# Minimizing Oscillations for Magnitude and Rate-Saturated Discrete-Time Systems by a $D_R$ Region Pole Placement

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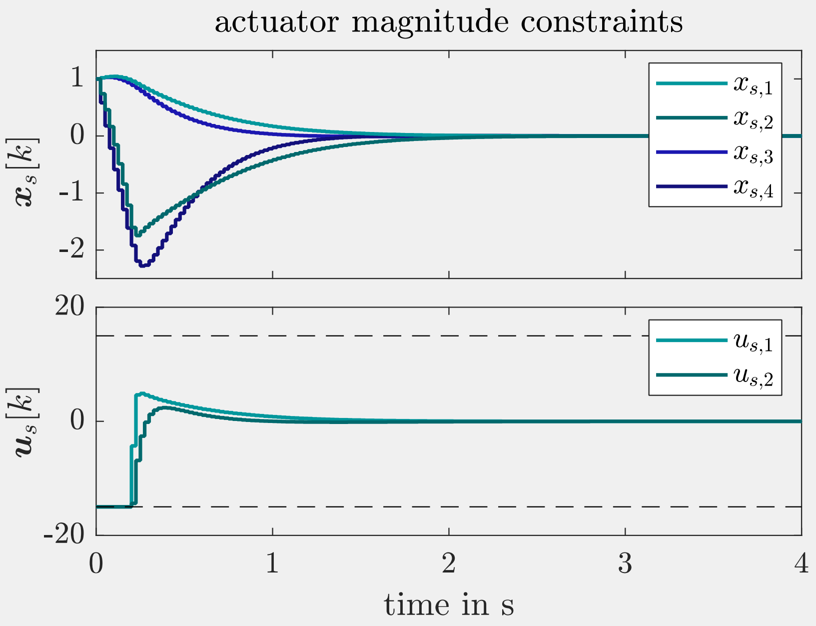
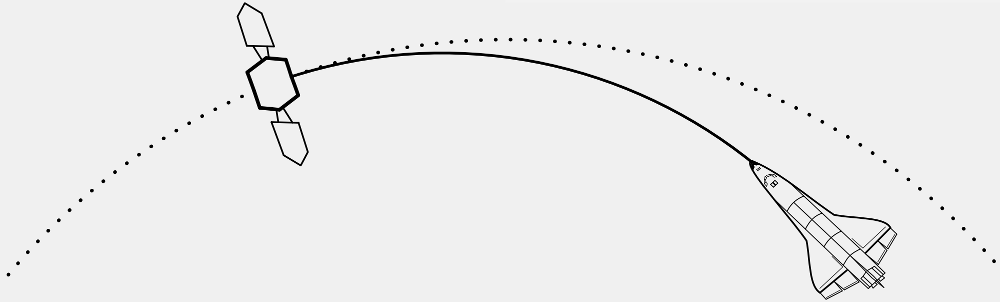
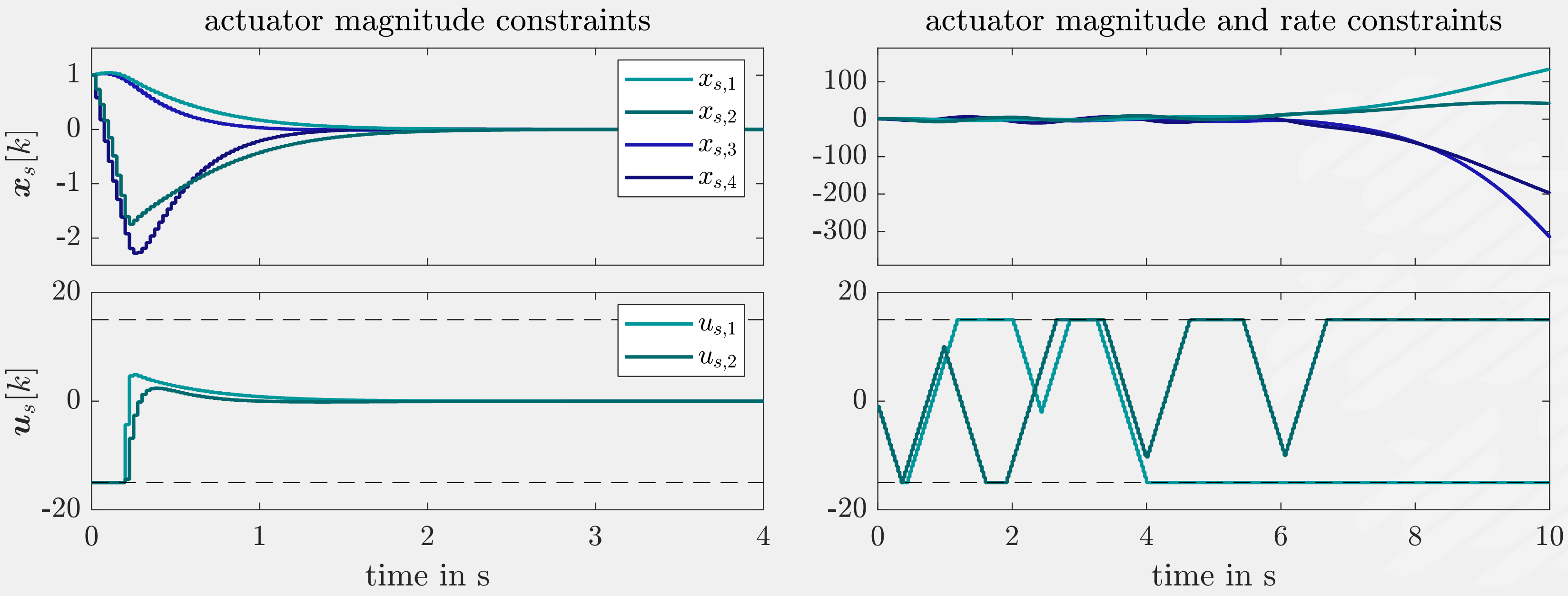
# Motivation: Stability of Systems with Actuator Constraints

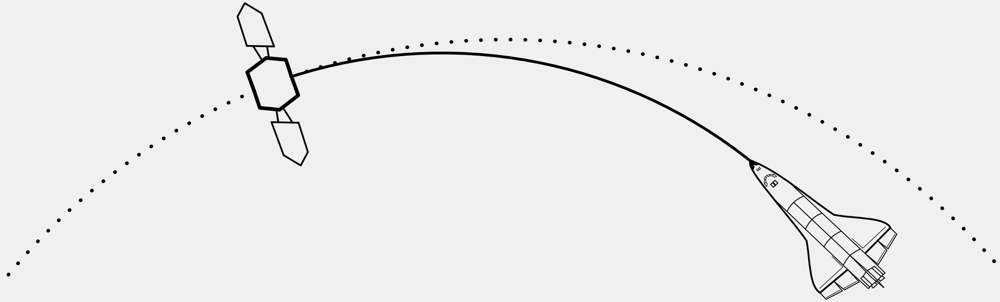




# Motivation: Stability of Systems with Actuator Constraints



Decay rate: maximum value of $\sigma$ such that

$$\|x[k]\| \leq m e^{-\sigma k} \|x[0]\|$$

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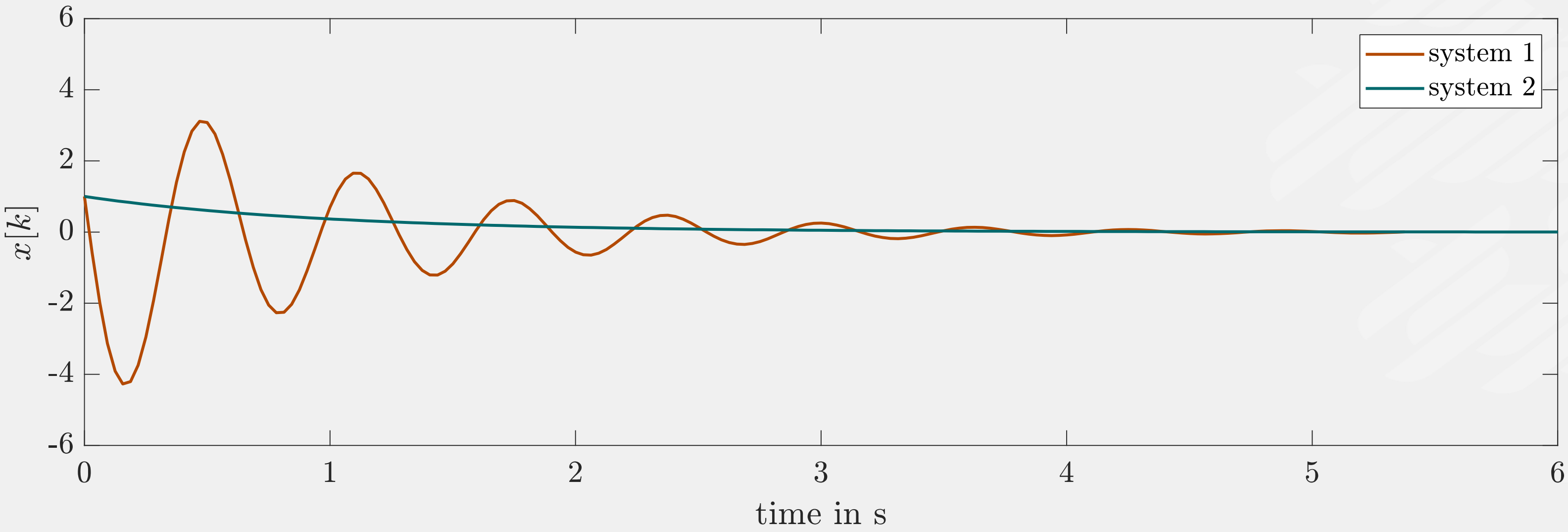

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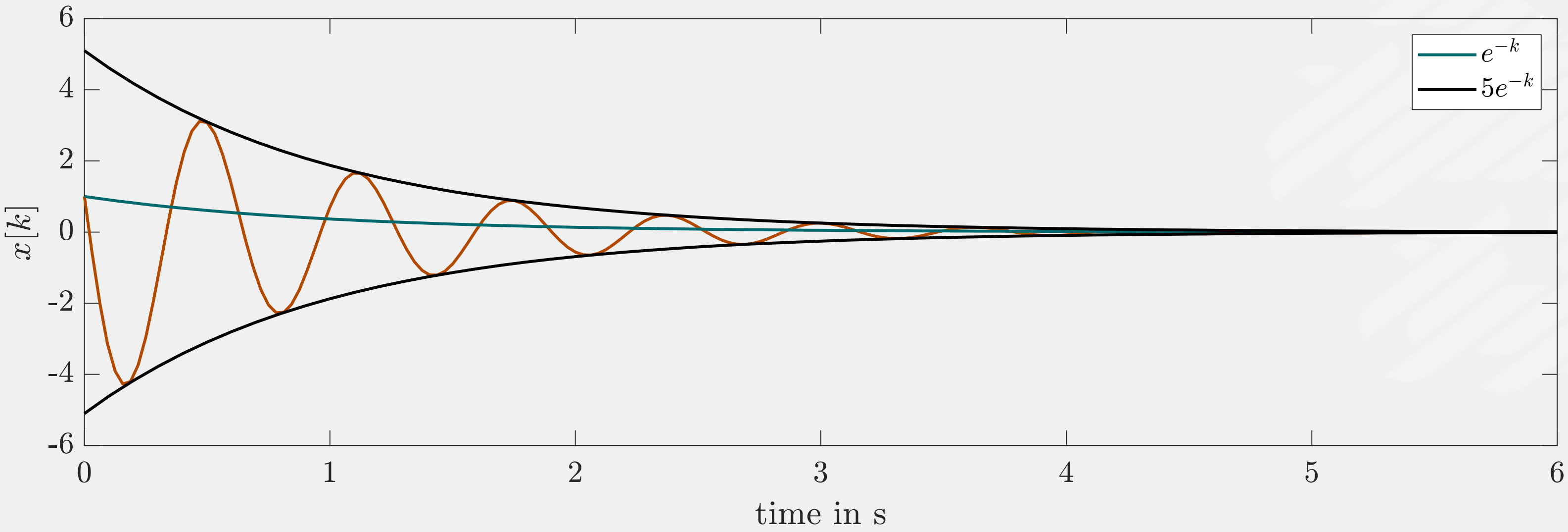
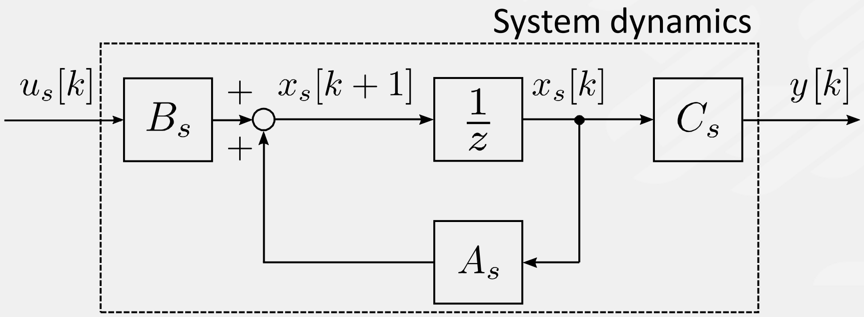


