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Interval Impulsive Observers: A Framework for Robust Estimation with Aperiodic or Event-Triggered Sampling

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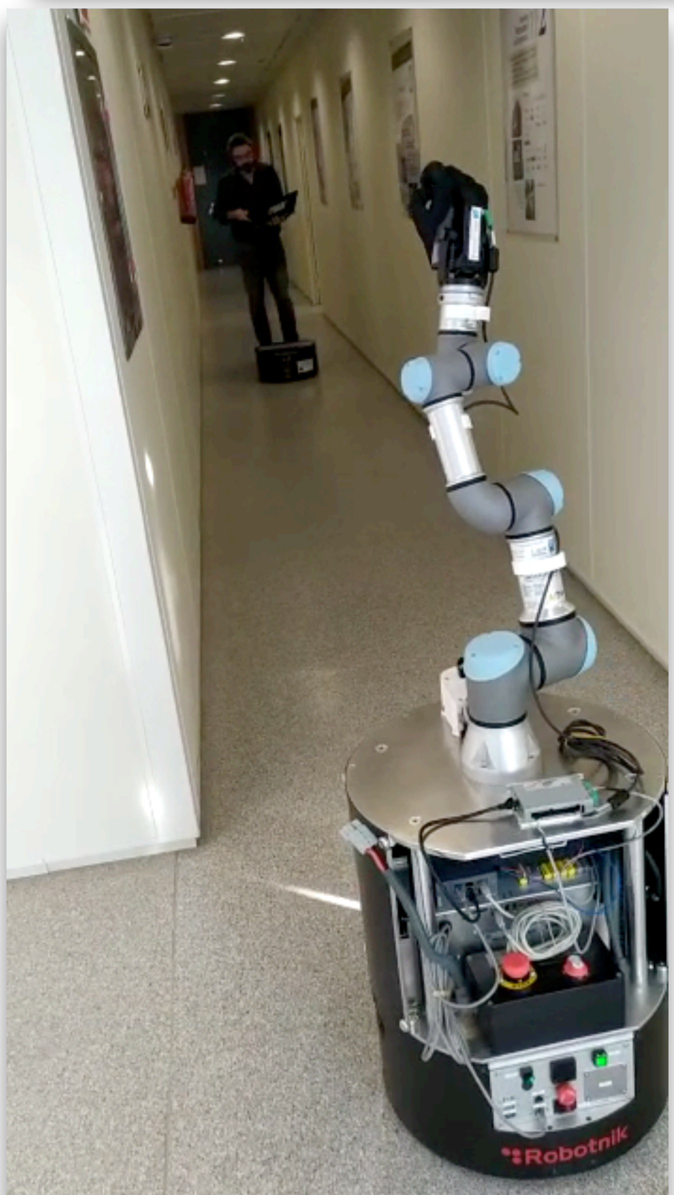
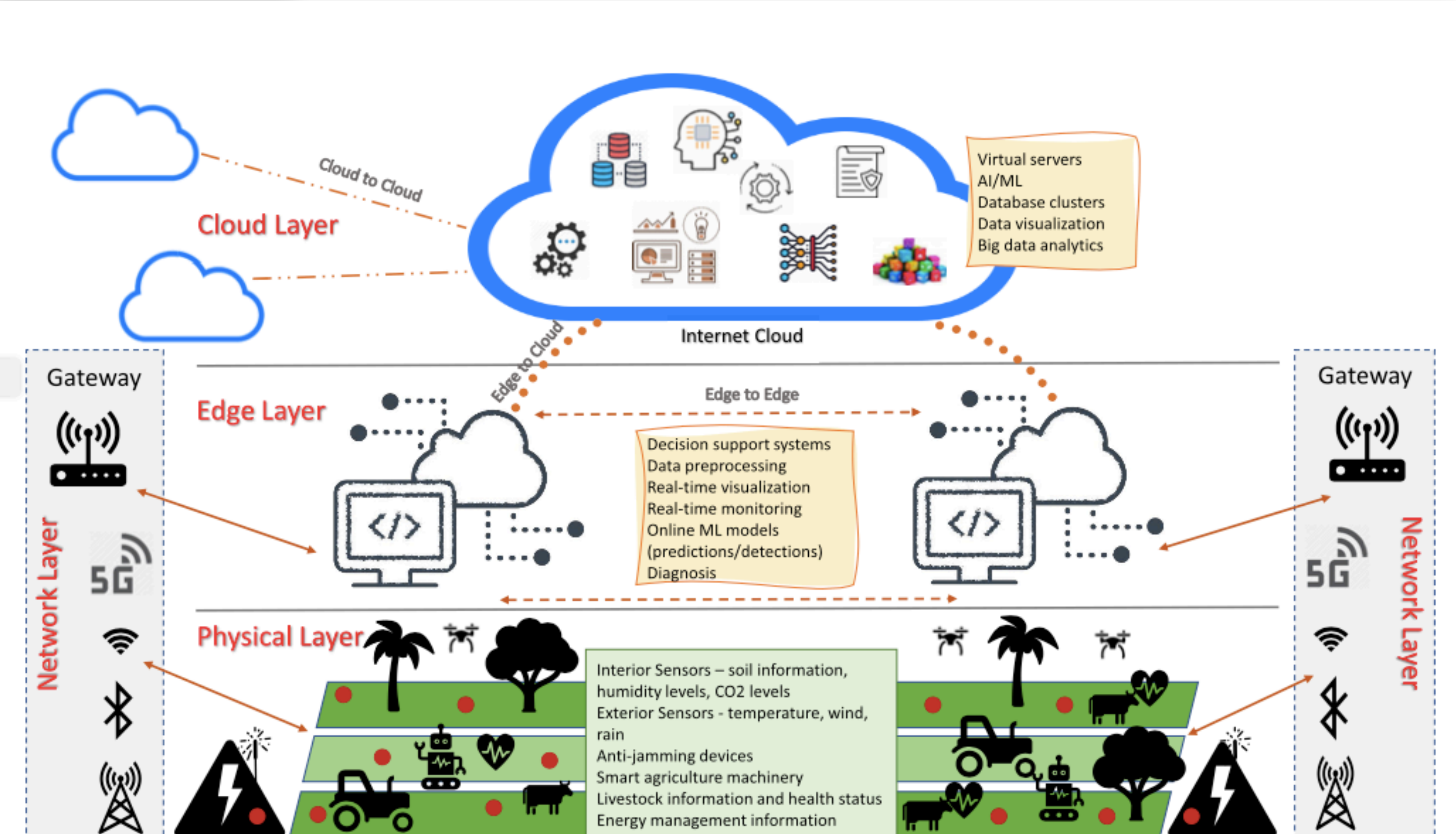
in collaboration with Djahid RABEHI and Nacim MESLEM

Cyber-Physical Systems

Critical Infrastructures
Network Control Systems
Distributed Systems

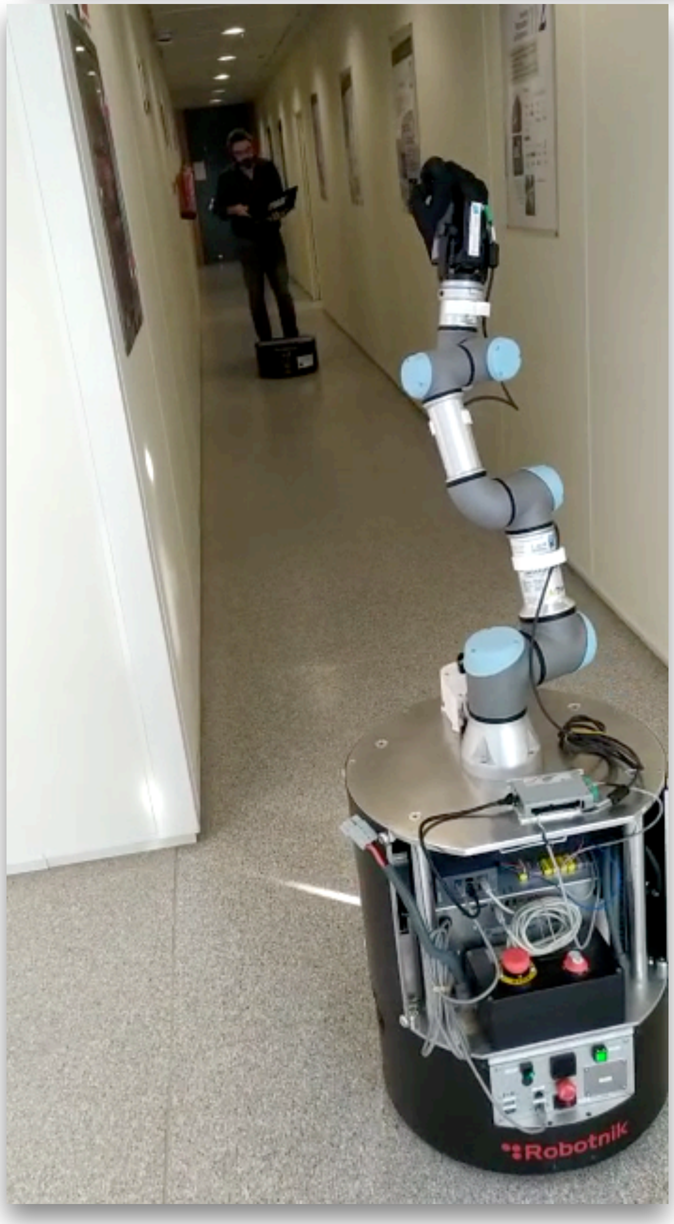


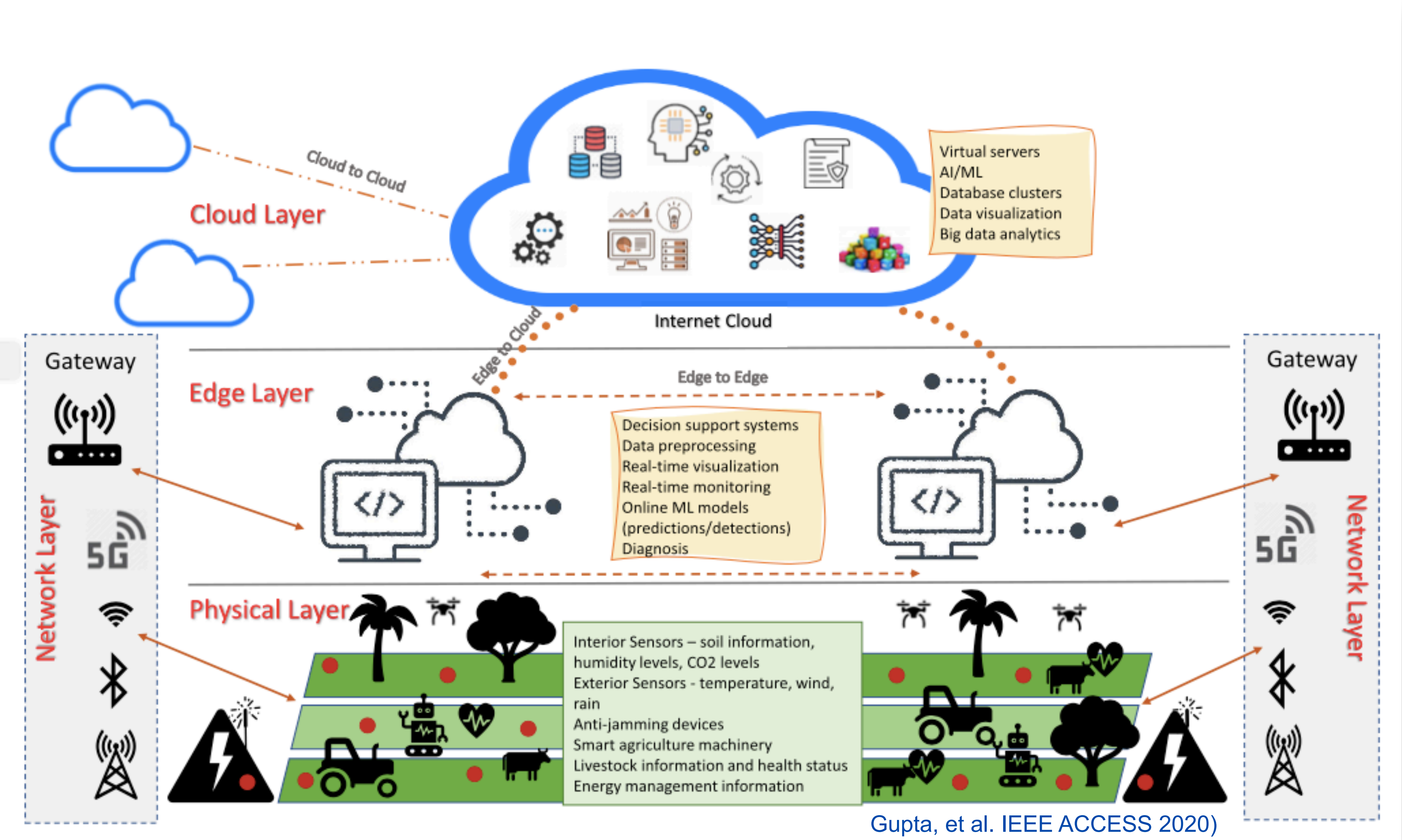


Gupta, et al. IEEE ACCESS 2020)

