Preliminary design of an interval-based autopilot

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Outline

What is an autopilot?

- An autopilot is a device usually combining inertial and magnetic sensors with multiple inputs and outputs designed to support additional navigation sensors such as GPS or communication devices
- Additionally, its embedded processor has algorithms to fuse the sensors data, report the best state estimation, and control predefined actuators to make typical autonomous missions (e.g. following waypoints) with configurable robot types

Figure: Example of autopilot typically found on low-cost aerial, surface and submarine drones

IMU vs AHRS vs INS vs autopilot

- IMU (Inertial Measurement Unit) : combination of 3 accelerometers (1 on each axis to get values in 3D), 3 gyrometers, 3 magnetometers, without data fusion
- AHRS (Attitude and Heading Reference System) : IMU with the addition of an embedded processor that fuses the sensors data to provide the best estimation of the 3 Euler angles, their derivatives, and linear accelerations. Usually not able to compute accurately the linear speeds and positions

IMU vs AHRS vs INS vs autopilot

- INS (Inertial Navigation System) : AHRS able to compute the linear speeds and positions, usually thanks to aiding devices such as GPS, barometer, odometers. Predefined profiles are usually available to specify assumptions on the robot model
- Autopilot : INS with control algorithms. Usually, more details are required to be configured in the predefined profiles especially to specify the actuators and robot model. Additionally, more sensors and advanced navigation algorithms are supported

Problems often encountered with autopilots

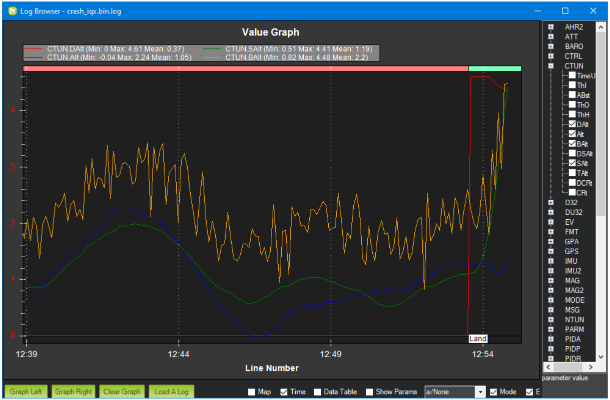

Figure: Example of wrong state estimation due to the failure of 1 sensor even though others were correct : a quadrotor climbs unexpectedly when activating autopilot land mode

Problems often encountered with autopilots

- Lots of autopilot failures are related to a wrong state estimation, especially the heading and the position, even though in practice sometimes multiple sensors are available to estimate them
- => We propose here to study more in details the heading and position estimation

Outline

Sensors used

Assuming a 2D movement :

- 1 gyrometer can provide the rotation speed around the z axis and can be integrated during a short delay from a known reference
- 1 compass (2 magnetometers) can measure the horizontal vector component of the Earth magnetic field and deduce an angle to the magnetic North
- A dual GPS can provide an angle to the geographic North
- Lever arms or other physical bias assumed to be known

Common sensors problems

- Gyrometer: integration errors accumulate over time
- Magnetometers: some objects might disturb the magnetic field, need also to know the magnetic declination to translate the measurements to geographic North
- Dual GPS: not always available, limited data output rate

Common data fusion problems

- Most commercial AHRS use probabilistic methods, precise in optimal conditions but not designed to detect inconsistencies and react accordingly
- On the contrary, interval methods are well suited to detect inconsistencies. By design, any interval fusion algorithm forces to think about what to do in case of an inconsistency (which appears as an empty interval during the computations)

Outline

Interval arithmetic

- An *interval* is a closed connected subset of $\mathbb{R}$
- $[-1,2]$,$\{4\}$,$[-\infty,1]$, $\mathbb{R}$ and $\emptyset$ are examples of intervals from the set of real-valued intervals denoted $\mathbb{IR}$
- Additions, divisions, $\sin$, $\exp$, etc. can be defined for intervals
- Usual set operations such as $\bigcap$, etc. can also be applied
- Interval vectors (usually named *boxes*) and intervals of trajectories (*tubes*) can also be manipulated in a similar way