# Content

1. System description
   - Linear discrete time system
   - Nonlinear actuator with magnitude and rate saturation (MRS)
   - Different controller types
2. Stability conditions for MRS Systems
3. Maximization of the decay rate
4. Maximization of the damping
5. Iterative LMI Algorithm
6. Examples
7. Conclusion

$$x_s[k+1] = A_s x_s[k] + B_s u_s[k] \quad \text{with } x_s[0] = x_{s,0}$$
$$y[k] = C_s x_s[k]$$



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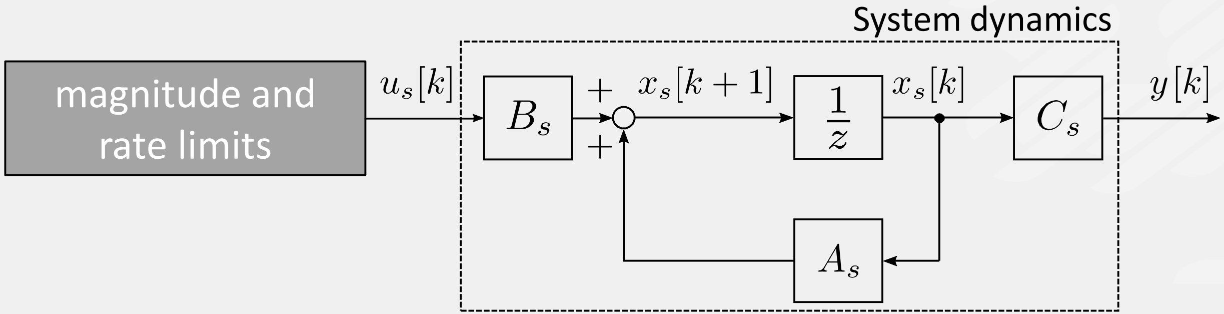
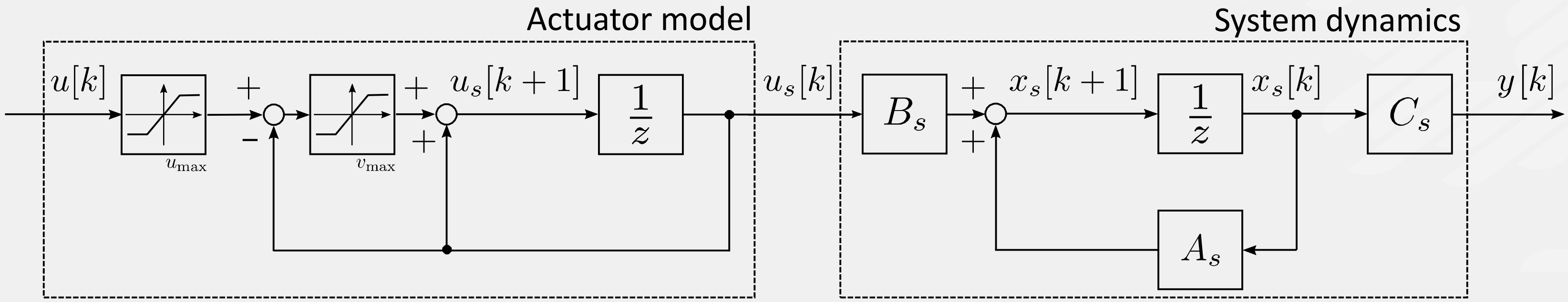


$$x_s[k+1] = A_s x_s[k] + B_s u_s[k] \quad \text{with } x_s[0] = x_{s,0}$$
$$y[k] = C_s x_s[k]$$

$$u_s[k+1] = u_s[k] + \text{sat}_V(\text{sat}_U(u[k]) - u_s[k])$$

$U$: magnitude saturation
$V$: rate saturation



# Augmented System of the Saturated Plant

$$x_s[k+1] = A_s x_s[k] + B_s u_s[k]$$
$$u_s[k+1] = u_s[k] + \text{sat}_V(\text{sat}_U(u[k]) - u_s[k])$$
$$y[k] = C_s x_s[k]$$

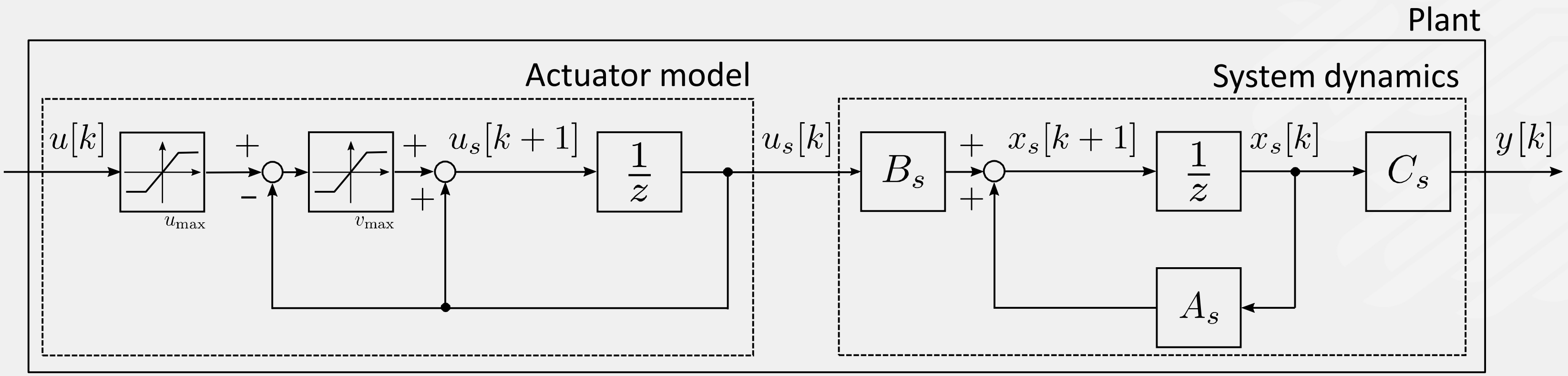
Augmented state vector $x[k] = \begin{pmatrix} x_s[k] \\ u_s[k] \end{pmatrix}$

Augmented system

$$x[k+1] = Ax[k] + B\text{sat}_V(\text{sat}_U(u[k]) + Fx[k])$$
$$y[k] = Cx[k]$$

$$A = \begin{pmatrix} A_s & B_s \\ 0 & I \end{pmatrix}, B = \begin{pmatrix} 0 \\ I \end{pmatrix}, F = \begin{pmatrix} 0 & -I \end{pmatrix}, C = \begin{pmatrix} C_s & 0 \end{pmatrix}$$

$$x[k+1] = Ax[k] + B\mathrm{sat}_V(\mathrm{sat}_U(u[k]) + Fx[k])$$
$$y[k] = Cx[k]$$



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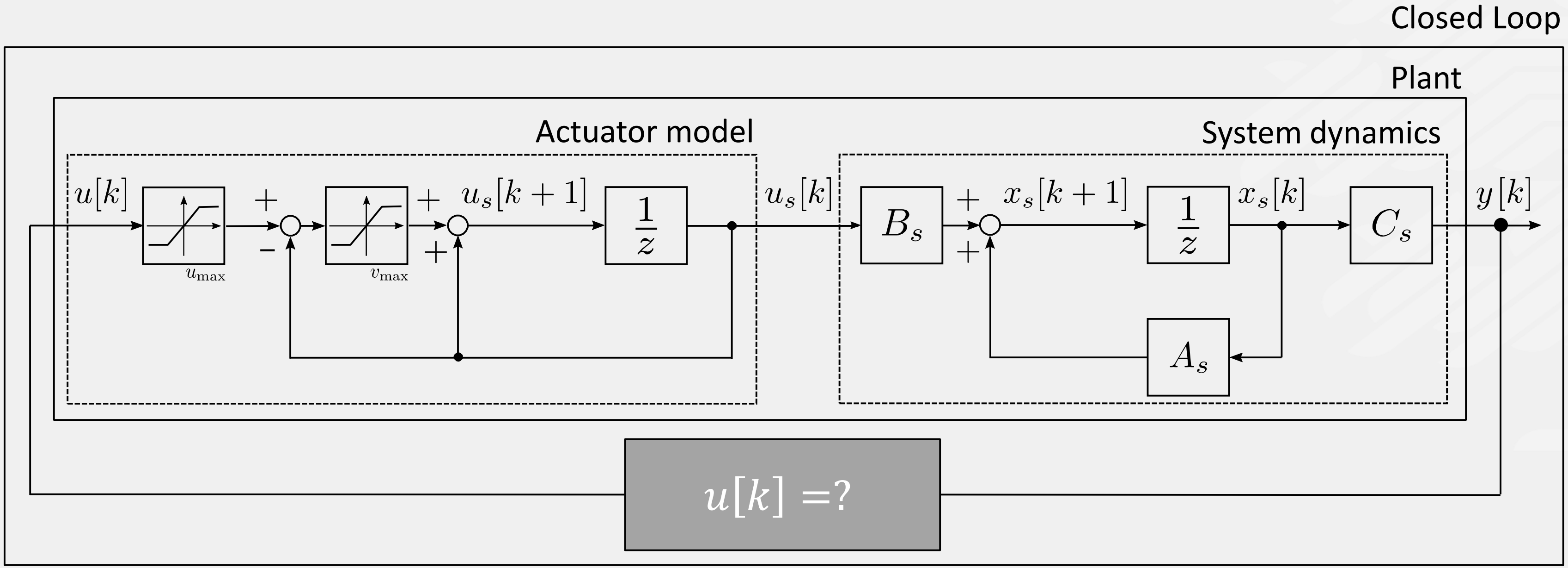
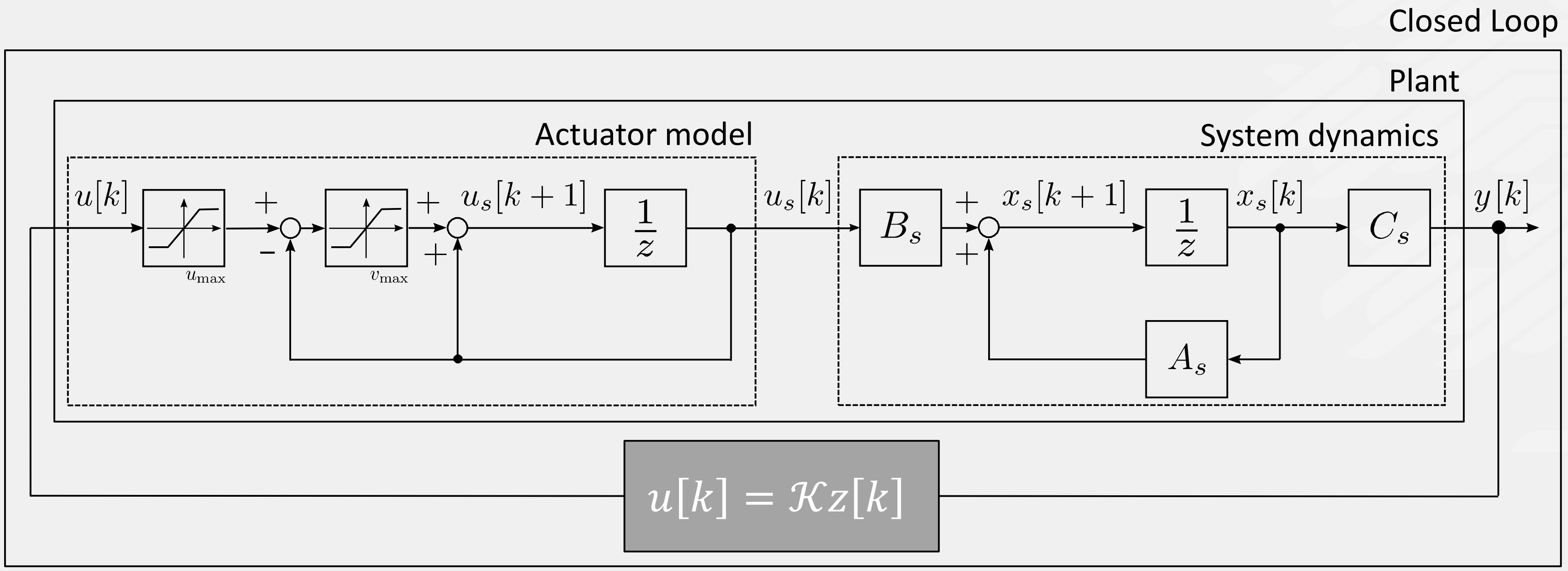