Cyber-Physical Systems

Critical Infrastructures
Network Control Systems
Distributed Systems

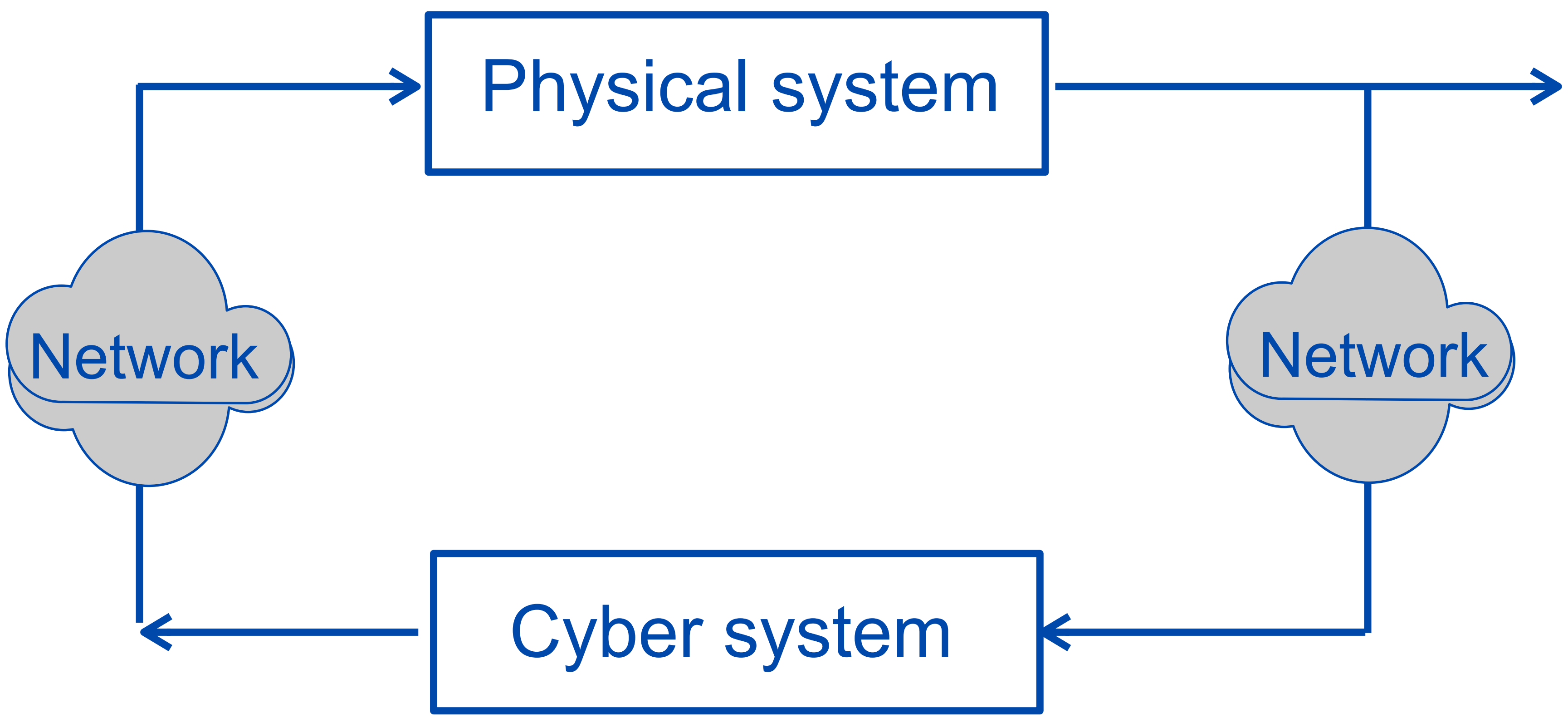




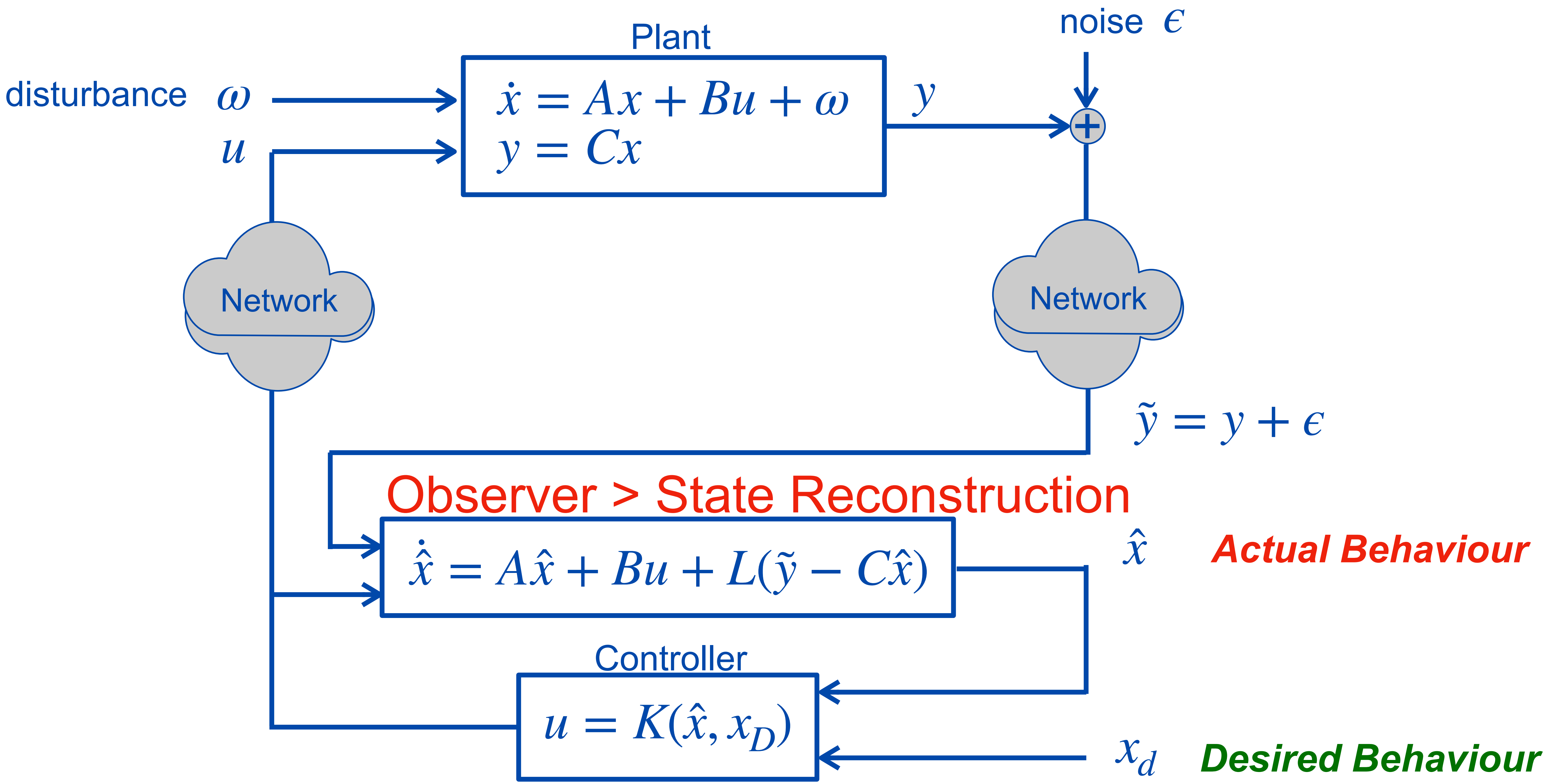
Gupta, et al. IEEE ACCESS 2020)

Cyber-Physical Systems

Networked controlled systems

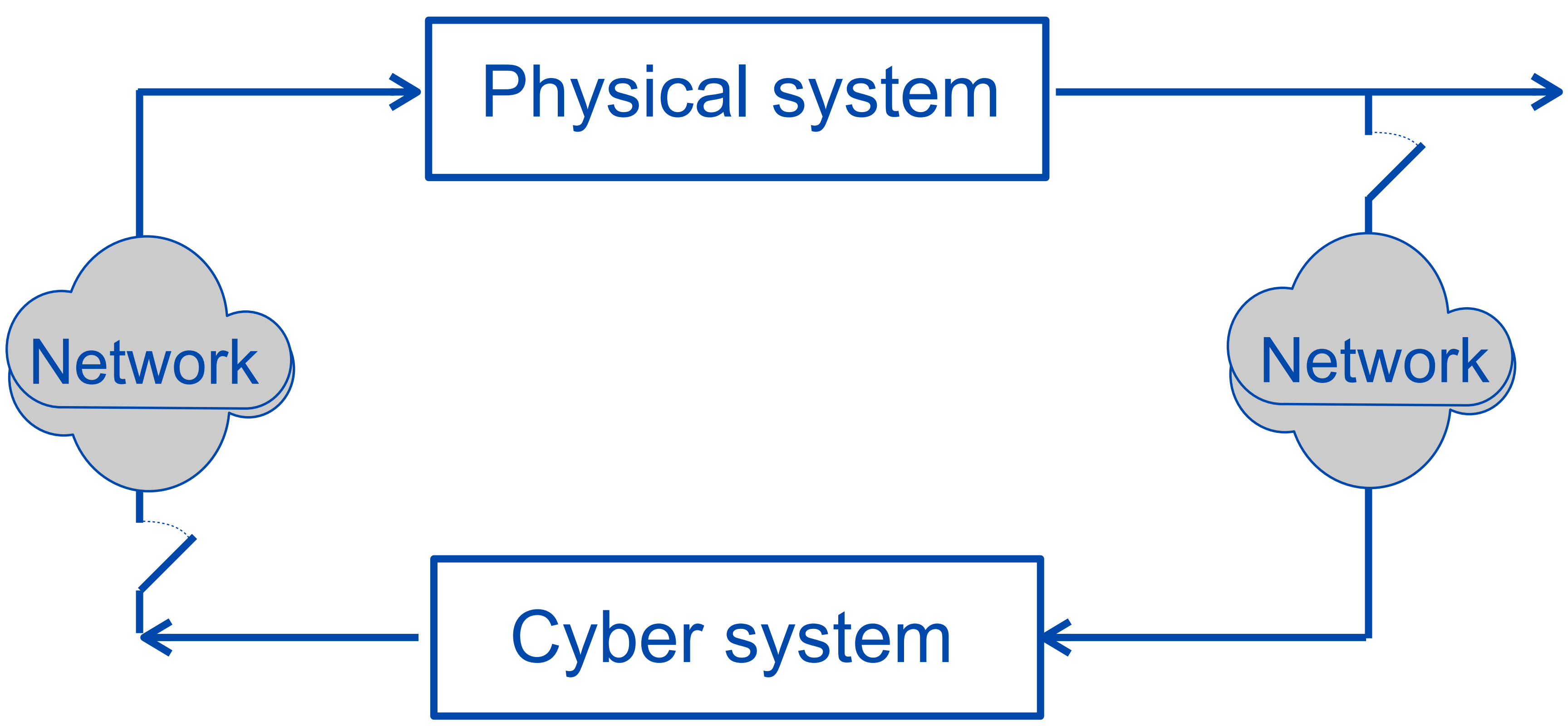


Networked Feedback Control



Cyber-Physical Systems

- **Networked controlled systems**
  - **Jitter, packet loss, …** > **Aperiodic sampling**
  - **Reduce network usage** > **Controlled sampling**



Problem Statement

$$t \in \mathbb{R}, \quad \dot{x}(t) = Ax(t) + Bu(t) + \omega(t)$$

$$k \in \mathbb{N}, \quad y(t_k) = Cx(t_k) + \epsilon(t_k)$$

Aperiodic sampling: $t_{k+1} - t_k \in [\tau_{min}, \tau_{max}]$

Controlled sampling: $t_{k+1} - t_k = \min\{\tau \mid \phi(t_k, \tau) \leq 0\}$

Talk Objective & Outline

- Outline
  - Robust state estimation
  - Interval Impulsive Observers
  - Robust state estimation with event-triggered sampling
  - Robust state estimation with sporadic measurements

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- **Robust state estimation**

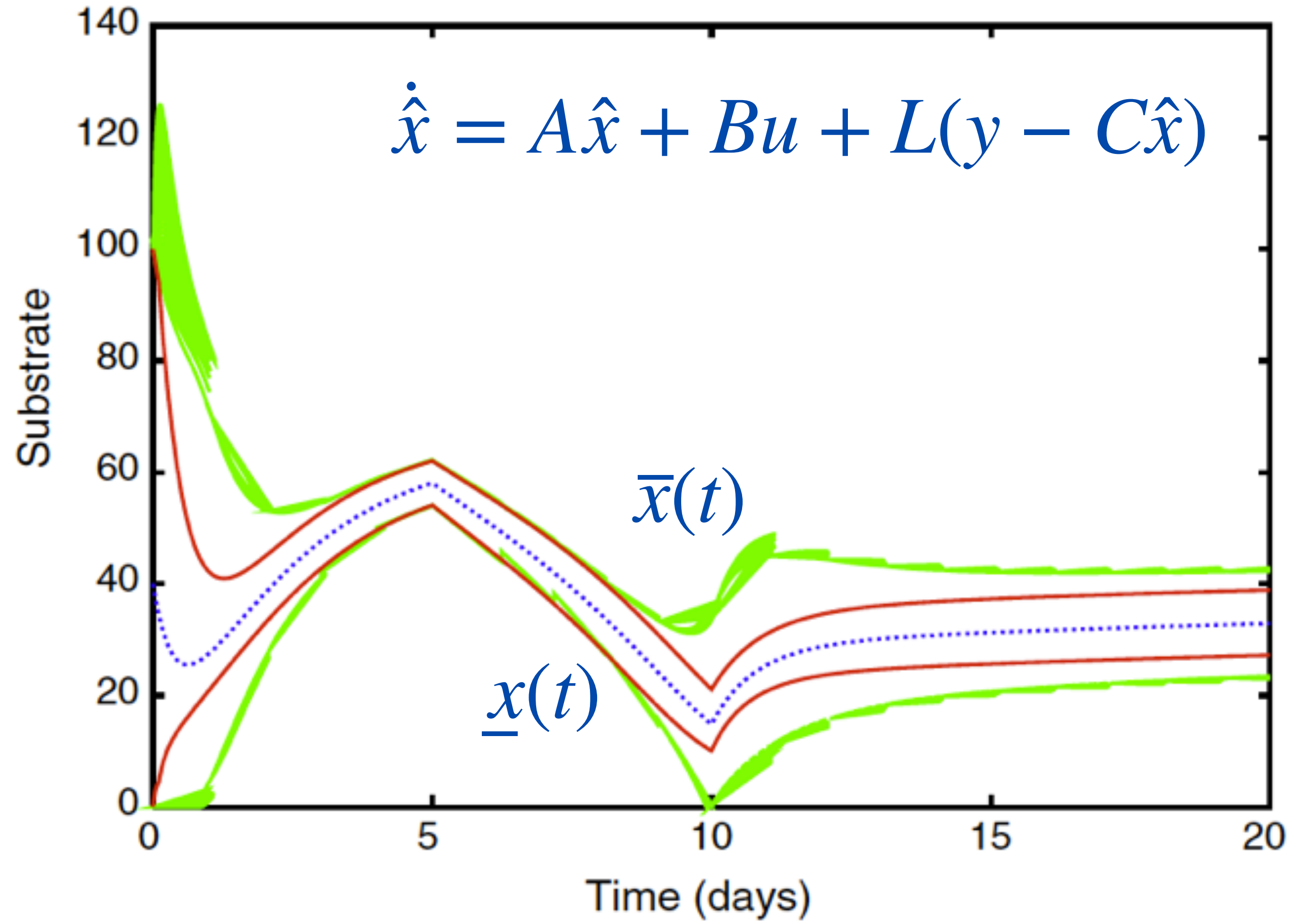
Robust State Estimation

- State estimation the bounded-error framework
  - **Interval observers**
  - **Set membership predictor-corrector algorithms**