Example of contraction procedure

- Assume that you have a compass and another algorithm able to estimate the heading of the robot
- The compass estimates an angle of 8 deg and its documentation suggests its error is likely to be within 2 deg, therefore we will represent its information with the interval $x_1 = [6, 10]$
- There is a bias between the compass and the robot of $x_2 = [0, 2]$ deg and the other algorithm estimates a heading for the robot of e.g. $x_3 = [4, 7]$ deg
- Therefore the variables x_1, x_2, x_3 are linked by the equation $x_1 + x_2 = x_3$

Example of contraction procedure

$$x_3 \in [4,7] \bigcap ([6,10]+[0,2]) = [4,7] \bigcap [6,12] = [6,7]$$

$$x_1 \in [6,10] \bigcap ([4,7]-[0,2]) = [6,10] \bigcap [2,7] = [6,7]$$

$$x_2 \in [0,2] \bigcap ([4,7]-[6,10]) = [-2,0] \bigcap [-6,1] = [0,1]$$

Outline

Assumptions

- Purpose : estimate the heading ψ of the boat as an interval $[\psi]$, using the boat's dual GPS which gives an estimation ψ_{GPS}, the vertical gyrometer of the AHRS ω, and the magnetometers of the AHRS which gives an estimation ψ_{mag}
- Magnetic declination and physical bias between the sensors assumed to be known or within the width of the intervals

Assumptions

- Since in 2D, the vertical gyrometer is measuring the heading velocity,
$$\dot{\psi} = \omega \tag{1}$$
- Integrating this eq. with regular (e.g. only available every s) $[\psi_{\mathrm{GPS}}]$ if the GPS is available and the distance between the 2 antennas is considered consistent with the user-provided antenna distance or $[\psi_{\mathrm{mag}}]$ measurements otherwise should enable to get an estimation of ψ at all time
- However, direct operations and comparisons on angles are not recommended due to modulo 2π equivalence of angles

Avoiding direct angles manipulations due to modulo 2π

- Instead, we can use the $\cos$ and $\sin$ of the angles so if $\mathbf{x}(t) = \begin{pmatrix} \cos(\psi(t)) \\ \sin(\psi(t)) \end{pmatrix}$, due to equation (1) we have $\dot{\mathbf{x}}(t) = \begin{pmatrix} -\omega(t)\sin(\psi(t)) \\ \omega(t)\cos(\psi(t)) \end{pmatrix}$ (evolution equation)
- This can be written as $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ with $\mathbf{A} = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix}$, which has an exact solution $\mathbf{x}(t) = k\exp(\mathbf{A}\cdot t)$, so $\mathbf{x}(t+dt) = \exp(\mathbf{A}dt)\,\mathbf{x}(t)$

Avoiding direct angles manipulations due to modulo 2π

- Then, interval arithmetic and intersections can be used to compute $[\mathbf{x}(t+dt)]$ from a known $[\mathbf{x}(t)]$
- $x_1^2 + x_2^2 = 1$ (consistency equation) can try to limit potential overestimation of the uncertainty when evaluating $[\cos(\psi(t))]$ and $[\sin(\psi(t))]$
- Then, the polar contractor from [1] can be used to contract $[\psi]$ from $[\cos(\psi(t))]$ and $[\sin(\psi(t))]$
- If an empty set appears at any time during the contraction procedure at a time step, $[\psi]$ is set to its last known value, but other choices could be possible

Problem formalization

$$\left\{ \begin{array}{lcll} \dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{x}) & \text{(evolution equation)} \\ \mathbf{y} & = & \mathbf{g}(\mathbf{x}) & \text{(observation equation)} \\ \mathbf{0} & = & \mathbf{h}(\mathbf{x}) & \text{(consistency equation)} \end{array} \right. \quad (2)$$

Problem formalization

where $\mathbf{x}(t) = \begin{pmatrix} \cos(\psi(t)) \\ \sin(\psi(t)) \end{pmatrix}$ is the state vector,