# Augmented Closed Loop

$$z[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

$$z[k] = \begin{pmatrix} x_s[k] \\ u_s[k] \\ x_c[k] \end{pmatrix}$$



# Augmented Closed Loop with Different Controller Types

$$z[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

| Controller type | $\mathcal{A}$ | $\mathcal{B}$ | $\mathcal{K}$ | $\mathcal{F}$ |
|---|---|---|---|---|
| Full state feedback | $A$ | $B$ | $K$ | $(0 \quad -I)$ |
| Static output feedback | $A$ | $B$ | $KC$ | $(0 \quad -I)$ |
| Observer-based feedback | $\begin{pmatrix} A & 0 \\ 0 & A_s - LC_s \end{pmatrix}$ | $\begin{pmatrix} B \\ 0 \end{pmatrix}$ | $(K \quad 0 - K)$ | $(0 \quad -I)$ |
| Dynamic output feedback | $\begin{pmatrix} A & 0 \\ B_c C & A_c \end{pmatrix}$ | $\begin{pmatrix} B \\ 0 \end{pmatrix}$ | $(D_c C \quad C_c)$ | $(0 \quad -I)$ |
| PID control | $\begin{pmatrix} A & 0 & 0 \\ -C & I & 0 \\ -C & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} B \\ 0 \\ 0 \end{pmatrix}$ | $((K_P + K_D)C \quad K_I \ - K_D)$ | $(0 \quad -I)$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

# Augmented Closed Loop with Different Controller Types

$$z[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

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| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

# Discrete-Time Stability Condition with a QLF

Quadratic Lyapunov Function $V(z[k]) = z^{\mathrm{T}}[k]Pz[k]$

Domain of Attraction $\mathcal{E}(P) = \{z \in \mathbb{R}^{n_z}: z^{\mathrm{T}}Pz \leq 1\}$

Stability Condition $z^{\mathrm{T}}[k+1]Pz[k+1] - z^{\mathrm{T}}[k]Pz[k] \prec 0$

Linear autonomous system $z[k+1] = \mathcal{A}z[k]$

Stability Condition

$$\mathcal{A}^{\mathrm{T}}P\mathcal{A} - P \prec 0$$

# Discrete-Time Stability Condition with a QLF

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Quadratic Lyapunov Function $V(z[k]) = z^{\mathrm{T}}[k]Pz[k]$

Domain of Attraction $\mathcal{E}(P) = \{z \in \mathbb{R}^{n_z}: z^{\mathrm{T}}Pz \leq 1\}$

Stability Condition $z^{\mathrm{T}}[k+1]Pz[k+1] - z^{\mathrm{T}}[k]Pz[k] \prec 0$

Nonlinear system $z[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$

Stability Condition

$$\left(\mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])\right)^{\mathrm{T}} P\left(\mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])\right) - z^{\mathrm{T}}[k]Pz[k] \prec 0$$

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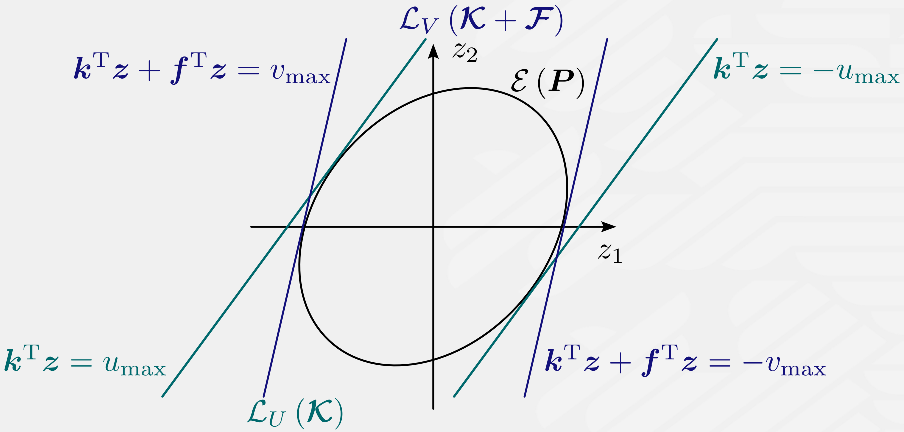
$$z[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

Linear when not saturated

$$\mathrm{sat}_U(\mathcal{K}z[k]) = \mathcal{K}z[k]$$

$$\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k]) = (\mathcal{K}+\mathcal{F})z[k]$$




$$\mathcal{L}_U(\mathcal{K}) = \left\{z \in \mathbb{R}^{n_z} \middle| \left|k_q^{\mathrm{T}} z\right| < u_{\max,q}, q = 1, \dots, m\right\}$$

$$\mathcal{L}_V(\mathcal{K}+\mathcal{F}) = \left\{z \in \mathbb{R}^{n_z} \middle| \left|k_q^{\mathrm{T}} z + f_q^{\mathrm{T}} z\right| < v_{\max,q}, q = 1, \dots, m\right\}$$

$$z[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

Linear when not saturated

$$\mathrm{sat}_U(\mathcal{K}z[k]) = \mathcal{K}z[k]$$

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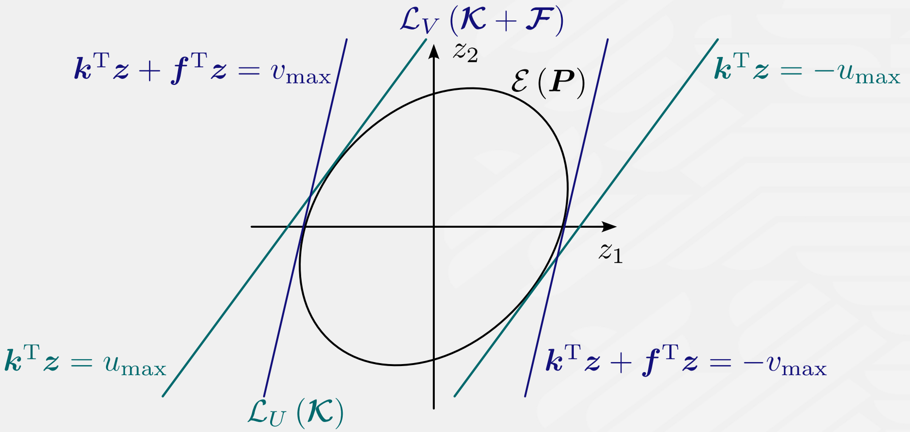


Stability conditions

$$\left(\mathcal{A}+\mathcal{B}(\mathcal{K}+\mathcal{F})\right)^{\mathrm{T}} P\left(\mathcal{A}+\mathcal{B}(\mathcal{K}+\mathcal{F})\right) - P \prec 0$$

$$\mathcal{E}(P) \subseteq \mathcal{L}_U(\mathcal{K}) \cap \mathcal{L}_V(\mathcal{K}+\mathcal{F})$$

$$\mathcal{L}_U(\mathcal{K}) = \{z \in \mathbb{R}^{n_z} \mid |k_q^{\mathrm{T}} z| < u_{\max,q}, q = 1, \dots, m\}$$

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Saturated controller

$$u_s[k] = \mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

Auxiliary controllers

$$u_s[k] = \mathrm{sat}_V(\mathrm{sat}_U(\mathcal{H}_1 z[k]) + \mathcal{F}z[k])$$

$$u_s[k] = \mathrm{sat}_V(\mathcal{H}_2 z[k])$$

with $\mathrm{sat}_U(\mathcal{H}_1 z[k]) = \mathcal{H}_1 z[k]$

$\mathrm{sat}_V(\mathcal{H}_2 z[k]) = \mathcal{H}_2 z[k]$

Saturated controller

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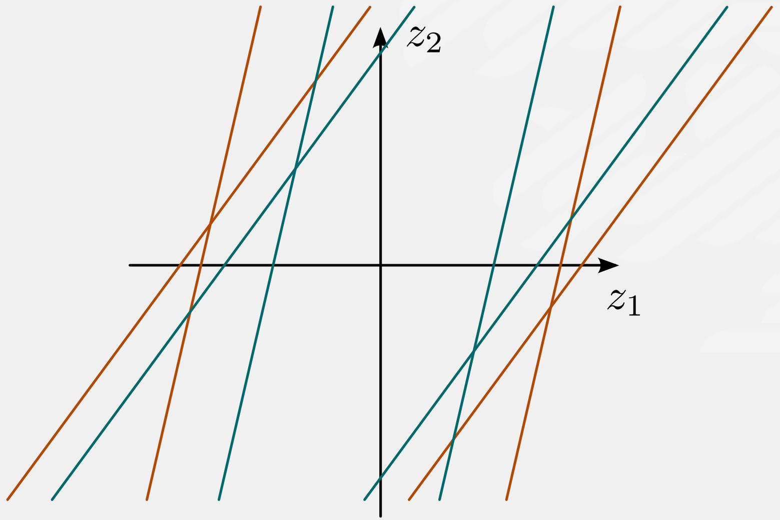
$\text{sat}_V(\mathcal{H}_2 z[k]) = \mathcal{H}_2 z[k]$

$$\mathcal{L}_U(\mathcal{K}) = \{z \in \mathbb{R}^{n_z} |\, |k_q^{\mathrm{T}} z| < u_{\max,q}, q = 1, \dots, m\}$$

$$\mathcal{L}_V(\mathcal{K} + \mathcal{F}) = \{z \in \mathbb{R}^{n_z} |\, |k_q^{\mathrm{T}} z + f_q^{\mathrm{T}} z| < v_{\max,q}, q = 1, \dots, m\}$$

$$\mathcal{L}_U(\mathcal{H}_1) = \{z \in \mathbb{R}^{n_z} |\, |h_{1,q}^{\mathrm{T}} z| < u_{\max,q}, q = 1, \dots, m\}$$

$$\mathcal{L}_V(\mathcal{H}_2) = \{z \in \mathbb{R}^{n_z} |\, |h_{2,q}^{\mathrm{T}} z| < v_{\max,q}, q = 1, \dots, m\}$$



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Saturated controller

$$u_s[k] = \mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

Auxiliary controllers

$$u_s[k] = \mathrm{sat}_V(\mathrm{sat}_U(\mathcal{H}_1 z[k]) + \mathcal{F}z[k])$$

$$u_s[k] = \mathrm{sat}_V(\mathcal{H}_2 z[k])$$

with $\mathrm{sat}_U(\mathcal{H}_1 z[k]) = \mathcal{H}_1 z[k]$

$\mathrm{sat}_V(\mathcal{H}_2 z[k]) = \mathcal{H}_2 z[k]$

$$\Rightarrow \mathcal{E}(P) \subseteq \mathcal{L}_V(\mathcal{H}_1) \cap \mathcal{L}_U(\mathcal{H}_2)$$

$$\mathcal{L}_U(\mathcal{K}) = \{z \in \mathbb{R}^{n_z} \mid |k_q^{\mathrm{T}} z| < u_{\max,q}, q = 1, \dots, m\}$$

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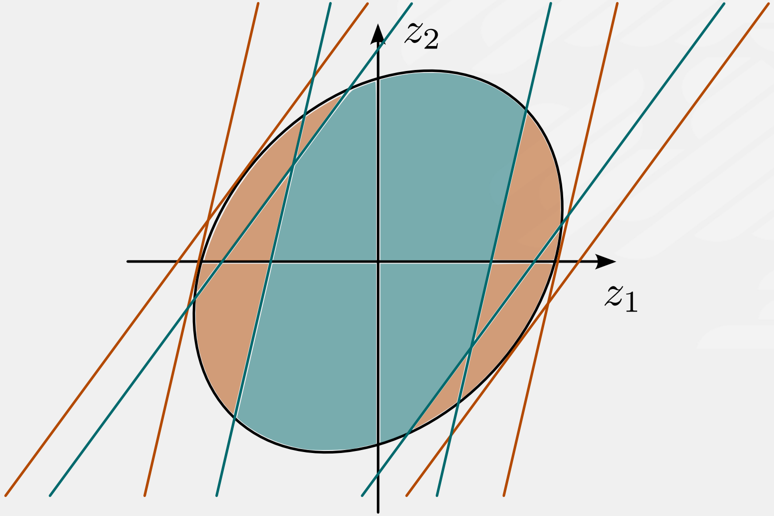

Saturated controller

$$u_s[k] = \text{sat}_V(\text{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

Linear case

$$u_s[k] = (\mathcal{K} + \mathcal{F})z[k]$$

Auxiliary controllers

$$u_s[k] = \text{sat}_V(\text{sat}_U(\mathcal{H}_1 z[k]) + \mathcal{F}z[k])$$

$$u_s[k] = \text{sat}_V(\mathcal{H}_2 z[k])$$

with $\text{sat}_U(\mathcal{H}_1 z[k]) = \mathcal{H}_1 z[k]$

$\text{sat}_V(\mathcal{H}_2 z[k]) = \mathcal{H}_2 z[k]$

$$u_s[k] = (\mathcal{H}_1 + \mathcal{F})z[k]$$

$$u_s[k] = \mathcal{H}_2 z[k]$$

Convex combination of linear feedbacks

$$\Xi_i = D_{i,1}(\mathcal{K} + \mathcal{F}) + D_{i,2}(\mathcal{H}_1 + \mathcal{F}) + D_{i,3}\mathcal{H}_2, i = 1, \dots, 3^m$$

$$\text{sat}_V(\text{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k]) \in \text{co}\{\Xi_i z[k], i = 1, \dots, 3^m\}$$