Interval Observers

- Interval estimation with **continuous-time data**
  - **Luenberger-like observers**: (Gouzé et al, 00),(Mazenc & Bernard, 10), (Meslem & Ramdani, 11), (Raïssi, et al., 12) …
  - Tune observer **gain** to ensure **Input-to-State Stability (practical stability)**
  - Build **framers** $\underline{x}(t)$ and $\overline{x}(t)$

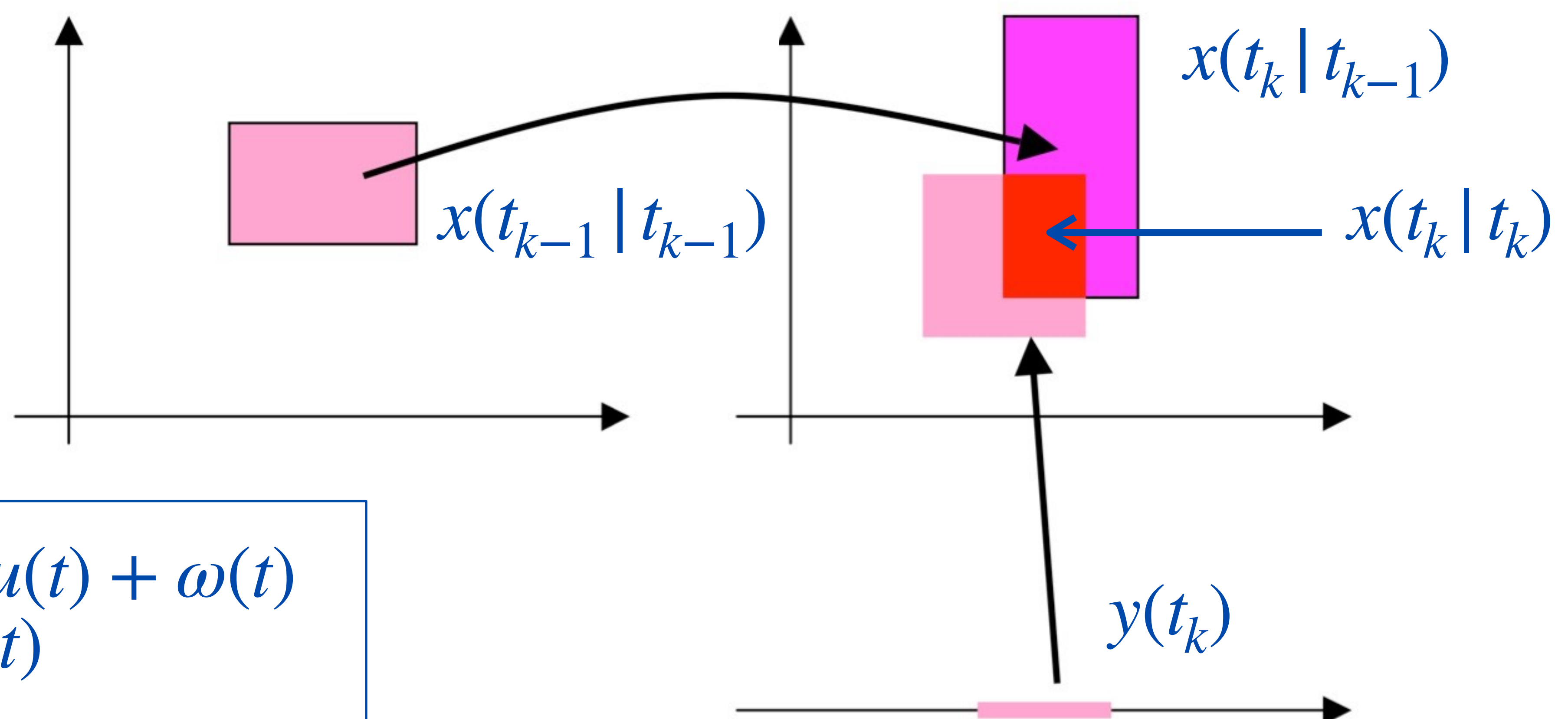
    $\underline{x}(t) \leq x(t) \leq \overline{x}(t)$



$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + \omega(t) \\ y(t) &= Cx(t) + \epsilon(t) \end{aligned}$$

Predictor-Corrector Algorithms

- Set membership estimation with **sampled data**
  - (Schweppe, 68) (Bertsekas & Rhodes, 71) (Kurzhanski & Vályi, 96), (Kieffer, et al., 02) (Jaulin, 02) (Raïssi et al., 04, 05) (Meslem, et al, 10), (Milanese & Novara, 11), (Kieffer & Walter, 11), (Combastel, 15) …
  - **Reachability** + **Set inversion +** Forward backward consistency



$$\dot{x}(t) = Ax(t) + Bu(t) + \omega(t)$$
$$y(t) = Cx(t) + \epsilon(t)$$

Predictor-Corrector Algorithms

- Set membership estimation with **sampled data**
  - (Schweppe, 68) (Bertsekas & Rhodes, 71) (Kurzhanski & Vályi, 96), (Kieffer, et al., 02) (Jaulin, 02) (Raïssi et al., 04, 05) (Meslem, et al, 10), (Milanese & Novara, 11), (Kieffer & Walter, 11), (Combastel, 15) …
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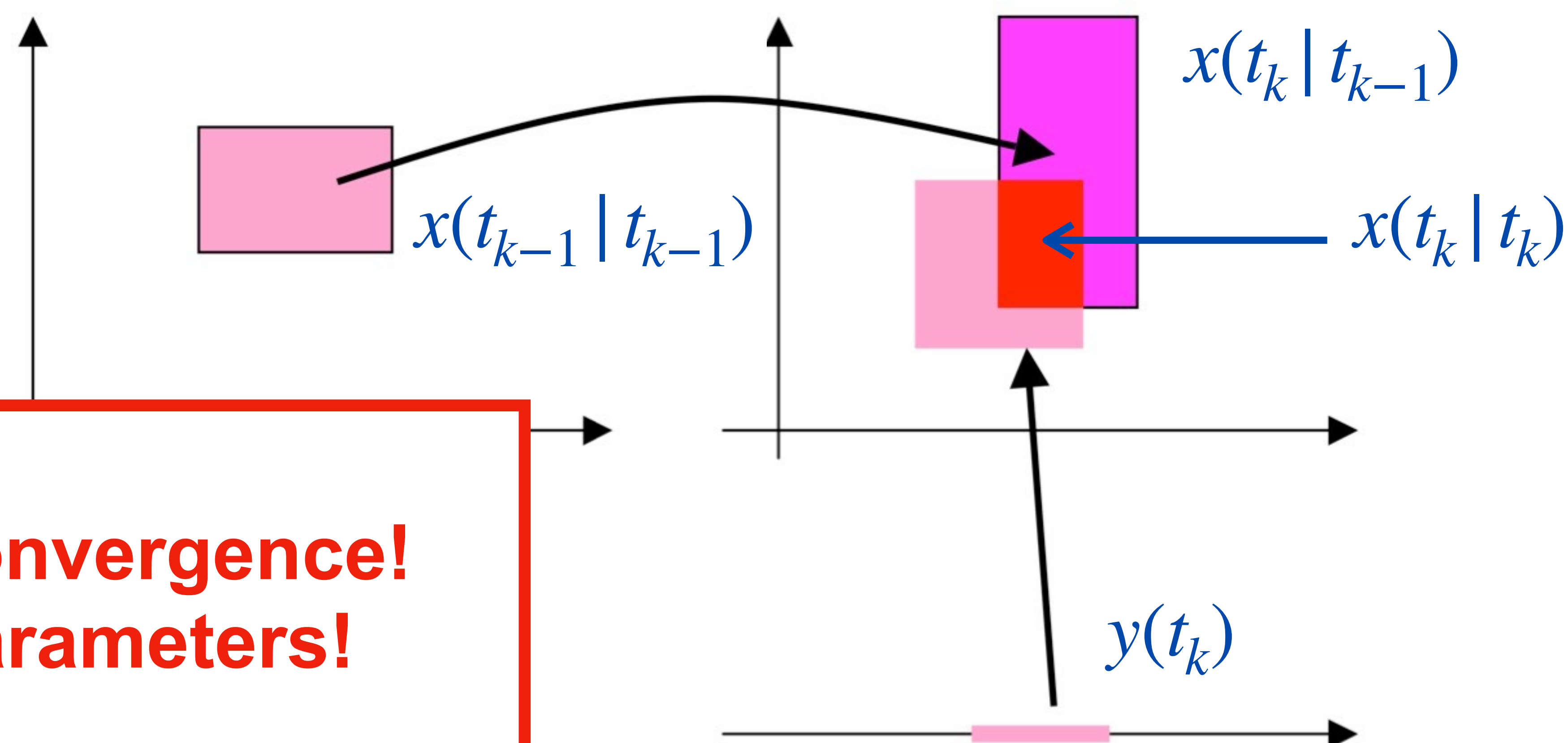

No proof of convergence!
No tuning parameters!

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Interval Impulsive Observers

Interval Impulsive Observer

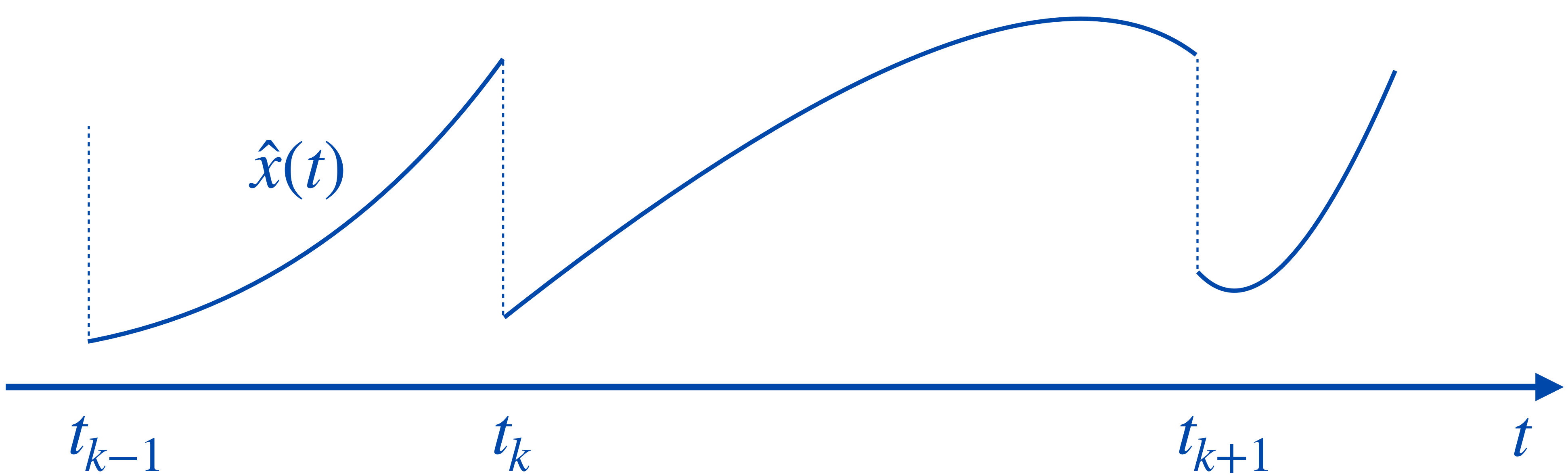
Impulsive observer: hybrid system

(Postoyan & Nesic, 2012), (Ferrante et al., 2016)

$$t \in [t_{k-1}, t_k], \quad \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t)$$

$$t = t_k, \quad \hat{x}(t_k^+) = \hat{x}(t_k) + L\left(C\hat{x}(t_k) + \epsilon(t_k) - y(t_k)\right)$$

$$\hat{x}(t_k^+) = (I + LC)\hat{x}(t_k) + L\epsilon(t_k) - Ly(t_k)$$



Interval Impulsive Observer

Interval bounds

$$A = A_M - A_N, \quad A_M \text{ Metzler}, \quad A_N \geq 0$$

$$I - LC = (I - LC)^+ - (I - LC)^-$$

Interval Impulsive Observer

$$t \in [t_{k-1}, t_k], \quad \dot{x}(t) = Ax(t) + Bu(t)$$
$$x(t_k^+) = (I + LC)x(t_k) + L\epsilon(t_k) - Ly(t_k)$$

(Djahid et al., 2021)

Interval Impulsive Observer

$$t \in [t_{k-1}, t_k], \quad \dot{x}(t) = Ax(t) + Bu(t)$$

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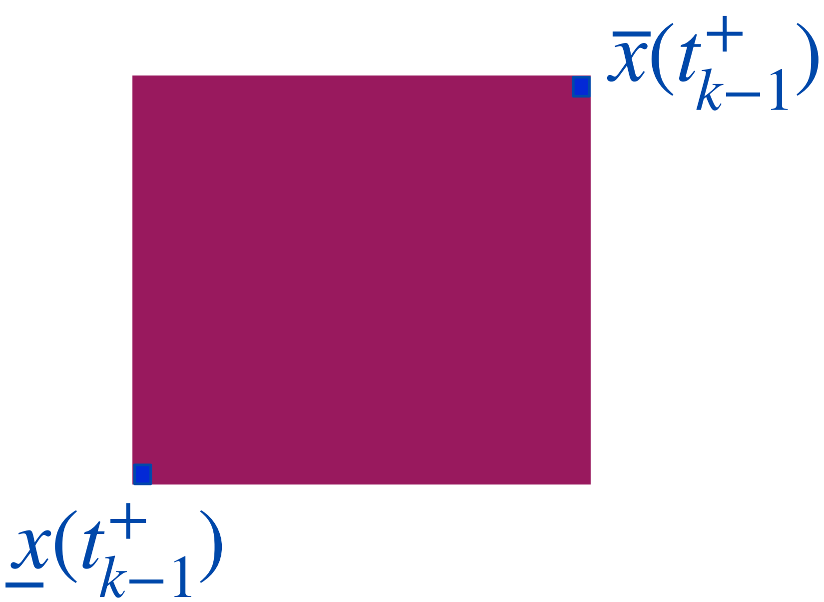