$\mathbf{y}(t) = \begin{pmatrix} \cos(\psi_{\text{GPS}}(t)) \\ \sin(\psi_{\text{GPS}}(t)) \\ \cos(\psi_{\text{mag}}(t)) \\ \sin(\psi_{\text{mag}}(t)) \end{pmatrix}$ is the output vector,

$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} \mathbf{x}$, $\mathbf{g}(\mathbf{x}) = \begin{pmatrix} x_1 \\ x_2 \\ x_1 \\ x_2 \end{pmatrix}$ and $\mathbf{h}(\mathbf{x}) = x_1^2 + x_2^2 - 1$

Problem formalization

We will assume that:

- For all $t \in [t_0, t_f]$, we have intervals enclosing $\omega(t)$:

$$\forall t \in [t_0, t_f], \omega(t) \in [\omega](t); \tag{3}$$

- For multiple time instants $t_i \in [t_0, t_f]$, we have intervals enclosing $\psi_{\text{GPS}}(t_i)$:

$$\exists t_i \in [t_0, t_f], \psi_{\text{GPS}}(t_i) \in [\psi_{\text{GPS}}](t_i); \tag{4}$$

- For all $t \in [t_0, t_f]$, we have intervals enclosing $\psi_{\text{mag}}(t)$:

$$\forall t \in [t_0, t_f], \psi_{\text{mag}}(t) \in [\psi_{\text{mag}}](t). \tag{5}$$

Outline

Comparisons in scenarios where inconsistencies between sensors occur

Systems tested:

- Interval-based INS : AHRS (SBG Ellipse2-A) to get the Euler angles and their derivatives with dual GPS (ArduSimple simpleRTK2B+heading), using CODAC library [2] on an embedded Raspberry Pi for the interval fusion
- Pixhawk 4 Mini autopilot (firmware ArduRover) with dual GPS
- SBG Ellipse3-D (INS which includes a dual GPS)

Comparisons in scenarios where inconsistencies between sensors occur

Test procedure:

- The INS is first left at a constant angle of 180 deg
- Then, it is turned by 90 deg towards West (270 deg) and left around 30 s before returning to the initial position
- To generate an inconsistency between the heading computed from the dual GPS antennas and the gyrometers, the IMU part of the system is rapidly moved to try to saturate the gyrometers and left at 270 deg during around 30 s, while the GPS antennas are not moved at all
- Finally, the IMU is put back in its original position to check how it recovers from the inconsistency

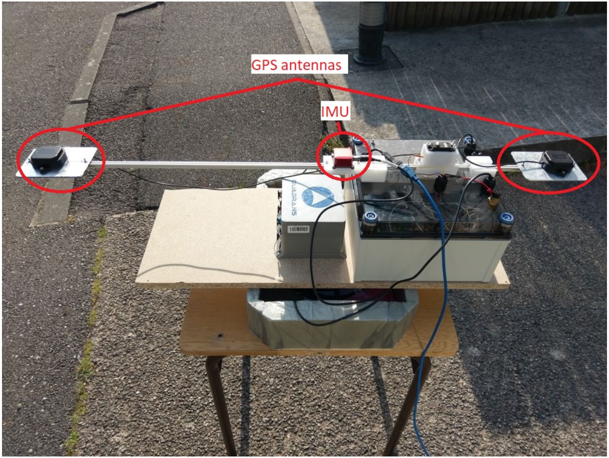

Figure: IMU part and GPS antennas of the interval-based INS.