Convex combination of linear feedbacks

$$\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k]) \,\epsilon\, \mathrm{co}\{\Xi_i z[k], i = 1, \dots, 3^m\}$$

with $\Xi_i = D_{i,1}(\mathcal{K} + \mathcal{F}) + D_{i,2}(\mathcal{H}_1 + \mathcal{F}) + D_{i,3}\mathcal{H}_2, i = 1, \dots, 3^m$

Nonlinear system

$$z[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

$$\epsilon\, \mathrm{co}\{\tilde{\mathcal{A}}_i z[k] = \mathcal{A}z[k] + \mathcal{B}\Xi_i z[k], i = 1, \dots, 3^m\}$$

Convex combination of linear feedbacks

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Nonlinear system

$$z[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

$$\epsilon\, \mathrm{co}\{\tilde{\mathcal{A}}_i z[k] = \mathcal{A}z[k] + \mathcal{B}\Xi_i z[k], i = 1, \ldots, 3^m\}$$

Stability conditions

$$\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i - P \prec 0, i = 1, \ldots, 3^m$$

$$\mathcal{E}(P) \subseteq \mathcal{L}_V(\mathcal{H}_1) \cap \mathcal{L}_U(\mathcal{H}_2)$$

Convex combination of linear feedbacks

$$\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k]) \,\epsilon\, \mathrm{co}\{\Xi_i z[k], i = 1, \dots, 3^m\}$$

with $\Xi_i = D_{i,1}(\mathcal{K} + \mathcal{F}) + D_{i,2}(\mathcal{H}_1 + \mathcal{F}) + D_{i,3}\mathcal{H}_2, i = 1, \dots, 3^m$

Nonlinear system

$$z[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(\mathcal{K}z[k]) + \mathcal{F}z[k])$$

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$$\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i - P \prec 0, i = 1, \dots, 3^m \qquad \Rightarrow \begin{pmatrix} P^{-1} & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0, i = 1, \dots, 3^m$$

$$\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i - P \prec 0, i = 1, \dots, 3^m \qquad \Longrightarrow \begin{pmatrix} P^{-1} & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0, i = 1, \dots, 3^m$$

Full state feedback: $\mathrm{z}[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(Kz[k]) + \mathcal{F}z[k])$

$$\begin{pmatrix} P^{-1} & \mathcal{A} + \mathcal{B}(D_{i,1}(K + \mathcal{F}) + D_{i,2}(\mathcal{H}_1 + \mathcal{F}) + D_{i,3}\mathcal{H}_2) \\ \star & P \end{pmatrix} \succ 0, i = 1, \dots, 3^m$$

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$$\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i - P \prec 0, i = 1, \dots, 3^m \qquad \Longrightarrow \begin{pmatrix} P^{-1} & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0, i = 1, \dots, 3^m$$

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Standard approach (e.g. Bateman and Lin (2002))

Change of variables $Q = P^{-1}, Y = KQ, G_1 = \mathcal{H}_1 Q, G_2 = \mathcal{H}_2 Q$

$$\Longrightarrow \begin{pmatrix} Q & \mathcal{A}Q + \mathcal{B}(D_{i,1}(Y + \mathcal{F}Q) + D_{i,2}(G_1 + \mathcal{F}Q) + D_{i,3}G_2) \\ \star & Q \end{pmatrix} \succ 0, i = 1, \dots, 3^m$$

Bateman, A. ; Lin, Z. : An analysis and design method for discrete-time linear systems under nested saturation. In: *IEEE Transactions on Automatic Control* 47 (2002), Nr. 8, S. 1305–1310

$$\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i - P \prec 0, i = 1, \ldots, 3^m \qquad \Longrightarrow \begin{pmatrix} P^{-1} & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0, i = 1, \ldots, 3^m$$

Static output feedback: $\mathrm{z}[k+1] = \mathcal{A}z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(KCz[k]) + \mathcal{F}z[k])$

$$\begin{pmatrix} P^{-1} & \mathcal{A} + \mathcal{B}(D_{i,1}(KC + \mathcal{F}) + D_{i,2}(\mathcal{H}_1 + \mathcal{F}) + D_{i,3}\mathcal{H}_2) \\ \star & P \end{pmatrix} \succ 0, i = 1, \ldots, 3^m$$

Bateman and Lin (2002): $Y = KCQ$

# Reformulation of the Lyapunov Condition for Static Output Feedback

$$\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i - P \prec 0, i = 1, \dots, 3^m \qquad \Rightarrow \begin{pmatrix} P^{-1} & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0, i = 1, \dots, 3^m$$

Static output feedback: $\mathrm{z}[k+1] = \mathcal{A} z[k] + \mathcal{B}\mathrm{sat}_V(\mathrm{sat}_U(KCz[k]) + \mathcal{F}z[k])$

$$\begin{pmatrix} P^{-1} & \mathcal{A} + \mathcal{B}(D_{i,1}(KC + \mathcal{F}) + D_{i,2}(\mathcal{H}_1 + \mathcal{F}) + D_{i,3}\mathcal{H}_2) \\ \star & P \end{pmatrix} \succ 0, i = 1, \dots, 3^m$$

Standard approach (Crusius and Trofino (1999))

Change of variables $Q = P^{-1}, Y = KN, G_1 = \mathcal{H}_1 Q, G_2 = \mathcal{H}_2 Q$

$$\Rightarrow \begin{pmatrix} Q & \mathcal{A}Q + \mathcal{B}(D_{i,1}(YC + \mathcal{F}Q) + D_{i,2}(G_1 + \mathcal{F}Q) + D_{i,3}G_2) \\ \star & Q \end{pmatrix} \succ 0, i = 1, \dots, 3^m$$

$$NC - CQ = 0$$

Crusius, C. ; Trofino, A. : Sufficient LMI conditions for output feedback control problems.
In: *IEEE Transactions on Automatic Control* 44 (1999), Nr. 5, S. 1053–1057

# Reformulation of the Lyapunov Condition for Different Controller Types

$$\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i - P \prec 0, i = 1, \dots, 3^m \qquad \Longrightarrow \begin{pmatrix} P^{-1} & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0, i = 1, \dots, 3^m$$

Linearization of $P^{-1}$ (Dehnert (2020))

$$L = \hat{P}^{-1}\left(2I - P\hat{P}^{-1}\right) \preccurlyeq P^{-1}$$

$$\begin{pmatrix} L & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0 \Rightarrow \begin{pmatrix} P^{-1} & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0$$

$$\begin{pmatrix} \hat{P}^{-1}\left(2I - P\hat{P}^{-1}\right) & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0, \qquad i = 1, \dots, 3^m$$

with $\hat{P} = \hat{P}^{\mathrm{T}} = \mathrm{const}$

$\hat{P} = P$

Dehnert, R. : *Entwurf robuster Regler mit Ausgangsrückführung für zeitdiskrete Mehrgrößensysteme*. Springer Vieweg Wiesbaden, 2020

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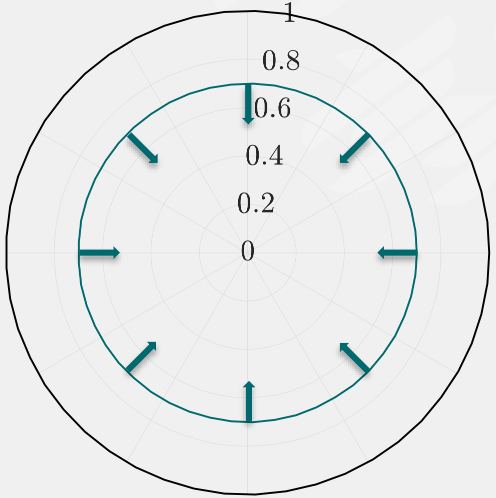
$$\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i - P \prec 0, i = 1, \dots, 3^m \quad \Longrightarrow \begin{pmatrix} P^{-1} & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0, i = 1, \dots, 3^m$$

Linearization of $P^{-1}$ (Dehnert (2020))

$$L = \hat{P}^{-1}\left(2I - P\hat{P}^{-1}\right) \preccurlyeq P^{-1} \qquad \begin{pmatrix} L & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0 \Rightarrow \begin{pmatrix} P^{-1} & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0$$

$$\begin{pmatrix} \hat{P}^{-1}\left(2I - P\hat{P}^{-1}\right) & \tilde{\mathcal{A}}_i \\ \star & P \end{pmatrix} \succ 0, \qquad i = 1, \dots, 3^m$$




Maximization of the decay rate

$$\min \bar{r}$$
$$\text{s.t.} \begin{pmatrix} \hat{P}^{-1}\left(2I - P\hat{P}^{-1}\right) & \tilde{\mathcal{A}}_i \\ \star & \bar{r}^2 P \end{pmatrix} \succ 0, \quad i = 1, \dots, 3^m$$

with $\hat{P} = \hat{P}^{\mathrm{T}} = \text{const}$

# Additional Stability Conditions

Linear auxiliary controllers

$$\mathcal{E}(P) \subseteq \mathcal{L}_V(\mathcal{H}_1) \cap \mathcal{L}_U(\mathcal{H}_2)$$

$$\begin{aligned}
&\begin{pmatrix} W_1 & \mathcal{H}_1 \\ \star & P \end{pmatrix} \succ 0, \\
&\begin{pmatrix} W_2 & \mathcal{H}_2 \\ \star & P \end{pmatrix} \succ 0, \\
&w_{1,qq} < u_{\max,q}^2, \qquad q = 1, \dots, m, \\
&w_{2,qq} < v_{\max,q}^2, \qquad q = 1, \dots, m
\end{aligned}$$

Linear auxiliary controllers

$$\mathcal{E}(P) \subseteq \mathcal{L}_V(\mathcal{H}_1) \cap \mathcal{L}_U(\mathcal{H}_2)$$

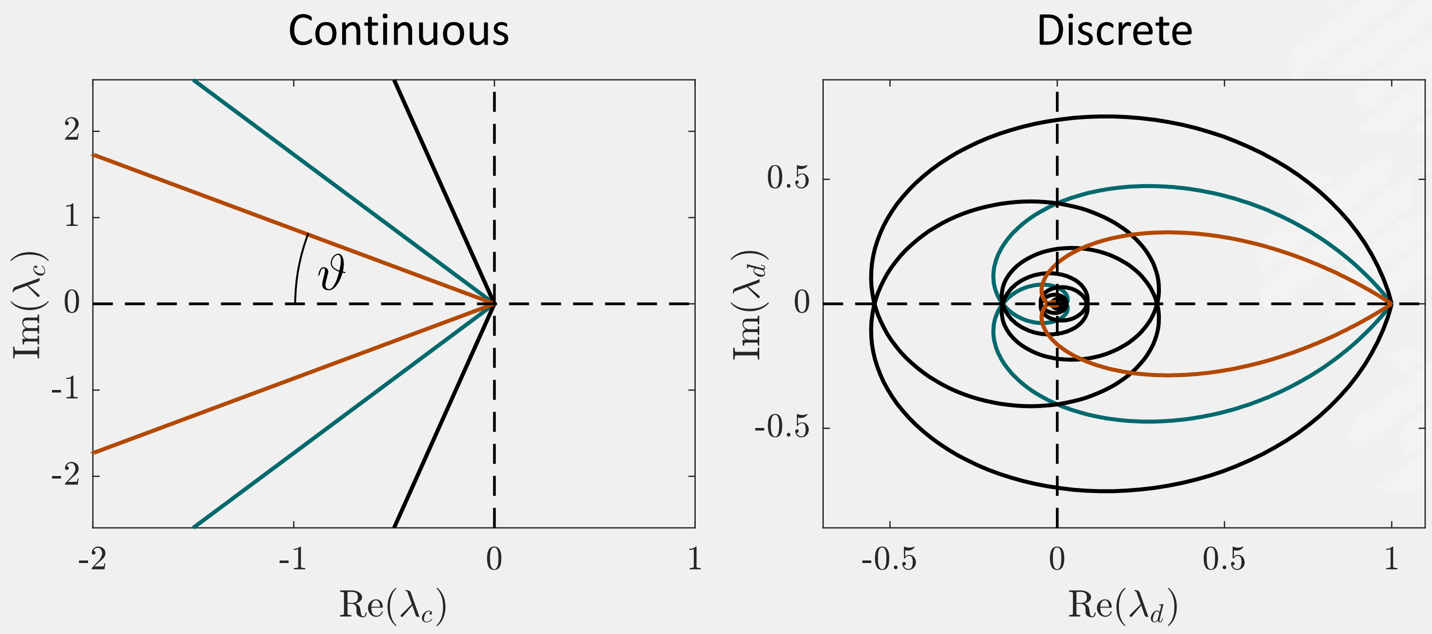
$$\begin{pmatrix} W_1 & \mathcal{H}_1 \\ \star & P \end{pmatrix} \succ 0,$$
$$\begin{pmatrix} W_2 & \mathcal{H}_2 \\ \star & P \end{pmatrix} \succ 0,$$
$$w_{1,qq} < u_{\max,q}^2, \qquad q = 1, \dots, m,$$
$$w_{2,qq} < v_{\max,q}^2, \qquad q = 1, \dots, m$$