(Djahid et al., 2021)

Interval Impulsive Observer

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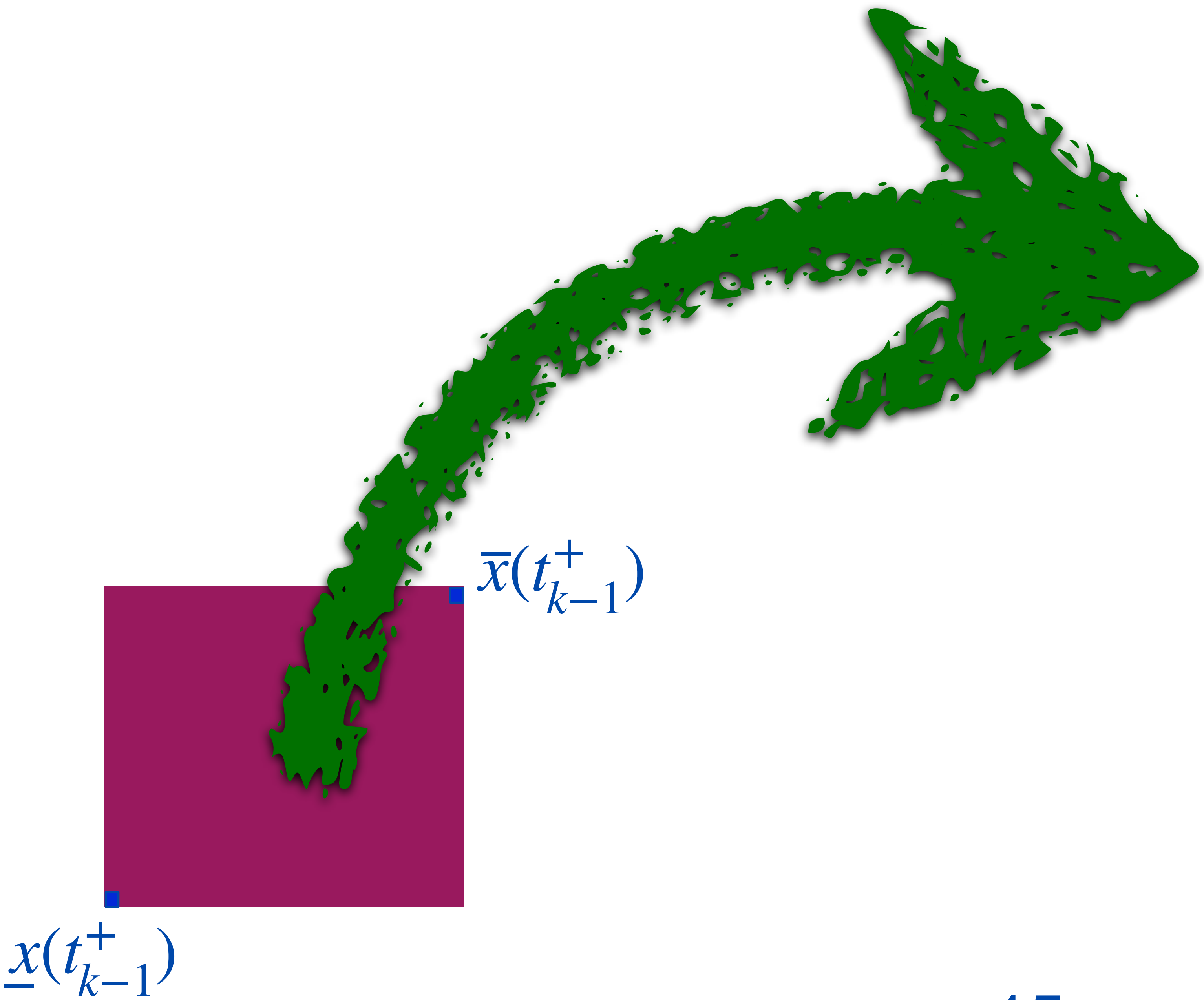

(Djahid et al., 2021)

Interval Impulsive Observer

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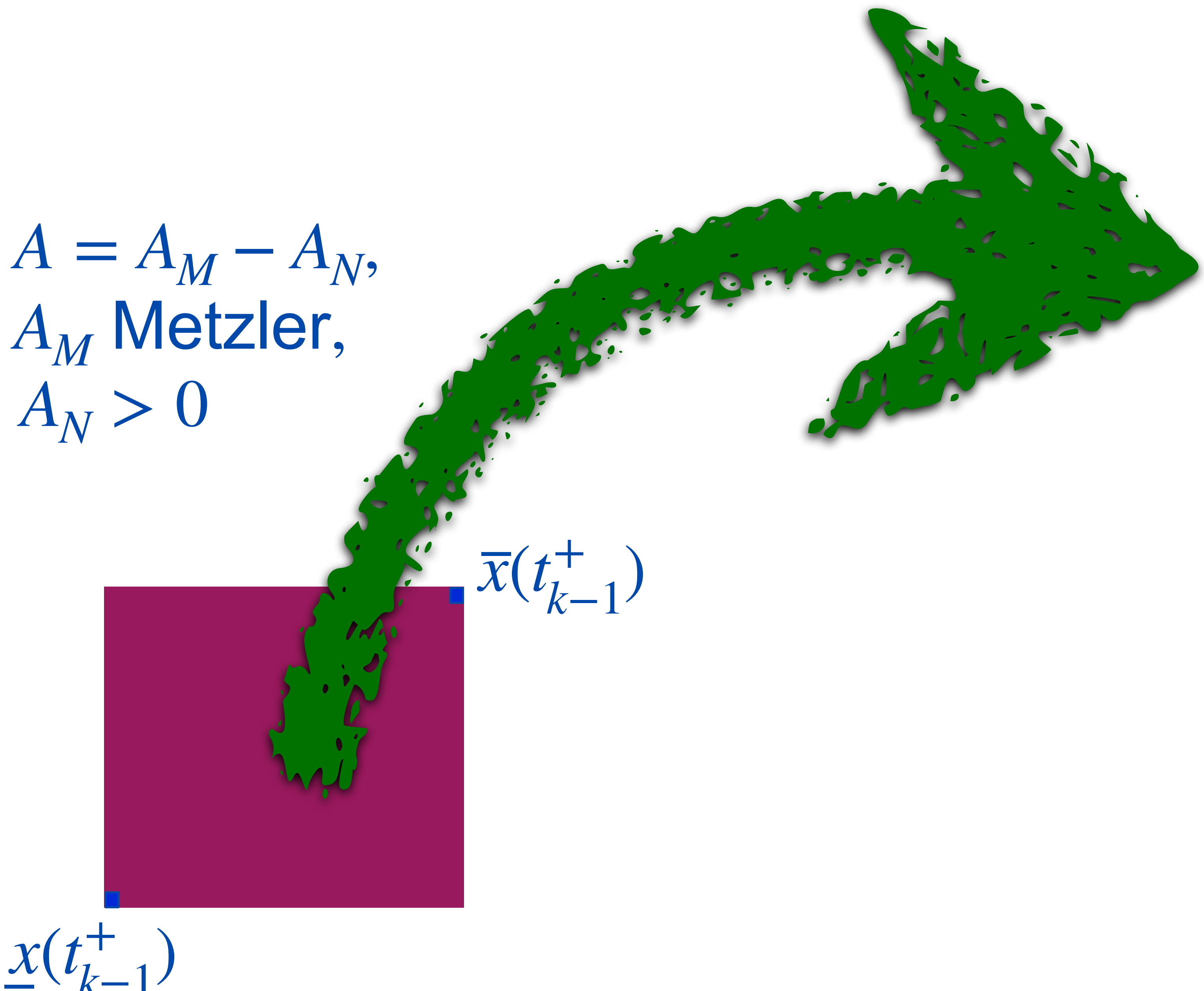
$A = A_M - A_N$,
A_M Metzler,
$A_N > 0$



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Interval Impulsive Observer

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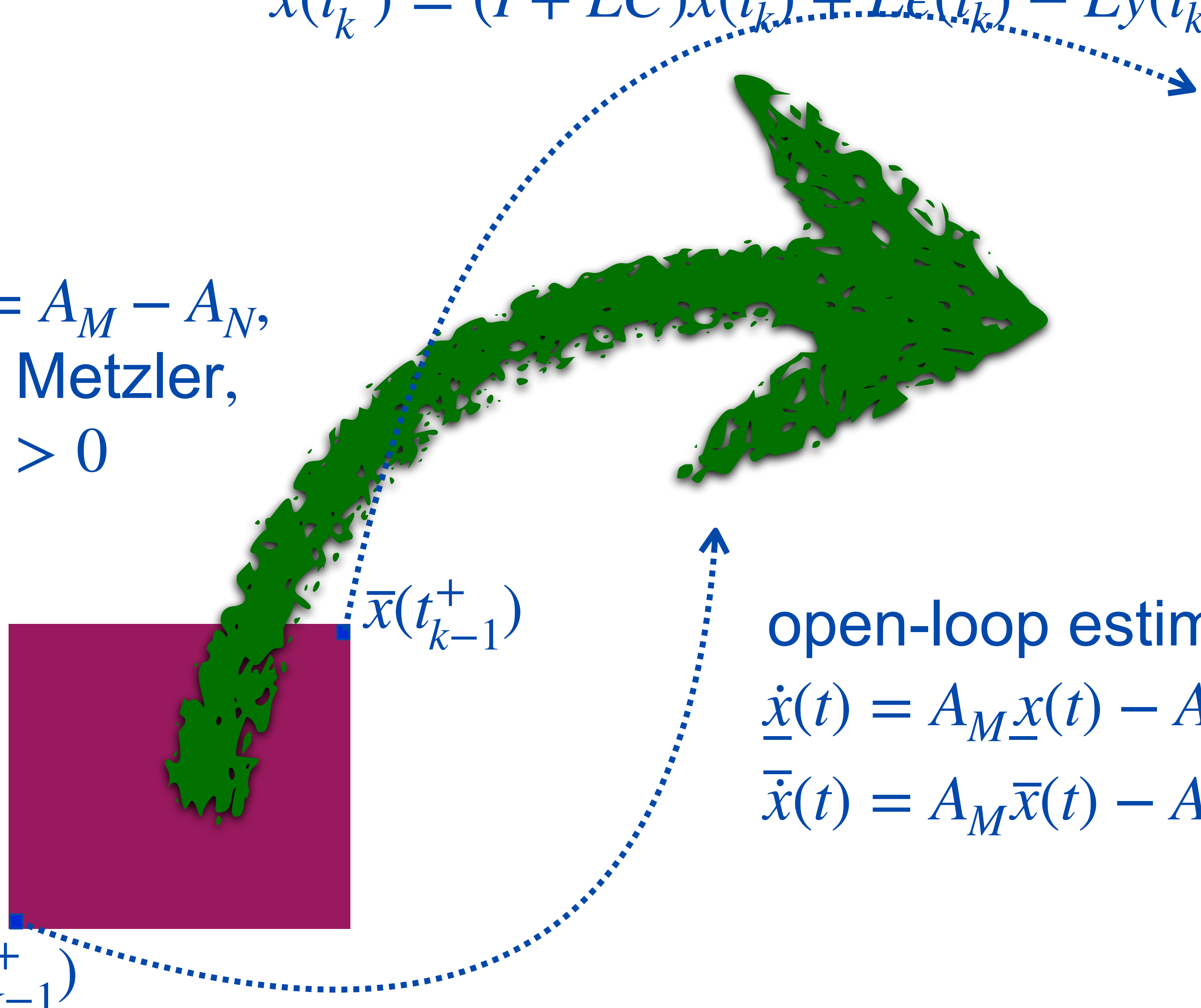


open-loop estimator $t \in [t_{k-1}, t_k]$

$$\dot{\underline{x}}(t) = A_M \underline{x}(t) - A_N \overline{x}(t) + Bu(t)$$

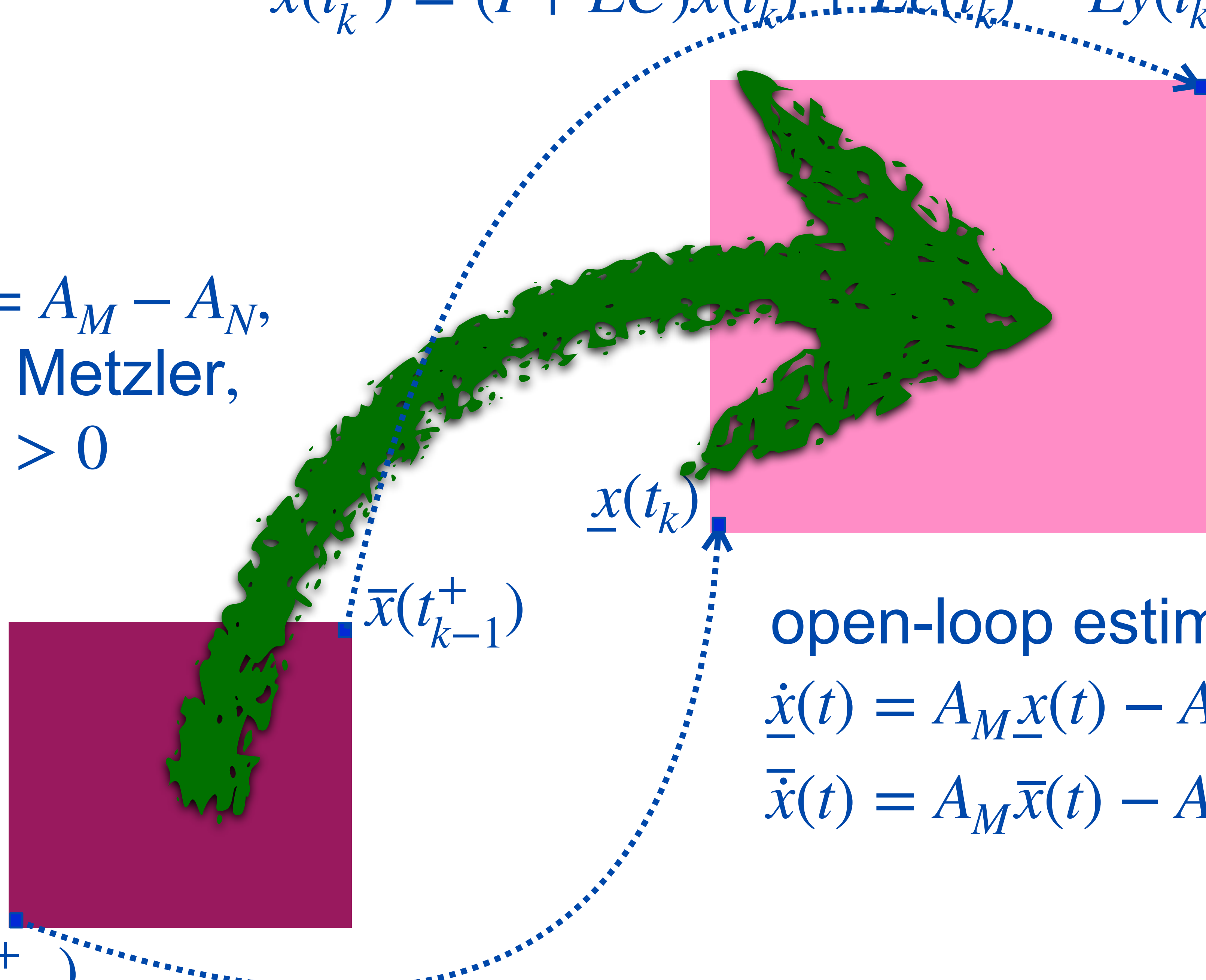
$$\dot{\overline{x}}(t) = A_M \overline{x}(t) - A_N \underline{x}(t) + Bu(t)$$

