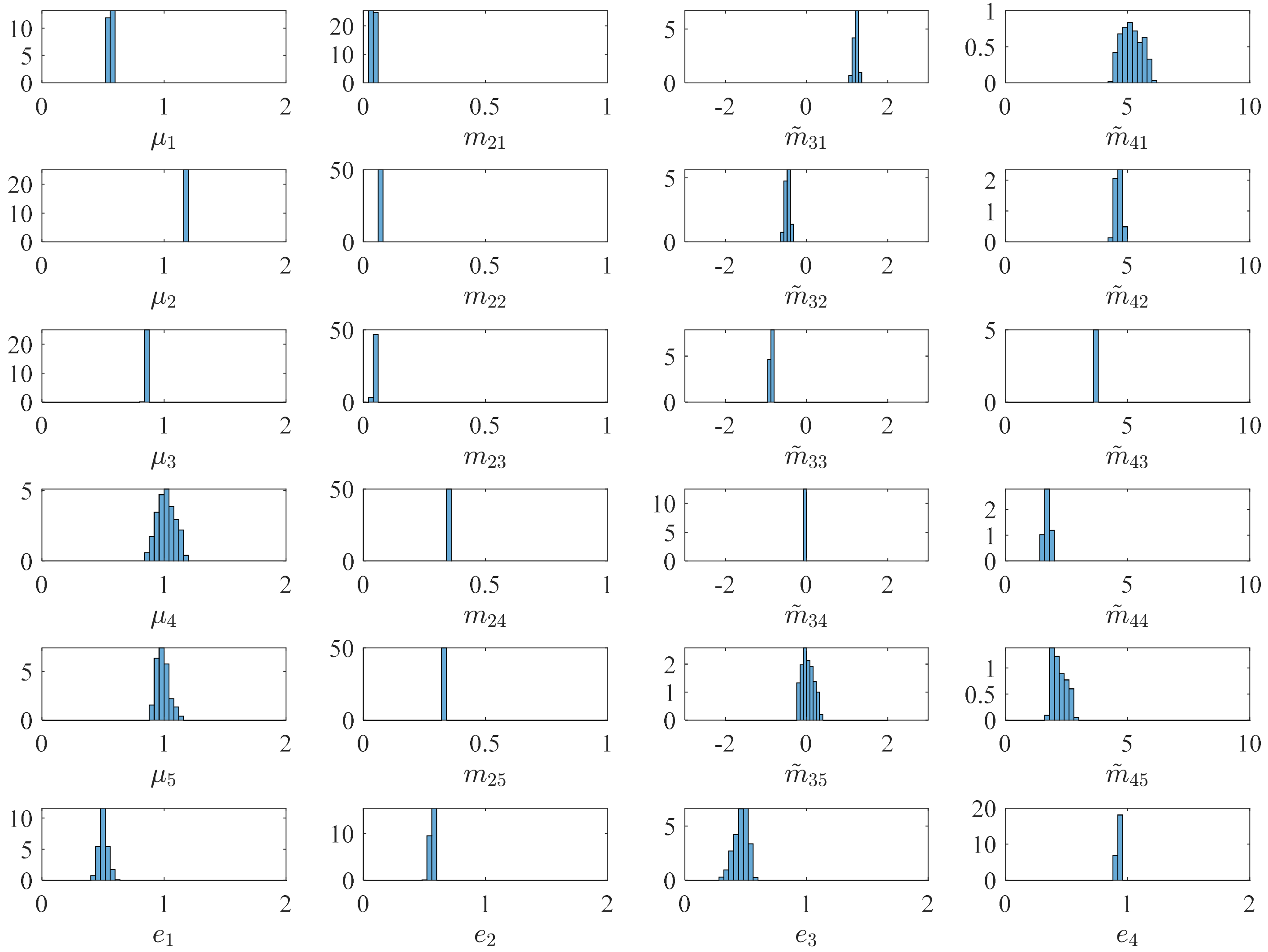
Model Calibration (Subproblem A)
Calibrated inputs/outputs