Initial state area

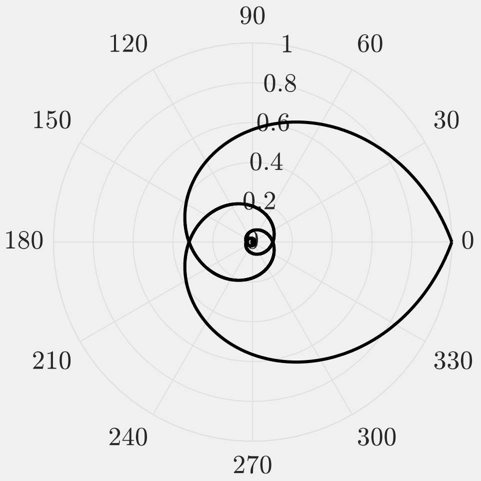
$$\mathcal{X}_0 = \mathrm{co}\left\{x_{0,1}, \dots, x_{0,N_{x_0}}\right\}$$

$$\mathcal{X}_0 \subseteq \mathcal{E}(P)$$

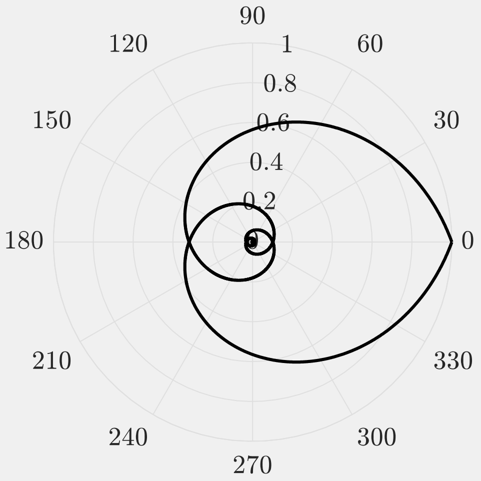
$$\begin{pmatrix} 1 & x_{0,s}^{\mathrm{T}} P \\ \star & P \end{pmatrix} \succ 0, \qquad s = 1, \dots, N_{x_0}$$

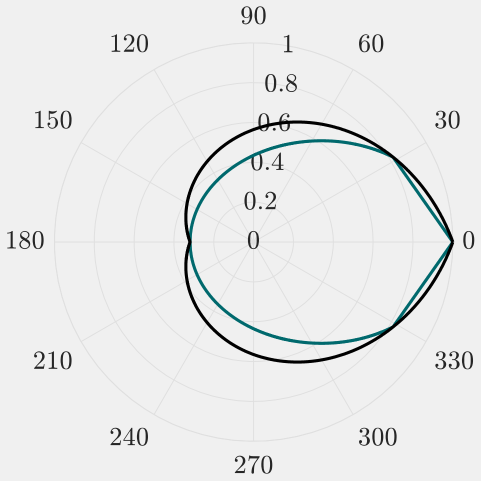

Continuous
Discrete
2
1
0
-1
-2
$\vartheta$
$\mathrm{Im}(\lambda_c)$
-2
-1
0
1
$\mathrm{Re}(\lambda_c)$
0.5
0
-0.5
$\mathrm{Im}(\lambda_d)$
-0.5
0
0.5
1
$\mathrm{Re}(\lambda_d)$

# Angle-Ellipse



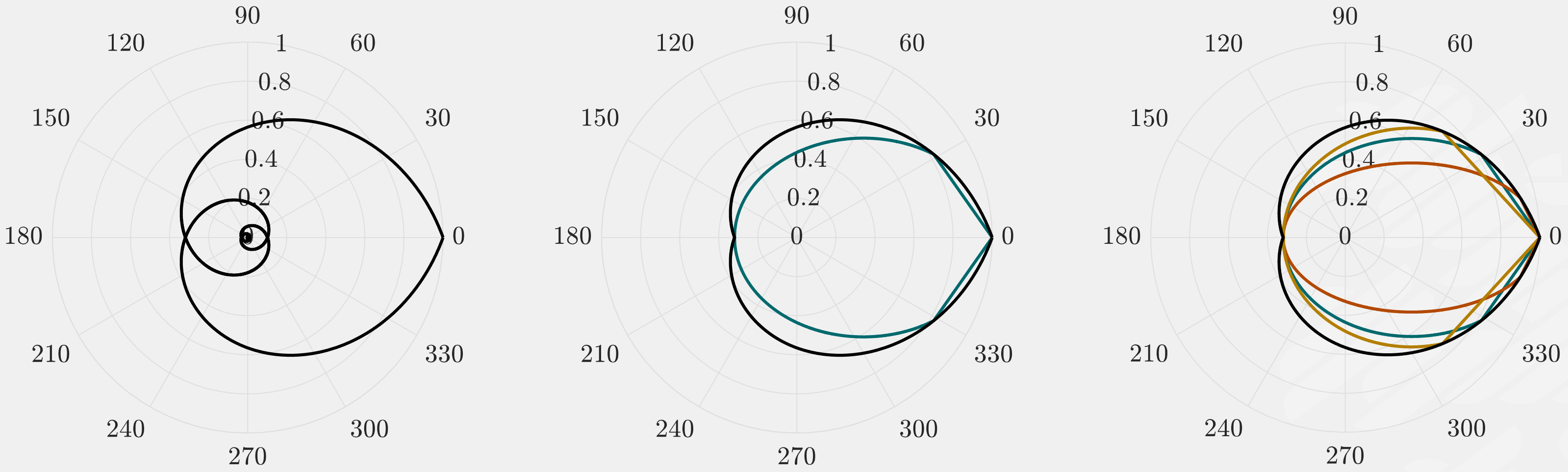
# Angle-Ellipse




Rosinova, D. ; Hypiusova, M. : LMI Pole Regions for a Robust Discrete-Time Pole Placement Controller Design. In: Algorithms 12 (2019), Nr. 8

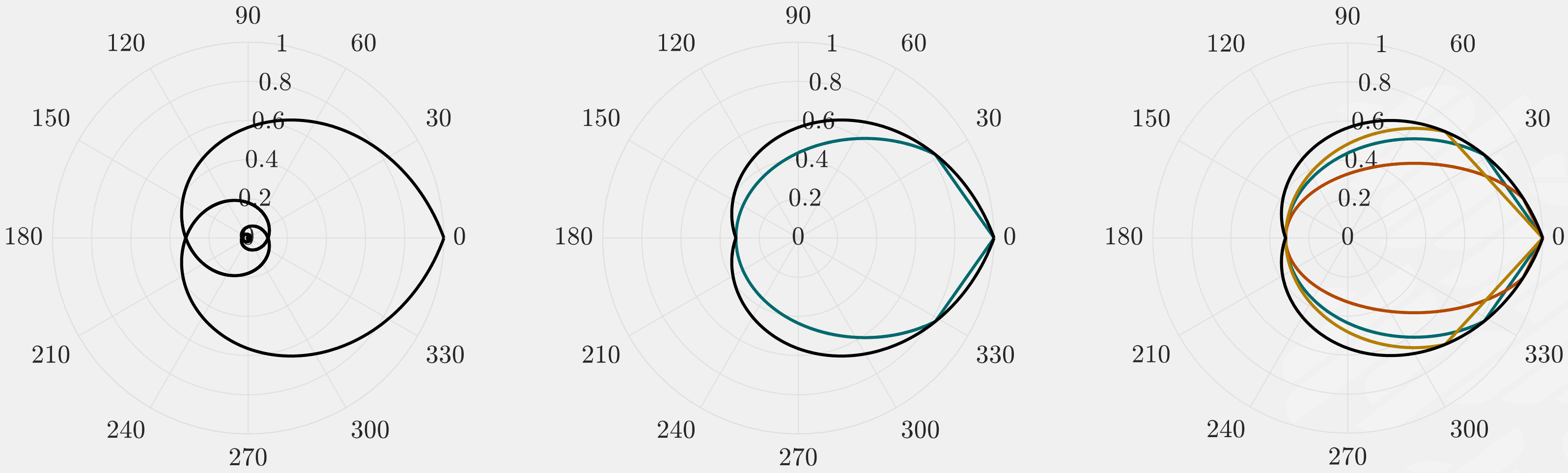
# Angle-Ellipse

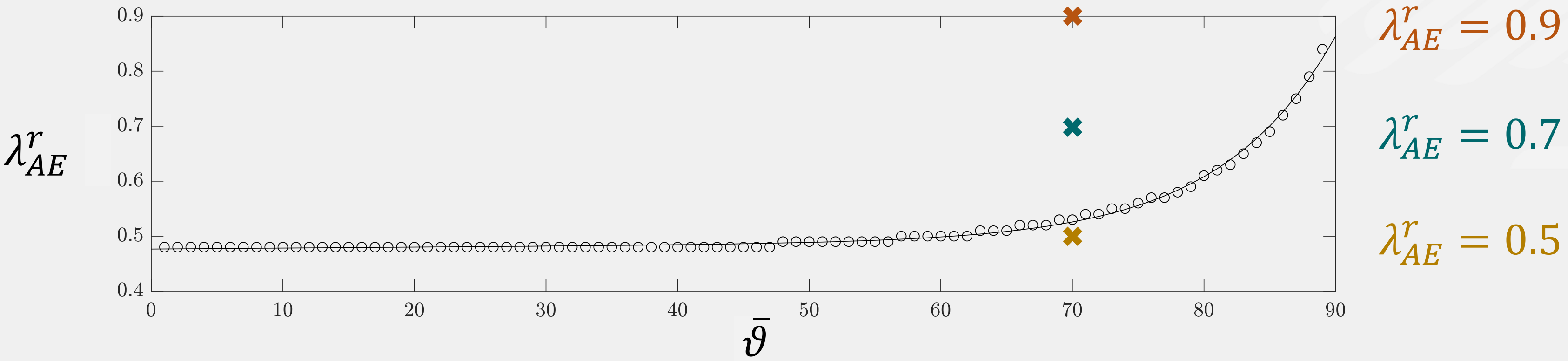



$\lambda_{AE}^{r} = 0.9$

$\lambda_{AE}^{r} = 0.7$

$\lambda_{AE}^{r} = 0.5$


90
120
60
150
30
180
0
210
330
240
300
270
1
0.8
0.6
0.4
0.2
0


$\lambda^r_{AE}$
$\bar{\vartheta}$
0.4
0.5
0.6
0.7
0.8
0.9
0
10
20
30
40
50
60
70
80
90
$\lambda^r_{AE} = 0.9$
$\lambda^r_{AE} = 0.7$
$\lambda^r_{AE} = 0.5$

$$D_R = \left\{ z \in \mathbb{C} : R_{11} + R_{12} z + R_{12}^{\mathrm{T}} z^* + R_{22} z z^* \prec 0 \right\}$$

$$z = \mathrm{Re}(z) + \mathrm{j}\,\mathrm{Im}(z), \qquad z^* = \mathrm{Re}(z) - \mathrm{j}\,\mathrm{Im}(z)$$

# $D_R$ Region

$$D_R = \left\{ z \in \mathbb{C} : R_{11} + R_{12} z + R_{12}^{\mathrm{T}} z^* + R_{22} z z^* \prec 0 \right\}$$

$$z = \mathrm{Re}(z) + \mathrm{j}\,\mathrm{Im}(z), \qquad z^* = \mathrm{Re}(z) - \mathrm{j}\,\mathrm{Im}(z)$$

Circle with upper bound
of specral radii $\bar{r}$

| |
|---|
| $R_{11} = -\bar{r}^{\,2}$ |
| $R_{12} = 0$ |
| $R_{22} = 1$ |

$$\Longrightarrow \mathrm{Re}^2 + \mathrm{Im}^2 < \bar{r}^{\,2}$$

# $D_R$ Region

$$D_R = \left\{ z \in \mathbb{C} : R_{11} + R_{12} z + R_{12}^{\mathrm{T}} z^* + R_{22} z z^* \prec 0 \right\}$$

$$z = \mathrm{Re}(z) + \mathrm{j}\,\mathrm{Im}(z), \qquad z^* = \mathrm{Re}(z) - \mathrm{j}\,\mathrm{Im}(z)$$