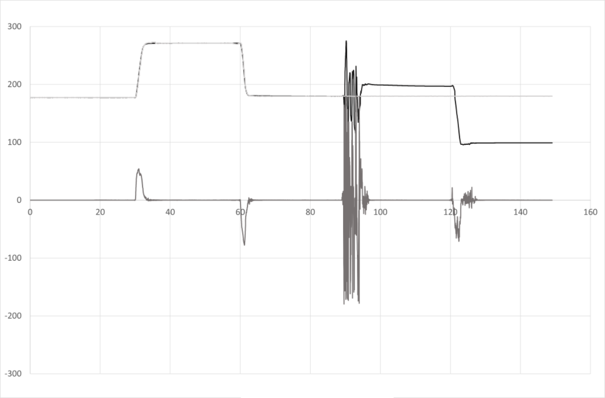

Figure: Evolution of the heading (in black) for the Ellipse-D (in deg w.r.t. arbitrary time unit). The raw heading from the GPS is in light gray (in deg) and the gyrometer data is in dark gray (in deg/s).

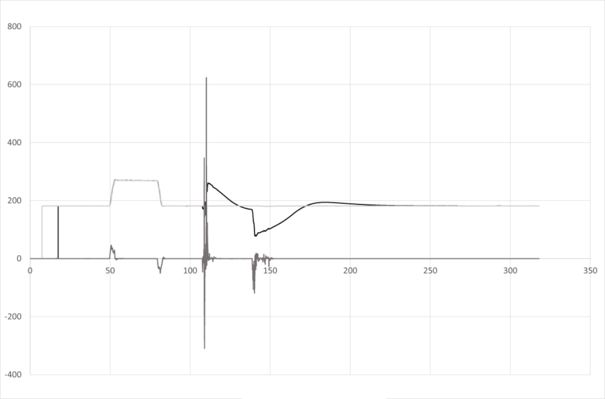

Figure: Evolution of the heading (in black) for the Pixhawk (in deg w.r.t. arbitrary time unit). The raw heading from the GPS is in light gray (in deg) and the gyrometer data is in dark gray (in deg/s).

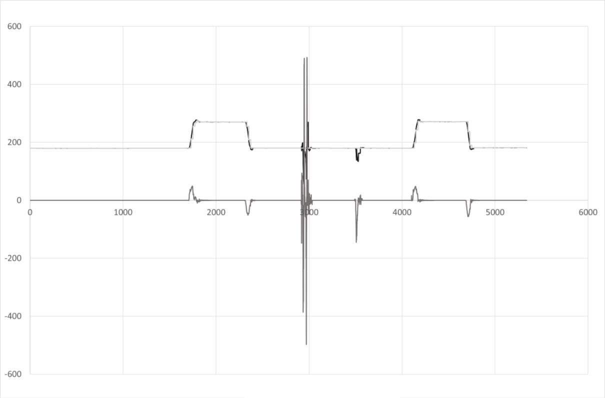

Figure: Evolution of the heading (in black) for the interval-based INS (in deg w.r.t. arbitrary time unit). The raw heading from the GPS is in light gray (in deg) and the gyrometer data is in dark gray (in deg/s).

Outline

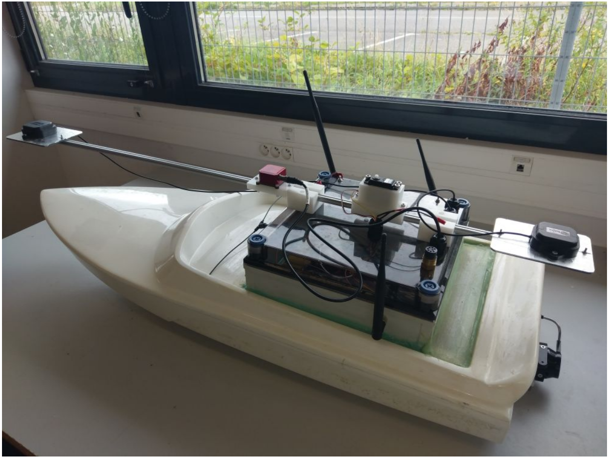

Figure: Autonomous boat with an interval-based INS.