(Djahid et al., 2021)

Interval Impulsive Observer

$t \in [t_{k-1}, t_k], \quad \dot{x}(t) = Ax(t) + Bu(t)$

$x(t_k^+) = (I + LC)x(t_k) + L\epsilon(t_k) - Ly(t_k)$



$A = A_M - A_N$,
A_M Metzler,
$A_N > 0$

open-loop estimator $\quad t \in [t_{k-1}, t_k]$

$\dot{\underline{x}}(t) = A_M \underline{x}(t) - A_N \overline{x}(t) + Bu(t)$

$\dot{\overline{x}}(t) = A_M \overline{x}(t) - A_N \underline{x}(t) + Bu(t)$

(Djahid et al., 2021)

Interval Impulsive Observer

$A = A_M - A_N$,
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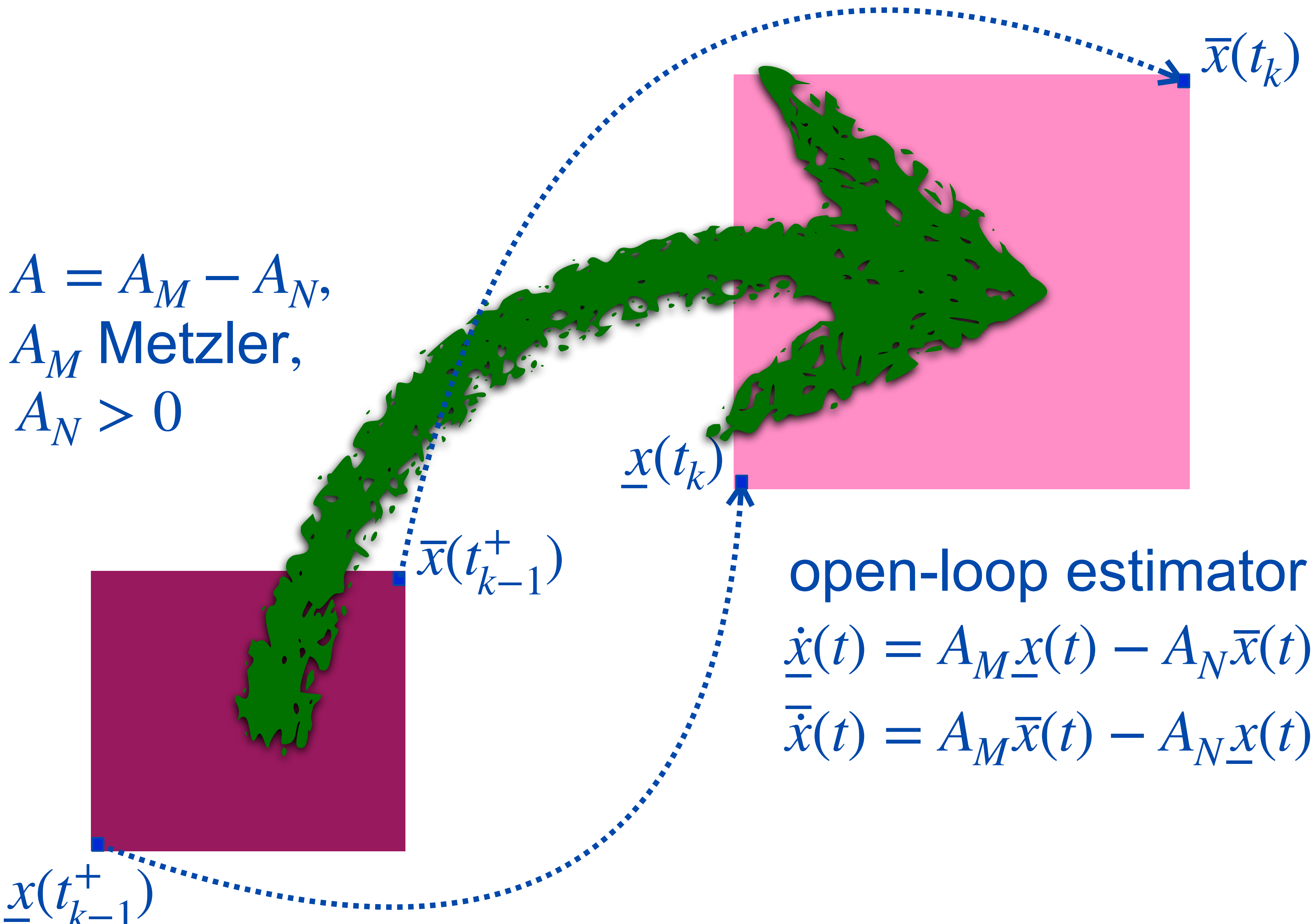

open-loop estimator $\quad t \in [t_{k-1}, t_k]$

$$\dot{\underline{x}}(t) = A_M \underline{x}(t) - A_N \overline{x}(t) + Bu(t)$$

$$\dot{\overline{x}}(t) = A_M \overline{x}(t) - A_N \underline{x}(t) + Bu(t)$$

(Djahid et al., 2021)

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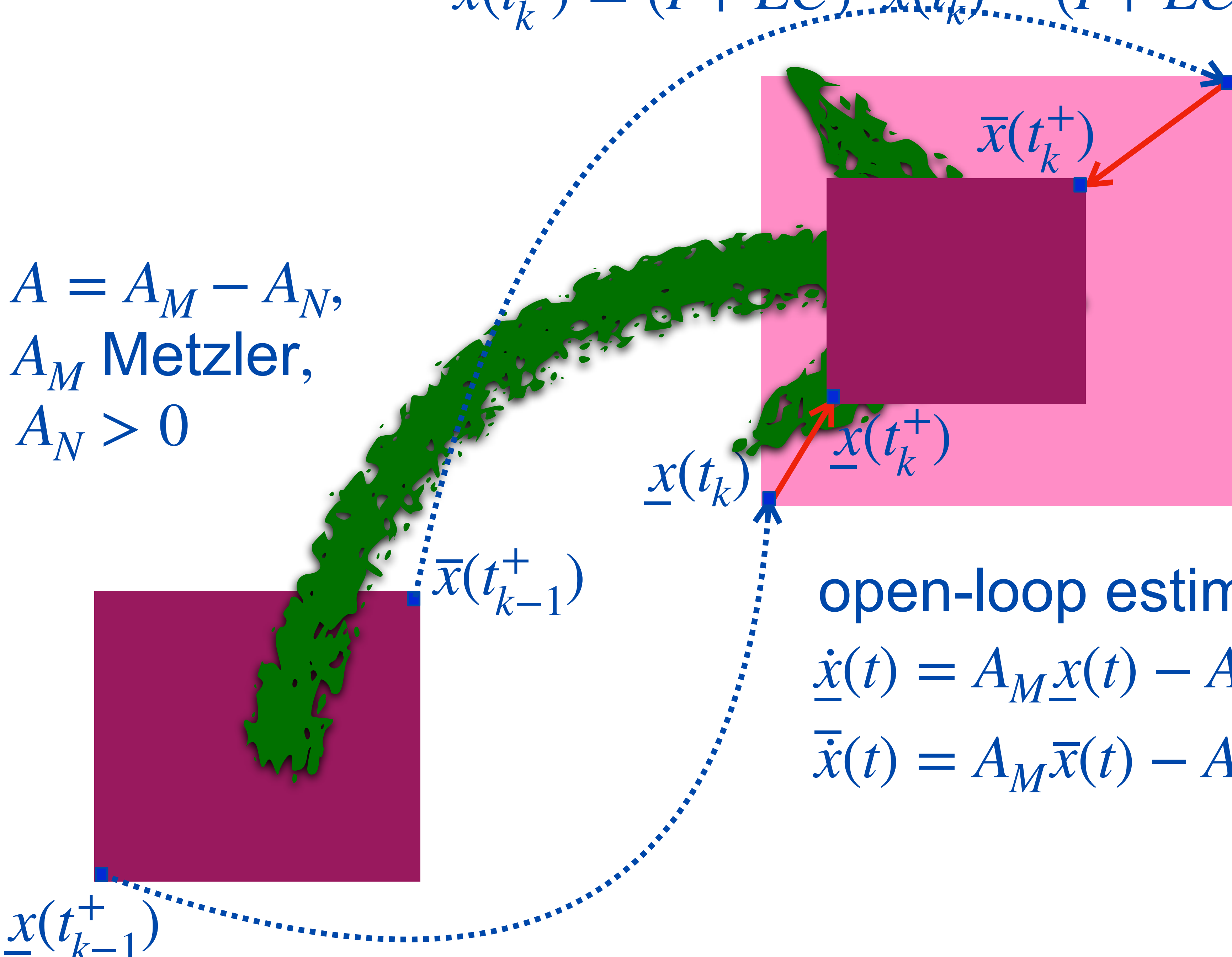
Interval Impulsive Observer

$$\underline{x}(t_k^+) = (I + LC)^+ \underline{x}(t_k) - (I + LC)^- \overline{x}(t_k) + |L| \underline{\epsilon}(t_k) - Ly(t_k)$$

$$\overline{x}(t_k^+) = (I + LC)^+ \overline{x}(t_k) - (I + LC)^- \underline{x}(t_k) + |L| \overline{\epsilon}(t_k) - Ly(t_k)$$



impulsive correction when measurement is available

$A = A_M - A_N$,
A_M Metzler,
$A_N > 0$

open-loop estimator $t \in [t_{k-1}, t_k]$

$$\dot{\underline{x}}(t) = A_M \underline{x}(t) - A_N \overline{x}(t) + Bu(t)$$

$$\dot{\overline{x}}(t) = A_M \overline{x}(t) - A_N \underline{x}(t) + Bu(t)$$

(Djahid et al., 2021)

Interval Impulsive Observer

Open-loop predictor

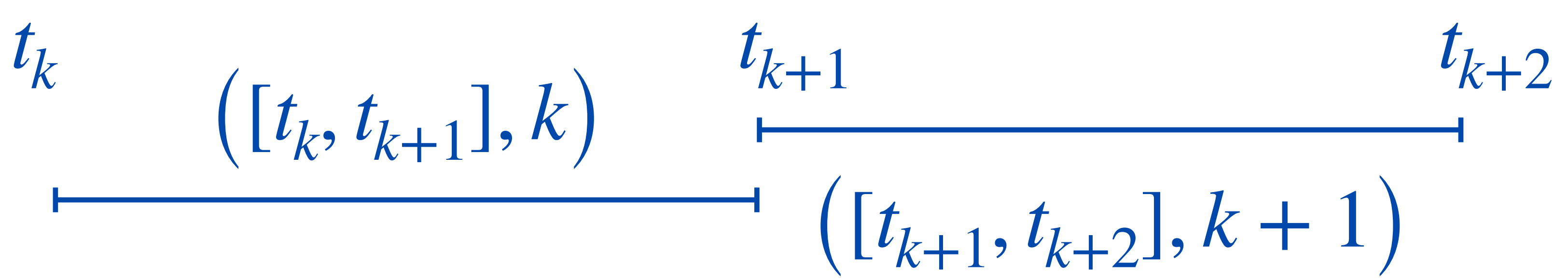
$$t \in [t_k, t_{k+1}], \quad \begin{aligned} \dot{\underline{x}}(t,k) &= A_M \underline{x}(t,k) - A_N \overline{x}(t,k) + Bu(t) \\ \dot{\overline{x}}(t,k) &= A_M \overline{x}(t,k) - A_N \underline{x}(t,k) + Bu(t) \end{aligned}$$

Impulsive correction when measurement is available

$$\underline{x}(t_{k+1}, k+1) = (I + LC)^+ \underline{x}(t_{k+1}, k) - (I + LC)^- \overline{x}(t_{k+1}, k) + |L| \underline{\epsilon}(t_{k+1}) - Ly(t_{k+1})$$

$$\overline{x}(t_{k+1}, k+1) = (I + LC)^+ \overline{x}(t_{k+1}, k) - (I + LC)^- \underline{x}(t_{k+1}, k) + |L| \overline{\epsilon}(t_{k+1}) - Ly(t_{k+1})$$

Hybrid time domain



Interval Impulsive Observer

Open-loop predictor

$$t \in [t_k, t_{k+1}], \quad \dot{\underline{x}}(t,k) = A_M \underline{x}(t,k) - A_N \overline{x}(t,k) + Bu(t)$$

$$\dot{\overline{x}}(t,k) = A_M \overline{x}(t,k) - A_N \underline{x}(t,k) + Bu(t)$$

Impulsive correction when measurement is available

$$\underline{x}(t_{k+1}, k+1) = (I + LC)^+ \underline{x}(t_{k+1}, k) - (I + LC)^- \overline{x}(t_{k+1}, k) + |L| \overline{\epsilon}(t_{k+1}) - Ly(t_{k+1})$$

$$\overline{x}(t_{k+1}, k+1) = (I + LC)^+ \overline{x}(t_{k+1}, k) - (I + LC)^- \underline{x}(t_{k+1}, k) + |L| \overline{\epsilon}(t_{k+1}) - Ly(t_{k+1})$$

Framing property

$$\underline{x}(t_0) \leq x(t) \leq \overline{x}(t_0) \quad \Rightarrow \quad \forall t \geq t_0,\ \underline{x}(t) \leq x(t) \leq \overline{x}(t)$$

Interval Impulsive Observer

Dynamics of the bounds of the estimation error

$$\underline{e} = x - \underline{x}, \quad \overline{e} = \overline{x} - x$$

$$\begin{bmatrix} \dot{\underline{e}} \\ \dot{\overline{e}} \end{bmatrix} = \mathcal{M}(A) \begin{bmatrix} \underline{e} \\ \overline{e} \end{bmatrix} + \tilde{E}\psi \qquad \mathcal{M}(A) = \begin{bmatrix} A_M & A_N \\ A_N & A_M \end{bmatrix}$$

$$\begin{bmatrix} \underline{e}^+ \\ \overline{e}^+ \end{bmatrix} = \Gamma(L) \begin{bmatrix} \underline{e} \\ \overline{e} \end{bmatrix} + \tilde{F}(L)\psi \qquad \Gamma(L) = \begin{bmatrix} (I_n + LC)^+ & (I_n + LC)^- \\ (I_n + LC)^- & (I_n + LC)^+ \end{bmatrix}$$

Stability property ?