Circle with upper bound of specral radii $\bar{r}$

$$R_{11} = -\bar{r}^{\,2}$$
$$R_{12} = 0$$
$$R_{22} = 1$$

$$\Longrightarrow \mathrm{Re}^2 + \mathrm{Im}^2 < \bar{r}^{\,2}$$

Angle-Ellipse with upper bound of damping angle $\bar{\vartheta}$

$$R_{11} = \begin{pmatrix} -1 & -\lambda_M/a & 0 & 0 \\ -\lambda_M/a & -1 & 0 & 0 \\ 0 & 0 & -2\sin\bar{\vartheta} & 0 \\ 0 & 0 & 0 & -2\sin\bar{\vartheta} \end{pmatrix}$$

$$R_{12} = \begin{pmatrix} 0 & (1/a - 1/b)/2 & 0 & 0 \\ (1/a + 1/b)/2 & 0 & 0 & 0 \\ 0 & 0 & \sin\bar{\vartheta} & \cos\bar{\vartheta} \\ 0 & 0 & -\cos\bar{\vartheta} & \sin\bar{\vartheta} \end{pmatrix}$$

$$R_{22} = 0$$

# LMI Formulation of $D_R$ Region Pole Placement

$$R_{11} \otimes P + \mathrm{He}(R_{12} \otimes (P\tilde{\mathcal{A}}_i)) + R_{22} \otimes (\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i) \prec 0$$

# LMI Formulation of $D_R$ Region Pole Placement

$$R_{11} \otimes P + \mathrm{He}(R_{12} \otimes (P\tilde{\mathcal{A}}_i)) + R_{22} \otimes (\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i) \prec 0$$

⇓ Peaucelle et. al (2000)

$$\begin{pmatrix} R_{11} \otimes P + \mathrm{He}(F(I \otimes \tilde{\mathcal{A}}_i)) & R_{12} \otimes P + \left(I \otimes \tilde{\mathcal{A}}_i^{\mathrm{T}}\right) G - F \\ \star & R_{22} \otimes P - G - G^{\mathrm{T}} \end{pmatrix} \prec 0$$

Peaucelle, D. ; Arzelier, D. ; Bachelier, J. ; Bernussou, J. : A New Robust D-Stability Condition for Real Convex Polytopic Uncertainty. In: *Systems and Control Letters* 40 (2000), S. 21–30

$$R_{11}\otimes P + \mathrm{He}(R_{12}\otimes(P\tilde{\mathcal{A}}_i)) + R_{22}\otimes(\tilde{\mathcal{A}}_i^{\mathrm{T}} P \tilde{\mathcal{A}}_i) \prec 0$$

⇓ Peaucelle et. al (2000)

$$\begin{pmatrix} R_{11}\otimes P + \mathrm{He}(F(I\otimes\tilde{\mathcal{A}}_i)) & R_{12}\otimes P + \left(I\otimes\tilde{\mathcal{A}}_i^{\mathrm{T}}\right)G - F \\ \star & R_{22}\otimes P - G - G^{\mathrm{T}} \end{pmatrix} \prec 0$$

⇓ Lerch et. al (2022)

$F = R_{12}\otimes\hat{P}$ and $G = R_{22}\otimes\hat{P}$ with $\hat{P} = \hat{P}^{\mathrm{T}} = \mathrm{const}$

$$\begin{pmatrix} R_{11}\otimes P + \mathrm{He}(R_{12}\otimes(\hat{P}\tilde{\mathcal{A}}_i)) & R_{12}\otimes(P-\hat{P}) + R_{22}\otimes\left(\hat{P}\tilde{\mathcal{A}}_i\right)^T \\ \star & R_{22}\otimes(P-2\hat{P}) \end{pmatrix} \prec 0$$

Lerch, S. ; Dehnert, R. ; Rosik, M. ; Tibken, B. : Minimizing Oscillations for Magnitude and Rate-Saturated Discrete-Time Systems by a $D_R$ Region Pole Placement. In: *2022 10th International Conference on Systems and Control (ICSC)*, 2022, S. 244–249

# LMI Formulation of $D_R$ Region Pole Placement

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$$R_{11}\otimes P + \mathrm{He}(R_{12}\otimes(P\tilde{\mathcal{A}}_i)) + R_{22}\otimes(\tilde{\mathcal{A}}_i^{\mathrm{T}} P\tilde{\mathcal{A}}_i) \prec 0$$

Peaucelle et. al (2000)

$$\begin{pmatrix} R_{11}\otimes P + \mathrm{He}(F(I\otimes\tilde{\mathcal{A}}_i)) & R_{12}\otimes P + (I\otimes\tilde{\mathcal{A}}_i^{\mathrm{T}})G - F \\ \star & R_{22}\otimes P - G - G^{\mathrm{T}} \end{pmatrix} \prec 0$$

Lerch et. al (2022)

$F = R_{12}\otimes\hat{P}$ and $G = R_{22}\otimes\hat{P}$ with $\hat{P} = \hat{P}^{\mathrm{T}} = \mathrm{const}$

$$\begin{pmatrix} R_{11}\otimes P + \mathrm{He}(R_{12}\otimes(\hat{P}\tilde{\mathcal{A}}_i)) & R_{12}\otimes(P - \hat{P}) + R_{22}\otimes(\hat{P}\tilde{\mathcal{A}}_i)^T \\ \star & R_{22}\otimes(P - 2\hat{P}) \end{pmatrix} \prec 0$$

$\hat{P} = P$


Lerch, S. ; Dehnert, R. ; Rosik, M. ; Tibken, B. : Minimizing Oscillations for Magnitude and Rate-Saturated Discrete-Time Systems by a $D_R$ Region Pole Placement. In: *2022 10th International Conference on Systems and Control (ICSC)*, 2022, S. 244–249

$$\begin{pmatrix} R_{11}\otimes P + \mathrm{He}(R_{12}\otimes(\hat{P}\tilde{\mathcal{A}}_i)) & R_{12}\otimes(P-\hat{P}) + R_{12}\otimes\left(\hat{P}\tilde{\mathcal{A}}_i\right)^T \\ \star & R_{22}\otimes(P-2\hat{P}) \end{pmatrix} \prec 0$$

Circle with radius $\bar{r}$

$R_{11} = -\bar{r}^{\,2}$
$R_{12} = 0$
$R_{22} = 1$

$$\begin{pmatrix} -\bar{r}^{\,2}P & \left(\hat{P}\tilde{\mathcal{A}}_i\right)^T \\ \star & P-2\hat{P} \end{pmatrix} \prec 0$$

$$\Rightarrow \begin{pmatrix} 0 & \hat{P}^{-1} \\ I & 0 \end{pmatrix} \begin{pmatrix} -\bar{r}^{\,2}P & \left(\hat{P}\tilde{\mathcal{A}}_i\right)^T \\ \star & P-2\hat{P} \end{pmatrix} \begin{pmatrix} 0 & I \\ \hat{P}^{-1} & 0 \end{pmatrix} \prec 0$$

$$\Rightarrow \begin{pmatrix} \hat{P}^{-1}\left(2I - P\hat{P}^{-1}\right) & \tilde{\mathcal{A}}_i \\ \star & \bar{r}^{\,2}P \end{pmatrix} \succ 0 \qquad \equiv \text{Linearization Method}$$

$$\begin{pmatrix} R_{11}\otimes P + \mathrm{He}(R_{12}\otimes(\hat{P}\tilde{\mathcal{A}}_i)) & R_{12}\otimes(P-\hat{P}) + R_{12}\otimes\left(\hat{P}\tilde{\mathcal{A}}_i\right)^T \\ \star & R_{22}\otimes(P-2\hat{P}) \end{pmatrix} \prec 0$$

$P$ is decoupled from $\tilde{\mathcal{A}}_i$: all considered controller types are possible

Optimum: $\hat{P} = P$

# New Method for Various Controller Types

$$\begin{pmatrix} R_{11} \otimes P + \mathrm{He}(R_{12} \otimes (\hat{P} \tilde{\mathcal{A}}_i)) & R_{12} \otimes (P - \hat{P}) + R_{12} \otimes \left(\hat{P} \tilde{\mathcal{A}}_i\right)^T \\ \star & R_{22} \otimes (P - 2\hat{P}) \end{pmatrix} \prec 0$$

$P$ is decoupled from $\tilde{\mathcal{A}}_i$: all considered controller types are possible

Optimum: $\hat{P} = P$

| Problem | Solution |
| --- | --- |
| $P$ is not known beforehand | |

# New Method for Various Controller Types

$$\begin{pmatrix} R_{11} \otimes P + \mathrm{He}(R_{12} \otimes (\hat{P}\tilde{\mathcal{A}}_i)) & R_{12} \otimes (P - \hat{P}) + R_{12} \otimes \left(\hat{P}\tilde{\mathcal{A}}_i\right)^T \\ \star & R_{22} \otimes (P - 2\hat{P}) \end{pmatrix} \prec 0$$

$P$ is decoupled from $\tilde{\mathcal{A}}_i$: all considered controller types are possible

Optimum: $\hat{P} = P$

| Problem | Solution |
|---|---|
| $P$ is not known beforehand | Initialization with $\hat{P} = I$<br>Iteration with $\hat{P}_{l+1} = P_l$ |

$$\begin{pmatrix} R_{11}\otimes P + \mathrm{He}(R_{12}\otimes(\hat{P}\tilde{\mathcal{A}}_i)) & R_{12}\otimes(P-\hat{P}) + R_{12}\otimes\left(\hat{P}\tilde{\mathcal{A}}_i\right)^T \\ \star & R_{22}\otimes(P-2\hat{P}) \end{pmatrix} \prec 0$$

$P$ is decoupled from $\tilde{\mathcal{A}}_i$: all considered controller types are possible

Optimum: $\hat{P} = P$

| Problem | Solution |
| --- | --- |
| $P$ is not known beforehand | Initialization with $\hat{P} = I$<br>Iteration with $\hat{P}_{l+1} = P_l$ |
| $\hat{P} = I$ might lead to infeasibility | |

$$\begin{pmatrix} R_{11} \otimes P + \mathrm{He}(R_{12} \otimes (\hat{P}\tilde{\mathcal{A}}_i)) & R_{12} \otimes (P - \hat{P}) + R_{12} \otimes (\hat{P}\tilde{\mathcal{A}}_i)^T \\ \star & R_{22} \otimes (P - 2\hat{P}) \end{pmatrix} \prec 0$$

$P$ is decoupled from $\tilde{\mathcal{A}}_i$: all considered controller types are possible

Optimum: $\hat{P} = P$

| Problem | Solution |
|---|---|
| $P$ is not known beforehand | Initialization with $\hat{P} = I$<br>Iteration with $\hat{P}_{l+1} = P_l$ |
| $\hat{P} = I$ might lead to infeasibility | Initialization with $\bar{r} > 1$ (unstable)<br>Iterative minimization of $\bar{r}$ until $\bar{r} = 1$ (stable) |