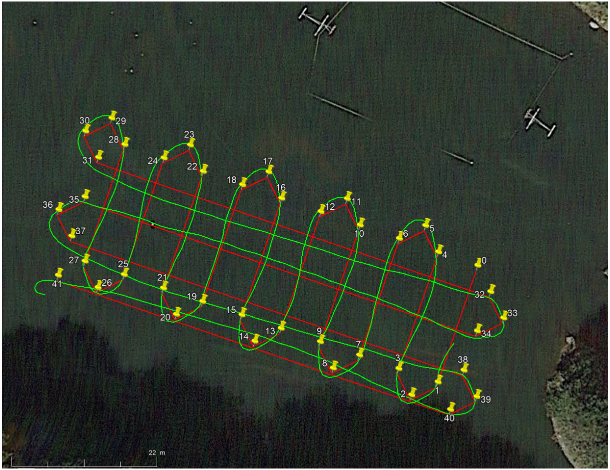

Figure: Real Boustrophedon track (in green) with the desired waypoints (in yellow, connected by red lines). Short legs are approximately 20 m and long ones are 70 m.

Outline

Our boat state equations

$$\begin{cases} \dot{x} & = & v_x \\ \dot{y} & = & v_y \\ \dot{\psi} & = & \frac{(v_x \cos(\psi)+v_y \sin(\psi)) \sin(\beta u_2)}{L/2} \\ \dot{v}_x & = & \alpha u_1 \cos\left(\beta u_2\right) \cos\left(\psi\right) - \alpha_f \left(1 + \left|\sin\left(\beta u_2\right)\right|\right) v_x \\ \dot{v}_y & = & \alpha u_1 \cos\left(\beta u_2\right) \sin\left(\psi\right) - \alpha_f \left(1 + \left|\sin\left(\beta u_2\right)\right|\right) v_y \end{cases} \quad (6)$$

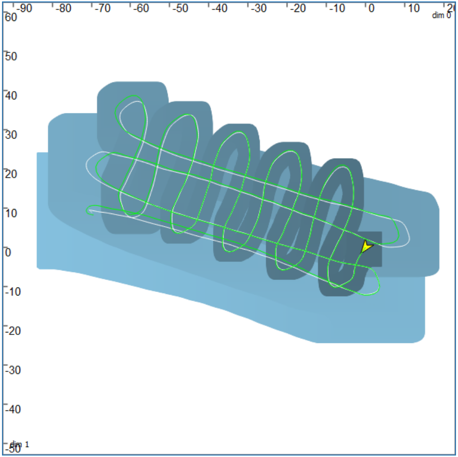

Figure: Trajectory estimation of the boat assuming the GPS cannot be used after 150 s (scale is in m).

Outline

If $\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix}$ is the position of the robot, $\mathbf{v}_r$ the speed vector measured directly by the DVL and $\mathbf{R}$ the Euler rotation matrix measured by the INS, we have

$$\dot{\mathbf{p}} = \mathbf{R} \cdot \mathbf{v}_r \tag{7}$$

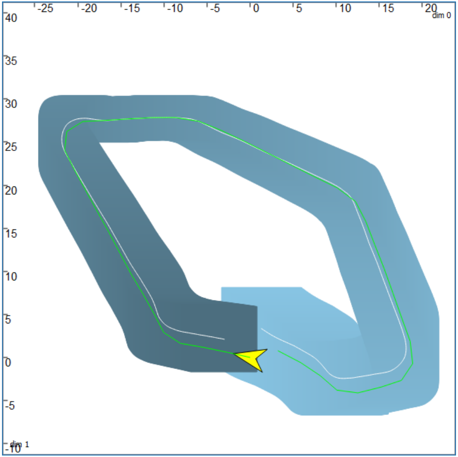

Figure: Trajectory estimation of the boat assuming the GPS cannot be used after 20 s, with DVL (scale is in m).

Summary

- The main contribution [3] is a first design of an interval-based INS, where the heading estimation is obtained from the fusion of dual GPS and gyrometers data in combination with a simple differential equation model
- Some scenarios show a better resilience to outliers compared to alternative systems
- The experiments made with an autonomous boat demonstrate the practical applicability of the approach

Summary

- Outlook
  - 3D
  - Fuzzy outliers handling as in [4]
  - Support better different types of disturbances, e.g. misleading GPS

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