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- **Finite-gain Lp Stability**

Lp Stability

Lp norm for hybrid signals

(Nesic, et al, 2013)

$$||z_{[T]}||_p := \left(\sum_{i=1}^{j(T)} |z(t_i, i)|^p + \sum_{i=0}^{j(T)} \int_{t_i}^{\sigma_i} |z(s, i)|^p ds \right)^{\frac{1}{p}}$$

$$||z||_p = \lim_{T \to T^*} ||z_{[T]}||_p,$$

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Lp Stability

Definition: Finite-gain Lp stable

(Nesic, et al, 2013)

Given $p \in [1, +\infty)$, the system

$$\begin{cases} \dot{x} = f(x, d), & \forall (x, d) \in \mathcal{C}_x \\ x^+ = g(x, d), & \forall (x, d) \in \mathcal{D}_x \\ y = h(x, d), & \end{cases} \tag{1}$$

is finite-gain $\mathcal{L}_p$ stable from d to y with gain upper bounded by $\gamma_p \geq 0$, if there exists a scalar $\beta \geq 0$ such that any solution to (1) satisfies

$$||y||_p \leq \beta |x(0,0)|_p + \gamma_p ||d||_p \tag{2}$$

for all $d \in \mathcal{L}_p^{n_d}$.

Lp Stability

Definition: Finite-gain Lp storage function (Nesic, et al, 2013)

Given $p \in [1, +\infty)$, a positive semi-definite continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}_+$ is a finite-gain $\mathcal{L}_p$ storage function for system (1) if there exist positive constants c_2, γ_{yf} and γ_{yg}, and nonnegative constants γ_{dg}, γ_{df}, such that

$$
\begin{aligned}
&\forall (x,d) \in \mathcal{C}_x \cup \mathcal{D}_x \qquad 0 \leq V(x) \leq c_2 |x|^p,\\
&\forall (x,d) \in \mathcal{C}_x \qquad \langle \nabla V(x), f(x,d) \rangle \leq -\gamma_{yf} |h(x,d)|^p + \gamma_{df} |d|^p,\\
&\forall (x,d) \in \mathcal{D}_x \quad V(g(x,d)) - V(x) \leq -\gamma_{yg} |h(x,d)|^p + \gamma_{dg} |d|^p.
\end{aligned}
$$

Lp Stability

Proposition: Finite-gain Lp stability (Nesic, et al, 2013)

Consider system (1), and suppose that there exists a finite-gain $\mathcal{L}_p$ storage function V. Then the system is finite gain $\mathcal{L}_p$ stable, and the gain of the operator $d \to y$ is upper bounded by $\gamma_p = \sqrt[p]{\gamma_d/\gamma_y}$, where $\gamma_d = \max\{\gamma_{df}, \gamma_{dg}\}$, $\gamma_y = \min\{\gamma_{yf}, \gamma_{yg}\}$.

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- **Event-triggered Interval Impulsive Observers**

Event-triggered Interval Observer

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Self-triggered mechanism

(Rabehi, et al, IJRNC 2021)

$$\begin{cases} \dot{\xi} = \mathcal{M}(A)\xi + \tilde{E}\psi & \forall(\xi,\psi) \in \mathcal{C}_\xi \\ \xi^+ = \Gamma(L)\xi + \tilde{F}(L)\psi & \forall(\xi,\psi) \in \mathcal{D}_\xi \end{cases}$$

$$\xi = (\underline{e}, \overline{e})$$

$$\psi = (\omega - \underline{\omega}, \overline{\omega} - \omega)$$

$$\mathcal{C}_\xi = \left\{(\xi,\psi) \in \mathbb{R}^n \times \mathbb{R}^{n_d} \; : \; |\xi|_1 \leq \beta|\psi|_1\right\}$$

$$\mathcal{D}_\xi = \left\{(\xi,\psi) \in \mathbb{R}^n \times \mathbb{R}^{n_d} \; : \; |\xi|_1 \geq \beta|\psi|_1\right\}$$

How to tune L and β to ensure stability ?

$$\omega(t,j) = \overline{x}(t,j) - \underline{x}(t,j) = \overline{e}(t,j) + \underline{e}(t,j)$$

$$\delta(t,j) = \overline{d}(t) - \underline{d}(t).$$

$$\mathcal{C}_\omega = \left\{(\omega,\delta) \in \mathbb{R}^n \times \mathbb{R}^{n_d} \; : \; |\omega|_1 \leq \beta|\delta|_1\right\}$$

$$\mathcal{D}_\omega = \left\{(\omega,\delta) \in \mathbb{R}^n \times \mathbb{R}^{n_d} \; : \; |\omega|_1 \geq \beta|\delta|_1\right\}$$

Event-triggered Interval Observer

Verification theorem : Algebraic inequalities

Positive systems

Copositive Lyapunov function $V(\xi) = x^\top \lambda$

S-procedure

Lp stability with p=1

Event-triggered Interval Observer

Verification theorem : Algebraic inequalities

Under bounded error assumptions. For a given matrix $L \in \mathbb{R}^{n \times n_y}$, *if there exist a positive vector* $\lambda \in \mathbb{R}^{2n}_{>0}$, *and positive scalars* $\zeta_{\mathcal{C}}$, $\zeta_{\mathcal{D}}$, $\gamma_{\delta f}$, $\gamma_{\delta g}$, $\gamma_{\omega f}$, $\gamma_{\omega g}$ *and* β, *satisfying the following inequalities*

$$
\begin{aligned}
\mathcal{M}^\top(A)\lambda + (\gamma_{\omega f} - \zeta_{\mathcal{C}})\mathbf{1}_{2n} &\leq 0 \\
\tilde{E}^\top \lambda - (\gamma_{\delta f} - \zeta_{\mathcal{C}}\beta)\mathbf{1}_{2n_d} &\leq 0 \\
\Gamma^\top(L)\lambda - \lambda + (\gamma_{\omega g} + \zeta_{\mathcal{D}})\mathbf{1}_{2n} &\leq 0 \\
\tilde{F}^\top(L)\lambda - (\gamma_{\delta g} + \zeta_{\mathcal{D}}\beta)\mathbf{1}_{2n_d} &\leq 0
\end{aligned}
$$

then, the event-triggered interval impulsive observer is a finite $\mathcal{L}_1$*-gain interval observer for the original system. Furthermore, the* $\mathcal{L}_1$*-gain from* δ *to* ω *is upper bounded by* $\gamma_{\mathcal{L}_1} = \gamma_\delta / \gamma_\omega$ *where* $\gamma_\delta = \max\{\gamma_{\delta f}, \gamma_{\delta g}\}$ *and* $\gamma_\omega = \min\{\gamma_{\omega f}, \gamma_{\omega g}\}$.

(Rabehi, et al, IJRNC 2021)

Event-triggered Interval Observer

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Dynamic self-triggered mechanism DETM

$$\xi = (\underline{e}, \overline{e}) \qquad \omega(t,j) = \overline{x}(t,j) - \underline{x}(t,j) = \overline{e}(t,j) + \underline{e}(t,j)$$

$$\psi = (\omega - \underline{\omega}, \overline{\omega} - \omega) \qquad \delta(t,j) = \overline{d}(t) - \underline{d}(t).$$

$$\begin{cases} \dot{\xi} = \mathcal{M}(A)\xi + \tilde{E}\psi & \forall(\xi,\psi) \in \mathcal{C}_\xi \\ \xi^+ = \Gamma(L)\xi + \tilde{F}(L)\psi & \forall(\xi,\psi) \in \mathcal{D}_\xi \end{cases}$$

$$\mathcal{C}_\eta = \left\{(\omega,\delta,\eta) \in \mathbb{R}^n \times \mathbb{R}^{n_d} \times \mathbb{R} \ : \ |\omega|_1 \leq \beta|\delta|_1 + \frac{\eta}{\theta}\right\}$$

$$\mathcal{D}_\eta = \left\{(\omega,\delta,\eta) \in \mathbb{R}^n \times \mathbb{R}^{n_d} \times \mathbb{R} \ : \ |\omega|_1 \geq \beta|\delta|_1 + \frac{\eta}{\theta}\right\}$$

$$\dot{\eta} = -\alpha\eta + \beta|\delta|_1 - |\omega|_1$$

$$\eta^+ = \eta,$$

Storage function:

$$W(\xi,\eta) = \xi^\top \lambda + \eta$$

(Rabehi, et al, IJRNC 2021)

Event-triggered Interval Observer

Under bounded error assumption and smooth error variations. For a given matrix $L \in \mathbb{R}^{n \times n_y}$, if there exist a positive vector $\lambda \in \mathbb{R}^{2n}_{>0}$, and positive scalars $\zeta_\mathcal{C}$, $\zeta_\mathcal{D}$, $\gamma_{\delta f}$, $\gamma_{\delta g}$, $\gamma_{\omega f}$, $\gamma_{\omega g}$, α, β and θ, satisfying the following inequalities

Corollary DETM

$$\left.\begin{array}{rl} \mathcal{M}^\top(A)\lambda + (-1 + \gamma_{\omega f} - \zeta_\mathcal{C})\mathbf{1}_{2n} & \leq 0 \\ \tilde{E}^\top \lambda + (\beta - \gamma_{\delta f} + \zeta_\mathcal{C}\beta)\mathbf{1}_{2n_d} & \leq 0 \\ -\alpha + \zeta_\mathcal{C}\frac{1}{\theta} & \leq 0 \end{array}\right\}$$

$$\left.\begin{array}{rl} \Gamma^\top(L)\lambda - \lambda + (\gamma_{\omega g} + \zeta_\mathcal{D})\mathbf{1}_{2n} & \leq 0 \\ \tilde{F}^\top(L)\lambda - (\gamma_{\delta g} + \zeta_\mathcal{D}\beta)\mathbf{1}_{2n_d} & \leq 0 \end{array}\right\}$$

then, the error hybrid dynamics is finite $\mathcal{L}_1$-gain stable. Thus, the event-triggered observer is a finite $\mathcal{L}_1$-gain interval observer for the original system. Furthermore, the $\mathcal{L}_1$-gain from δ to ω is upper bounded by $\gamma_{\mathcal{L}_1} = \gamma_\delta / \gamma_\omega$ where $\gamma_\delta = \max\{\gamma_{\delta f}, \gamma_{\delta g}\}$, $\gamma_\omega = \min\{\gamma_{\omega f}, \gamma_{\omega g}\}$.

(Rabehi, et al, IJRNC 2021)

Event-triggered Interval Observer

Minimal Inter-Event Time MIET (Rabehi, et al, IJRNC 2021)

The conditions in corollary DETM with the storage function $W(\xi, \eta) = \xi^\top \lambda + \eta$ *that satisfies*

$$\gamma_{\delta g} - \beta \gamma_{\omega g} \leq \beta(\epsilon_{min} - \epsilon_{max}),$$
$$\epsilon_{min} \leq \min(\lambda), \max(\lambda) \leq \epsilon_{max},$$

guarantee the existence of minimum inter-event time.