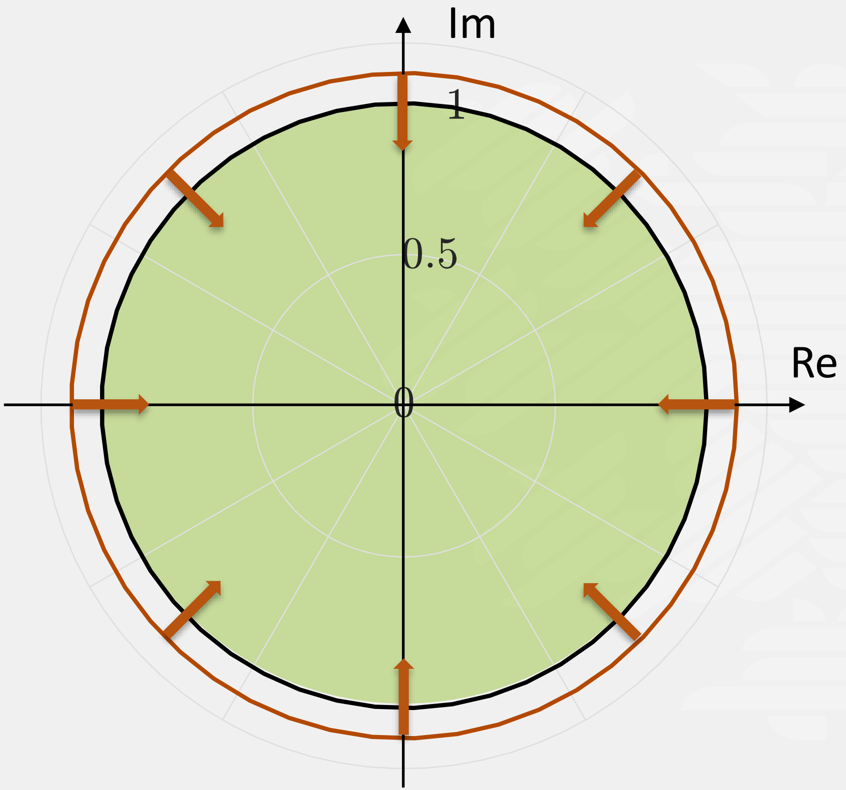
## Stage 1 Stabilize the closed loop

- Start with $\hat{P} = I$ and $\bar{r} > 1$
- End when $\bar{r} = 1$

## Stage 2 Minimize $\bar{r}$

## Stage 3 Minimize $\bar{\vartheta}$

| | | |
|---|---|---|
| $R_{11} = -\bar{r}^2$ | $R_{12} = 0$ | $R_{22} = 1$ |



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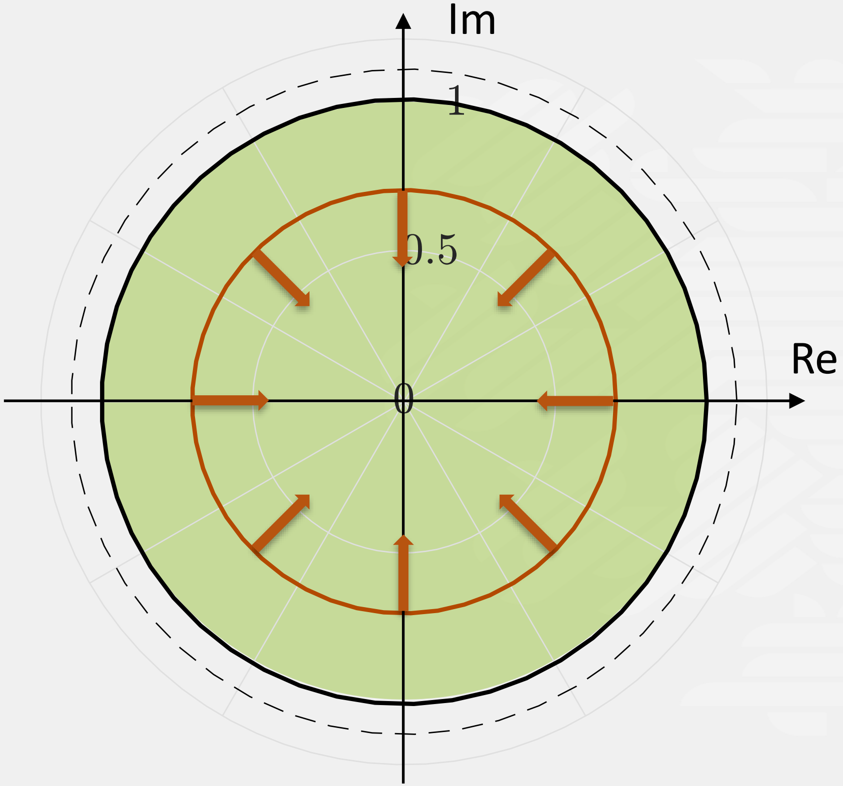
## Stage 1 Stabilize the closed loop

## Stage 2 Minimize $\bar{r}$

- Start with $\bar{r} = 1$
- End when $\bar{r}$ cannot be reduced further

## Stage 3 Minimize $\bar{\vartheta}$

$R_{11} = -\bar{r}^2 \qquad R_{12} = 0 \qquad R_{22} = 1$



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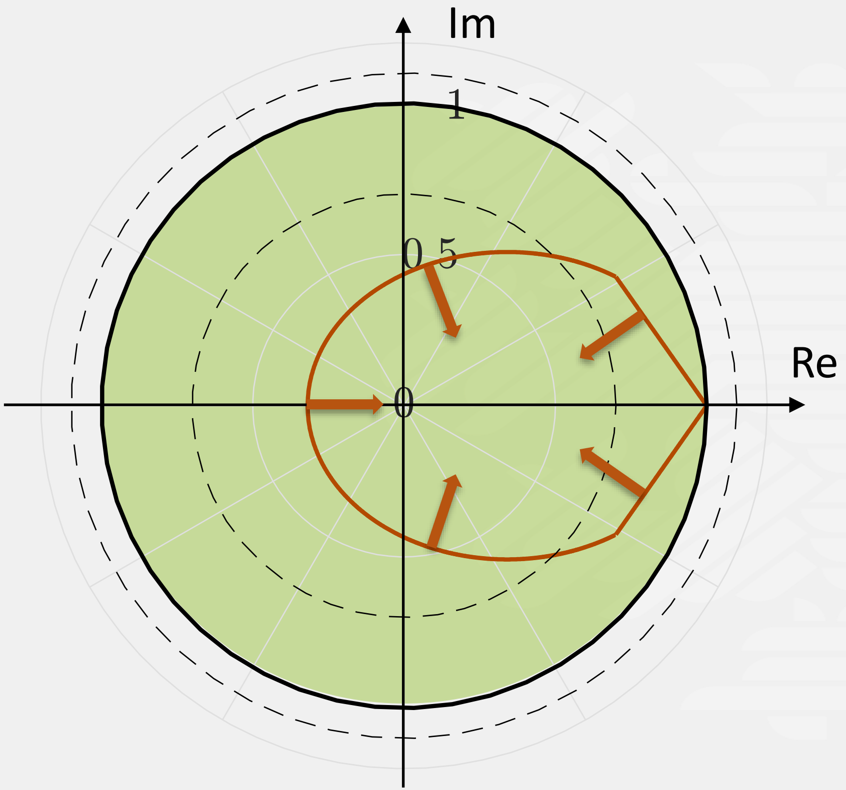
# Stage 1 Stabilize the closed loop

# Stage 2 Minimize $r$

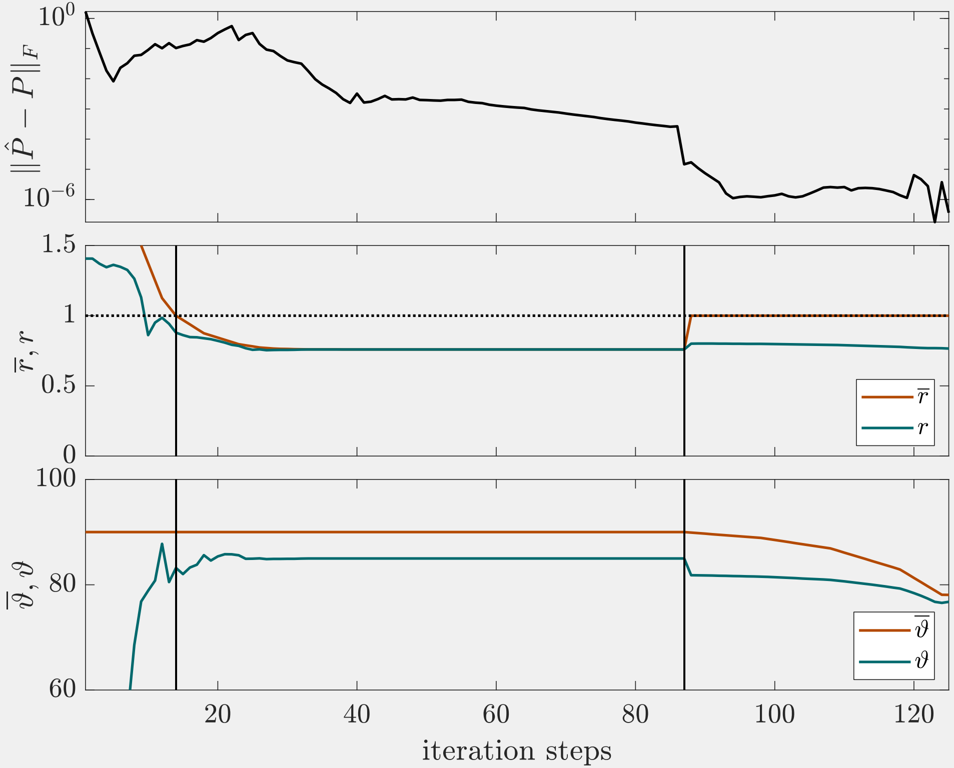
# Stage 3 Minimize $\bar{\vartheta}$

- Start with $\bar{\vartheta} = 90°$
- End when $\bar{\vartheta}$ cannot be reduced further




$$R_{11} = \begin{pmatrix} -1 & -\lambda_M/a & 0 & 0 \\ -\lambda_M/a & -1 & 0 & 0 \\ 0 & 0 & -2\sin\bar{\vartheta} & 0 \\ 0 & 0 & 0 & -2\sin\bar{\vartheta} \end{pmatrix}$$

$$R_{12} = \begin{pmatrix} 0 & (1/a - 1/b)/2 & 0 & 0 \\ (1/a + 1/b)/2 & 0 & 0 & 0 \\ 0 & 0 & \sin\bar{\vartheta} & \cos\bar{\vartheta} \\ 0 & 0 & -\cos\bar{\vartheta} & \sin\bar{\vartheta} \end{pmatrix} \qquad R_{22} = 0$$




Numerical example (Benzaouia (2006))

$$A_s = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad x_0 = \begin{pmatrix} 1 & 1 \end{pmatrix}^{\mathrm{T}}$$

$$B_s = \begin{pmatrix} 0.5 \\ 1 \end{pmatrix} \qquad u_{\max} = 1$$

$$C_s = \begin{pmatrix} 1 & 1 \end{pmatrix} \qquad v_{\max} = 1$$

Design of observer-based state feedback

$$K = \begin{pmatrix} -0.0489 & -0.3358 \end{pmatrix}$$

$$L = \begin{pmatrix} 0.6441 \\ 0.7594 \end{pmatrix}$$

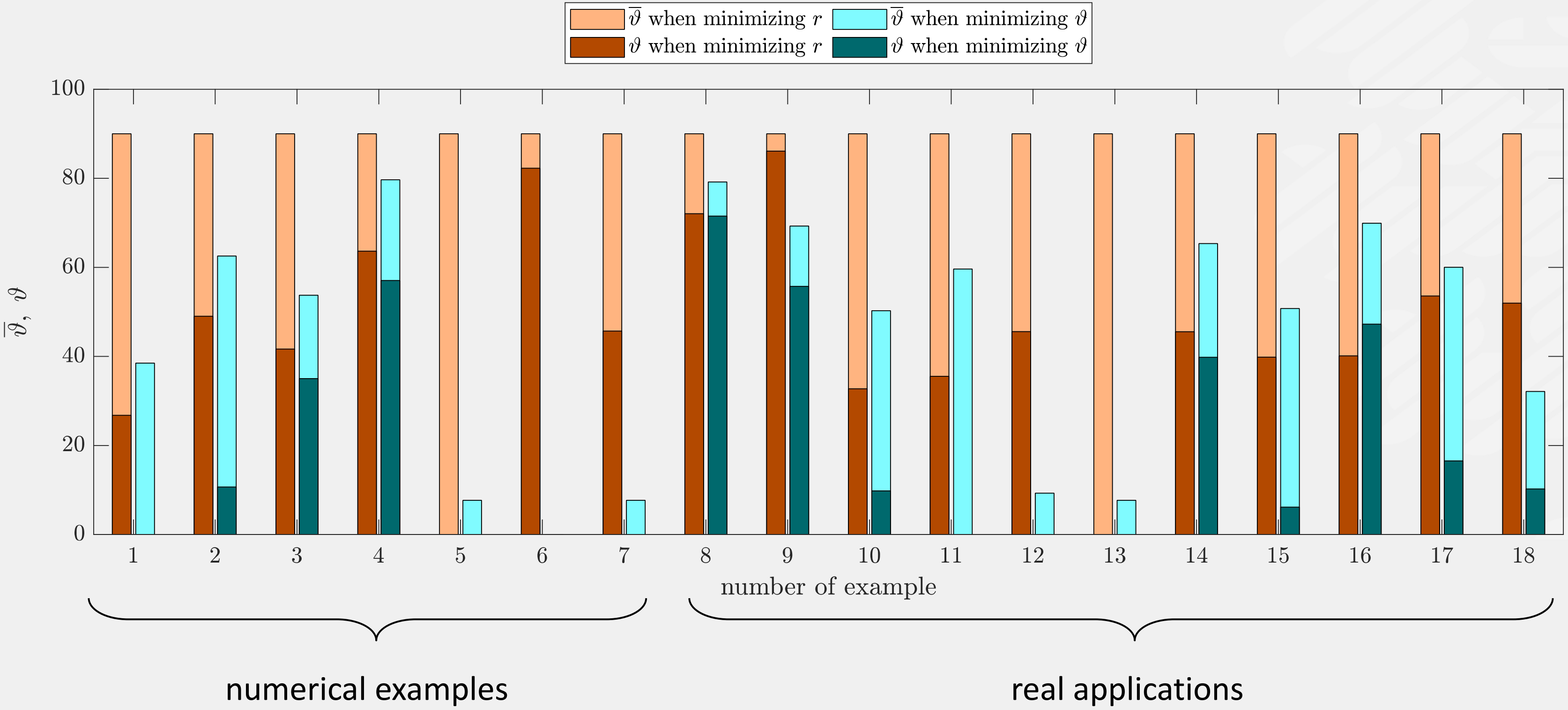
$$r = 0.7678 \qquad \vartheta = 76.5533^\circ$$