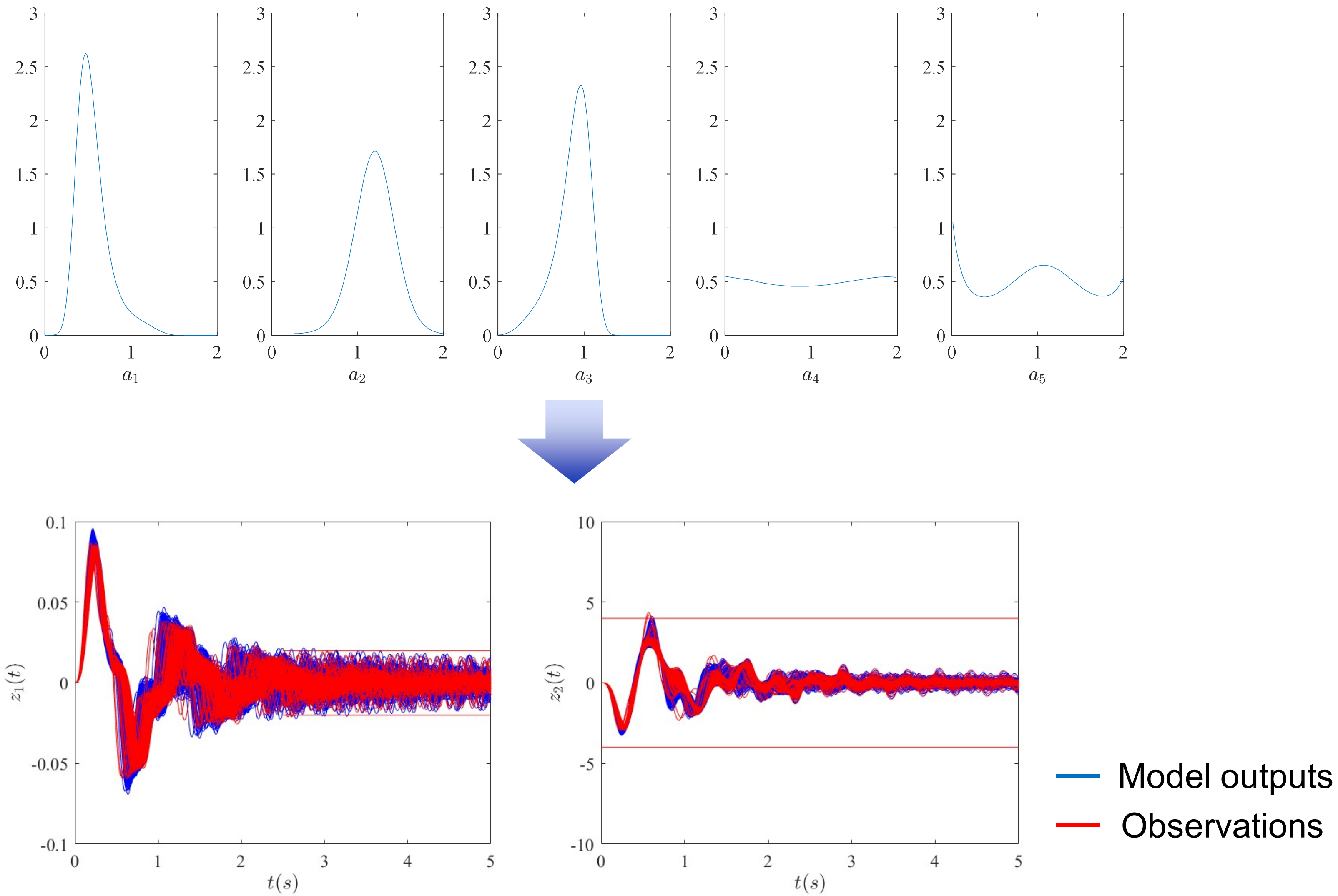




Model Updating (Subproblem E)
Posterior distributions



Model Updating (Subproblem E)
Calibrated inputs/outputs



Conclusions

- TMCMC provides high flexibility for model updating of complex problems
- Multiple likelihood can capture different level of information from the experiments
- Classic MU likelihood assumptions cannot be used with hybrid uncertainties
- Stochastic metric + ABC provide numerically efficient methods
- Staircase distributions frees from the assumption of a distribution family
- Epistemic and aleatory uncertainties must be treated separately

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