Event-triggered Interval Observer

Co-design of ETM and gains (Rabehi, et al, IJRNC 2021)

Positive realisation

$$\Gamma(L) = \begin{bmatrix} (I_n + LC)^+ & (I_n + LC)^- \\ (I_n + LC)^- & (I_n + LC)^+ \end{bmatrix}$$

$$G_p - G_n = I_n + LC \qquad \Gamma(G_p, G_n) = \begin{bmatrix} G_p & G_n \\ G_n & G_p \end{bmatrix}$$

Event-triggered Interval Observer

Co-design of ETM and gains (Rabehi, et al, IJRNC 2021)

Positive realisation

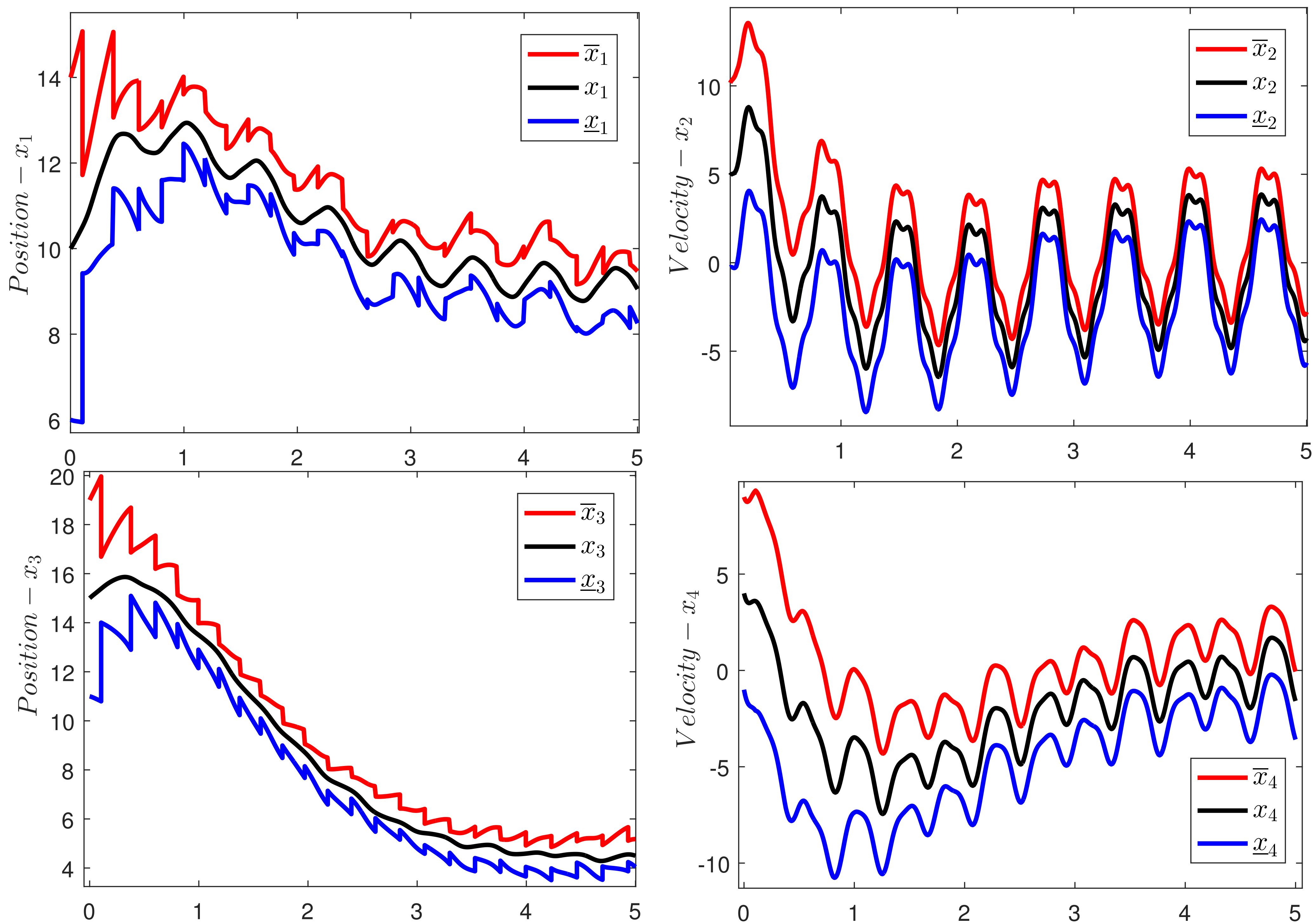
$$G_p - G_n = I_n + LC, \qquad \Gamma(G_p, G_n) = \begin{bmatrix} G_p & G_n \\ G_n & G_p \end{bmatrix}$$

$$R_p - R_n = LF \qquad \tilde{F}(R_p, R_n) = \begin{bmatrix} R_p & R_n \\ R_n & R_p \end{bmatrix}$$

$$\left.\begin{aligned} \Gamma^\top(G_p, G_n)\lambda - \lambda + (\gamma_{\omega g} + \zeta_D)\mathbb{1}_{2n} \leq 0 \\ \tilde{F}^\top(R_p, R_n)\lambda - (\gamma_{\delta g} + \zeta_D \beta)\mathbb{1}_{2n_d} \leq 0 \end{aligned}\right\}$$

Event-triggered Interval Observer

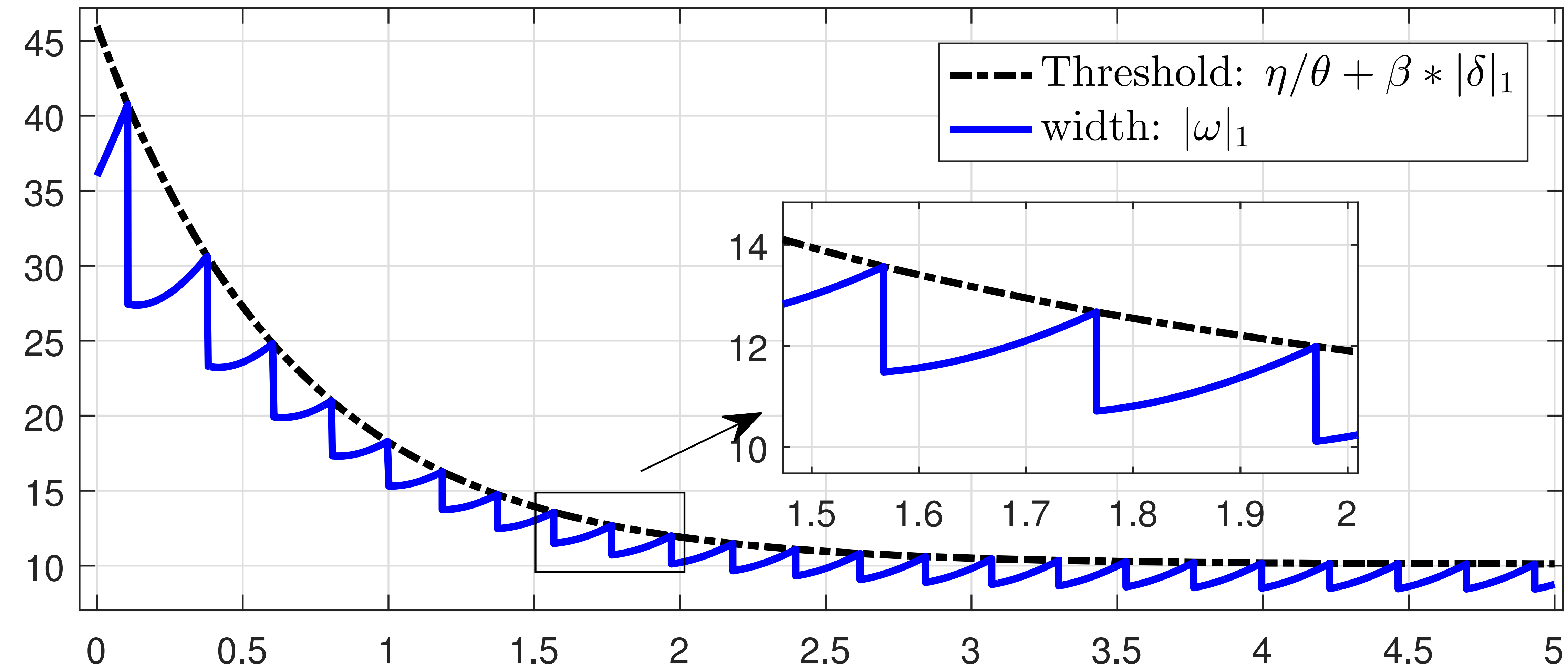
Evaluation: double spring-mass-damper system

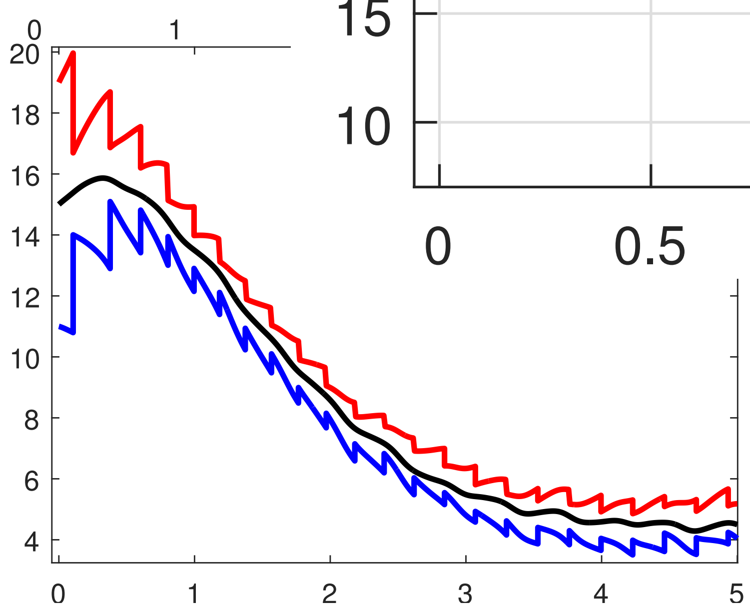


(Rabehi, et al, IJRNC 2021)

Event-triggered Interval Observer

Evaluation: the dynamic triggering conditions

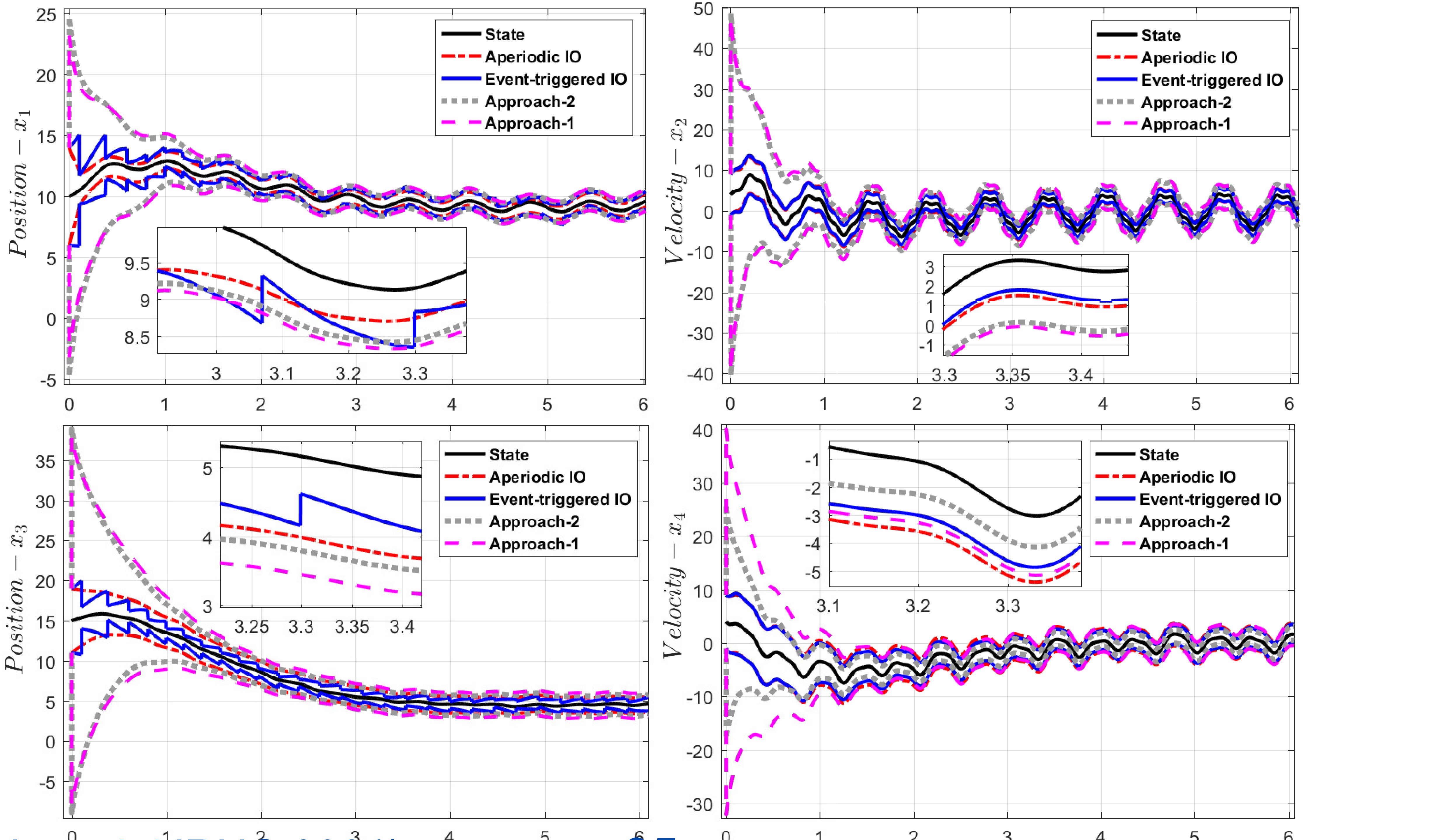




(Rabehi, et al, IJRNC 2021)

Event-triggered Interval Observer

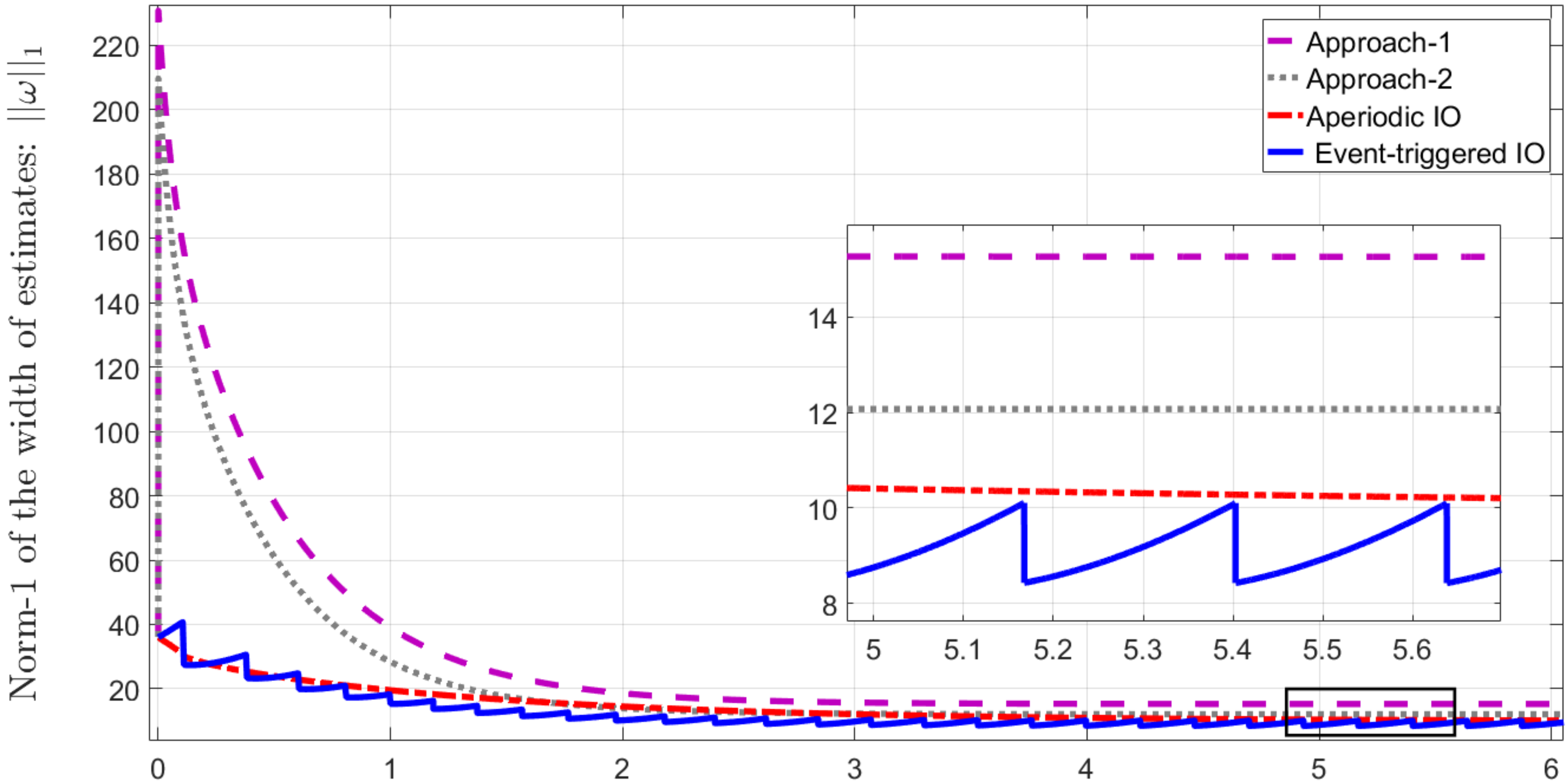
Evaluation: comparison with other approaches