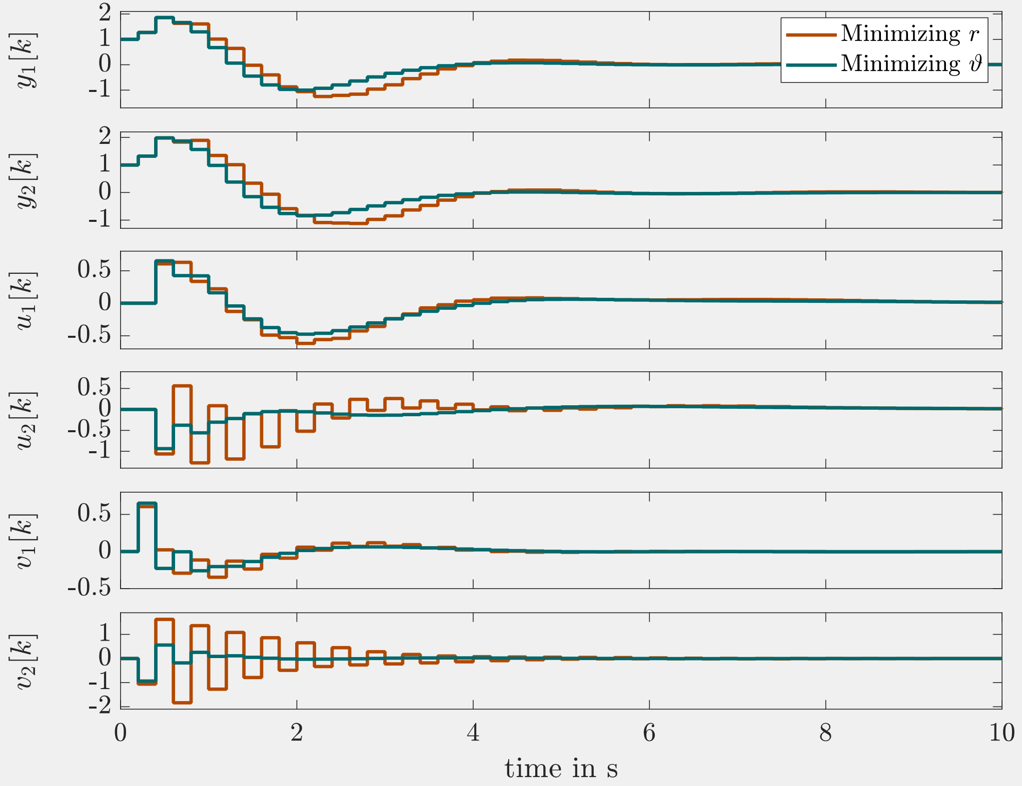
$$\left\|\hat{P} - P\right\|_F = 3.6335 \cdot 10^{-7}$$

Benzaouia, A. ; Mesquine, F. ; Hmamed, A. ; Aoufoussi, H. : Stability and control synthesis for discrete-time linear systems subject to actuator saturation by output feedback. In: *Mathematical Problems in Engineering* (2006)

18 Systems

Design of full state feedback

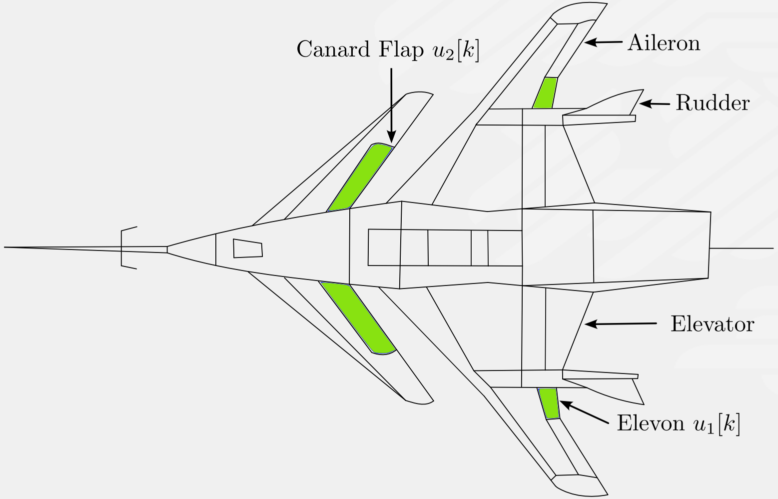




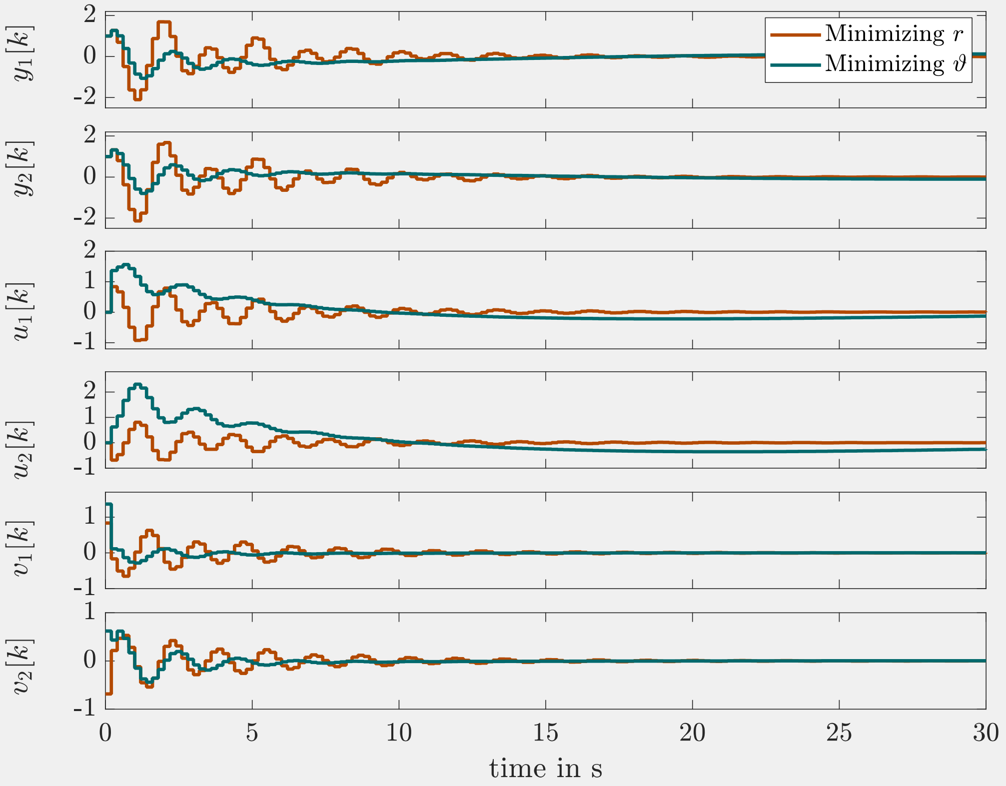

## Design of observer-based state feedback

$r = 0.9021$ $\qquad$ $r = 0.9023$

$\vartheta = 86.8503°$ $\qquad$ $\vartheta = 70.6795°$



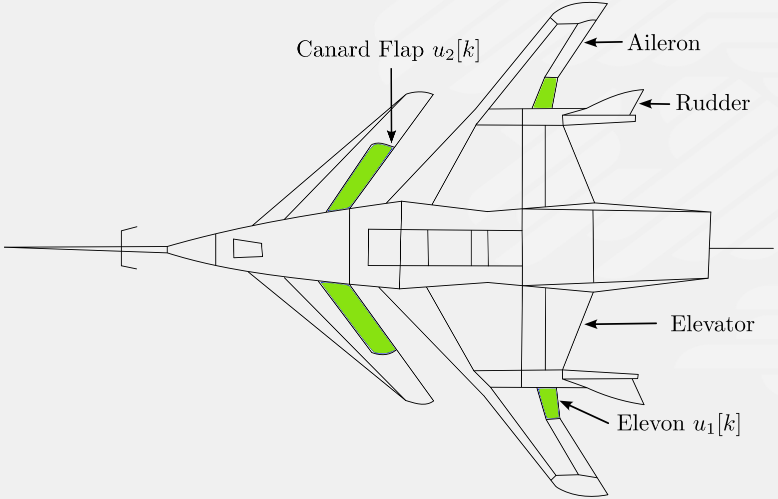

Safonov, M. ; Laub, A. ; Hartmann, G. : Feedback properties of multivariable systems: The role and use of the return difference matrix. In: *IEEE Transactions on Automatic Control* 26 (1981), Nr. 1, S. 47–65



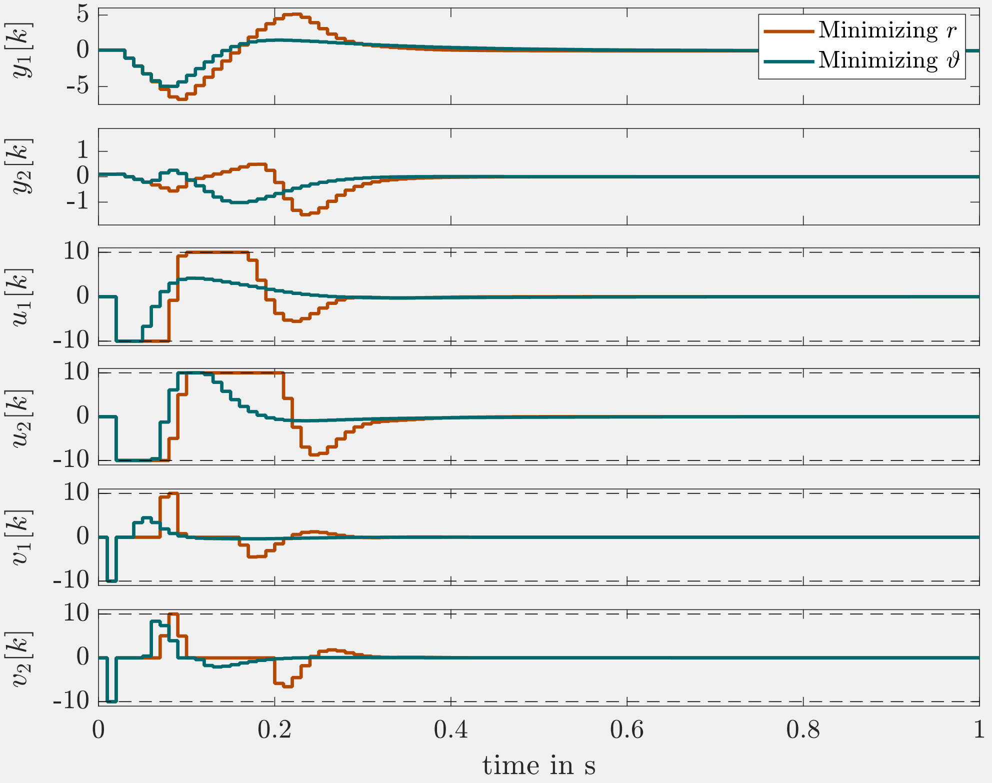

## Design of PID controller

$r = 0.9952$ $\quad$ $r = 0.9854$

$\vartheta = 87.0185°$ $\quad$ $\vartheta = 81.2153°$



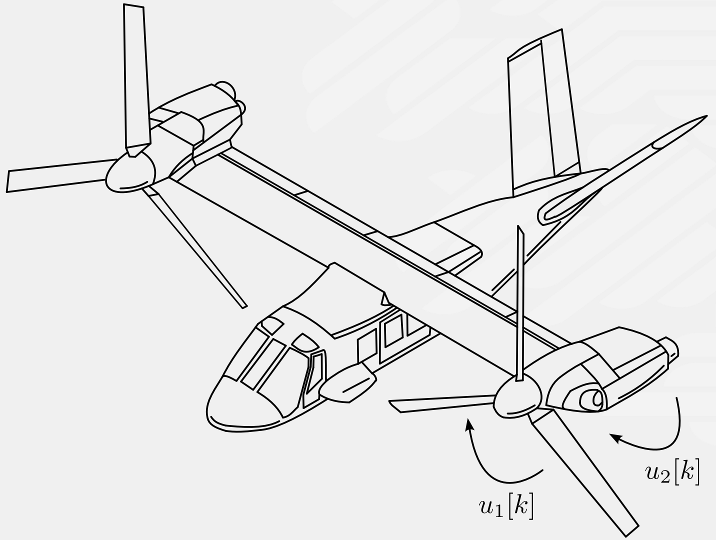

Safonov, M. ; Laub, A. ; Hartmann, G. : Feedback properties of multivariable systems: The role and use of the return difference matrix. In: *IEEE Transactions on Automatic Control* 26 (1981), Nr. 1, S. 47–65




## Design of observer-based state feedback

$r = 0.7871$ $r = 0.9308$

$\vartheta = 70.2408°$ $\vartheta = 27.1399°$

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# Conclusion

Summary

- Minimization of oscillations ($D_R$ region pole placement)
- Magnitude and rate saturation
- Iterative LMI method
- Design of different controller types with one method

Outlook

- Other $D_R$ regions for other objectives or combinations
- More accurate convex approximation of cardioid
- Tests on real applications
- Uncertainties, quasilinear forms, …

Thank you for your attention!

Any questions?