(Rabehi, et al, IJRNC 2021)

Event-triggered Interval Observer

Evaluation: comparison with other approaches



(Rabehi, et al, IJRNC 2021)

UNIVERSITE D'ORLEANS

- **Interval Impulsive Observers with Sporadic Sampling**

Sporadic/Aperiodic Sampling

$$t \in \mathbb{R}, \quad \dot{x}(t) = Ax(t) + Bu(t) + \omega(t)$$

$$k \in \mathbb{N}, \quad y(t_k) = Cx(t_k) + \epsilon(t_k)$$

Aperiodic sampling: $t_{k+1} - t_k = \tau \in [\tau_{min}, \tau_{max}]$

Interval impulsive observer:

$$t \in [t_{k-1}, t_k], \quad \dot{x}(t) = Ax(t) + Bu(t)$$
$$x(t_k^+) = x(t_k) + L\left(Cx(t_k) + \epsilon(t_k) - y(t_k)\right)$$

(Rabehi, et al, IEEE TAC 2021)

Aperiodic Sampling

Dynamics of the bounds of the estimation error

$$\begin{cases} \dot{z}_0 = \underbrace{\begin{bmatrix} \bar{A}\xi_0 \\ -1 \end{bmatrix}}_{f(z_0)=\mathscr{F}(z_0)} & \forall z_0 \in \mathscr{C} \\ z_0^+ = \underbrace{\begin{bmatrix} \Gamma(L)\xi_0 + \Upsilon_2 \\ \mu \end{bmatrix}}_{g(z_0)} \in \underbrace{\begin{bmatrix} \Gamma(L)\xi_0 + \Upsilon_2 \\ [\tau_{min}, \tau_{max}] \end{bmatrix}}_{\mathscr{G}(z_0)} & \forall z_0 \in \mathscr{D} \end{cases}$$

$$\mathscr{C} = \{(\xi_0, \tau) \in \mathbb{R}^{2n} \times \mathbb{R}_{\geq} \quad | \quad \tau \in [0, \tau_{max}]\}$$
$$\mathscr{D} = \{(\xi_0, \tau) \in \mathbb{R}^{2n} \times \mathbb{R}_{\geq} \quad | \quad \tau = 0\}.$$

Lyapunov function

$$V(z_0) = \xi_0^\top e^{\bar{A}^\top \tau} P e^{\bar{A}\tau} \xi_0.$$

$$\langle \nabla V(z_0), f(z_0) \rangle = 0 \;\; \forall z_0 \in \mathscr{C}$$

(Rabehi, et al, IEEE TAC 2021)

Aperiodic Sampling

Verification theorem : SDP & NLMI (Rabehi, et al, IEEE TAC 2021)

Under bounded error assumptions. For a given gain matrix $L \in \mathbb{R}^{n\times p}$, if there exist a symmetric positive definite matrix $P \in \mathbb{R}^{2n\times 2n}$ such that

$$\Gamma(L)^\top e^{\overline{A}^\top \mu} P e^{\overline{A}\mu} \Gamma(L) - P \prec 0 \quad \forall \mu \in [\tau_{min}, \tau_{max}]$$

is satisfied, then the interval impulsive system is Input-to-State-Stable (ISS), thus is an interval observer for the original system.

ISS & Hybrid Systems (Cai & Teel, 2009)

Aperiodic Sampling

Design procedure : SDP/NLMI relaxed to set of BMI.

- Positive realisation

$$G_p - G_n = I_n + LC$$

$$\Gamma(G_p, G_n) = \begin{bmatrix} G_p & G_n \\ G_n & G_p \end{bmatrix}$$

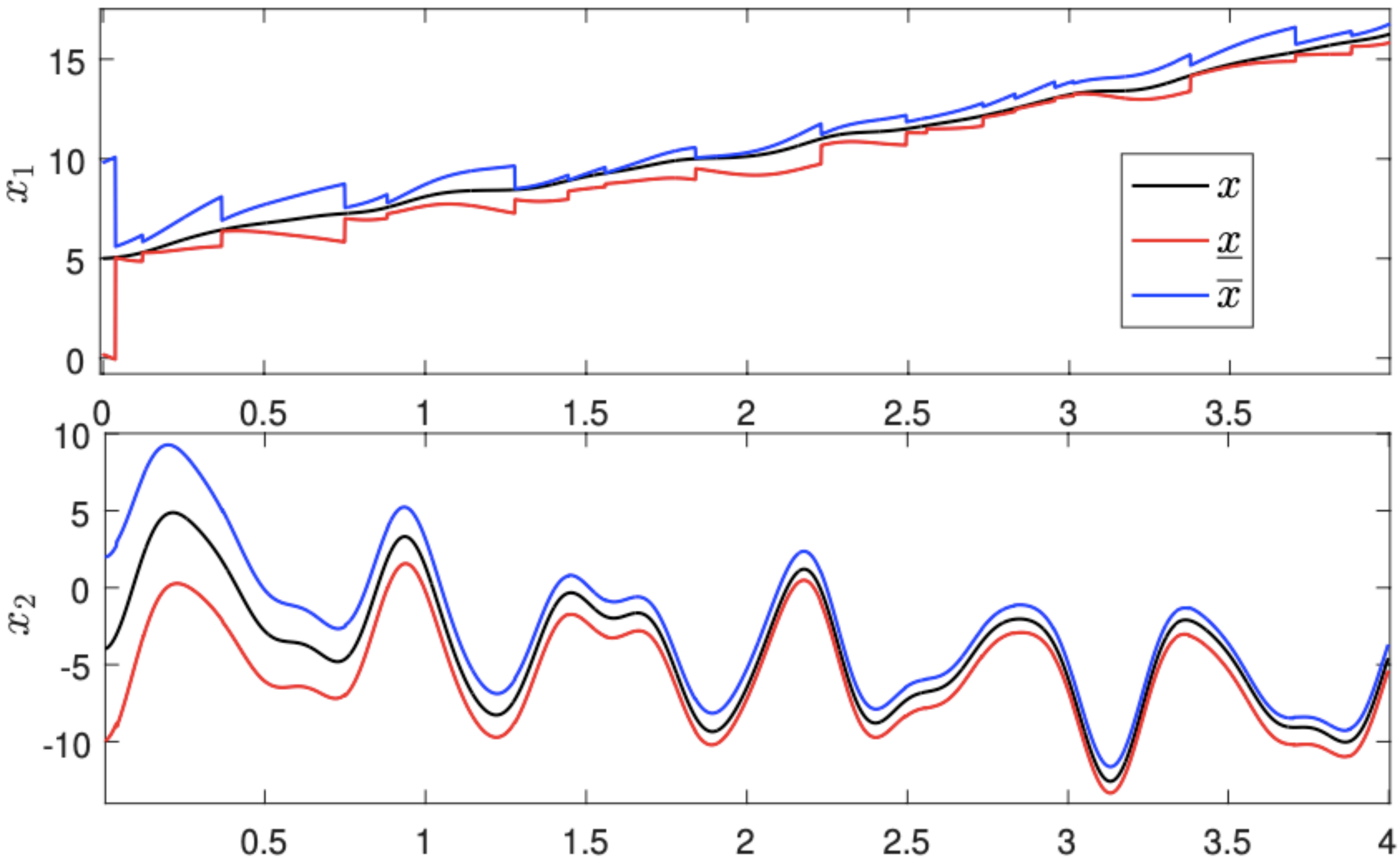
- Projection Lemma (Pipeleers, et al., 2009)
- Polytopic over approximation using Taylor series (L. Hetel, et al., 2007)

$$\begin{bmatrix} -F - F^\top & F\Gamma(G_1, G_2) & M_i^\top P \\ \star & -P & \mathbf{0} \\ \star & \star & -P \end{bmatrix} \prec 0 \ \forall i \in \{1, \ldots, v\}$$

(Rabehi, et al, IEEE TAC 2021)

Aperiodic Sampling

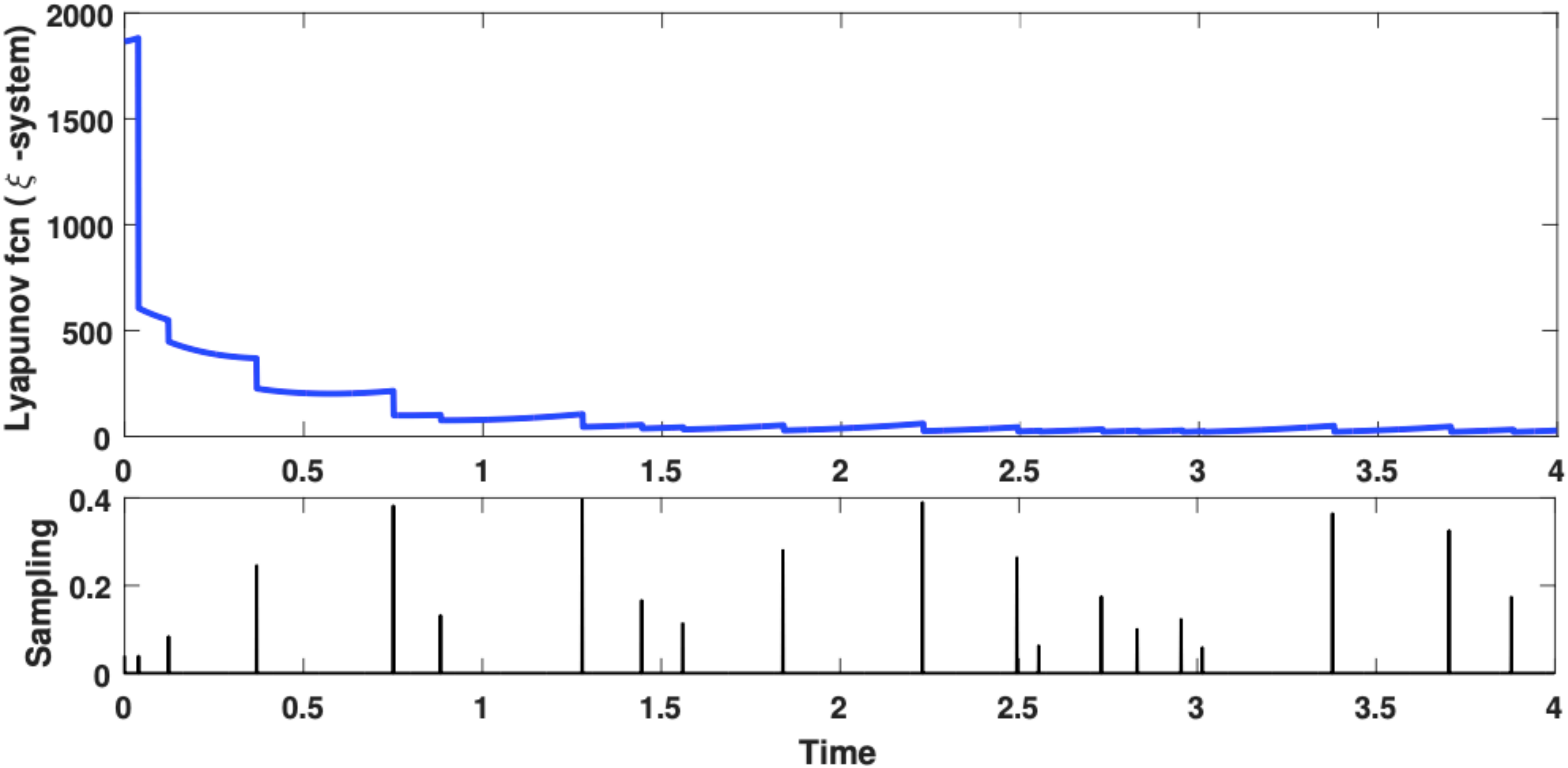
Evaluation: unstable system



(Rabehi, et al, IEEE TAC 2021)

Aperiodic Sampling

Evaluation: unstable system



(Rabehi, et al, IEEE TAC 2021)

- **Concluding remarks**

Concluding remarks

- Interval Impulsive Observer Framework
  - Dynamic Event-triggered sampling
  - Aperiodic/Sporadic sampling
- L1-gain synthesis
- SDP/NLMI

Main References

- Djahid Rabehi, Nacim Meslem, Nacim Ramdani.
  Finite-gain L1 Event-triggered Interval Observers design for Continuous-time Linear Systems.
  Int. Journal of Robust and Nonlinear Control 31, 4131-4153, 2021

- Djahid Rabehi, Nacim Meslem, Adnen Amraoui, & Nacim Ramdani.
  Interval impulsive observer for linear systems with aperiodic discrete measurements.
  IEEE Trans. Automatic Control 66(11), pp. 5407-5413, 2021

- Thank you !