Set Separation and Exclusion Tendency-Based Design Framework for Active Fault Diagnosis

—— A Talk in Online Seminar on Interval Methods

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Outline

Part 1: Introduction to Active Fault Diagnosis

- set separation-based active fault diagnosis

Part 2: Observer-Based Active Fault Diagnosis:

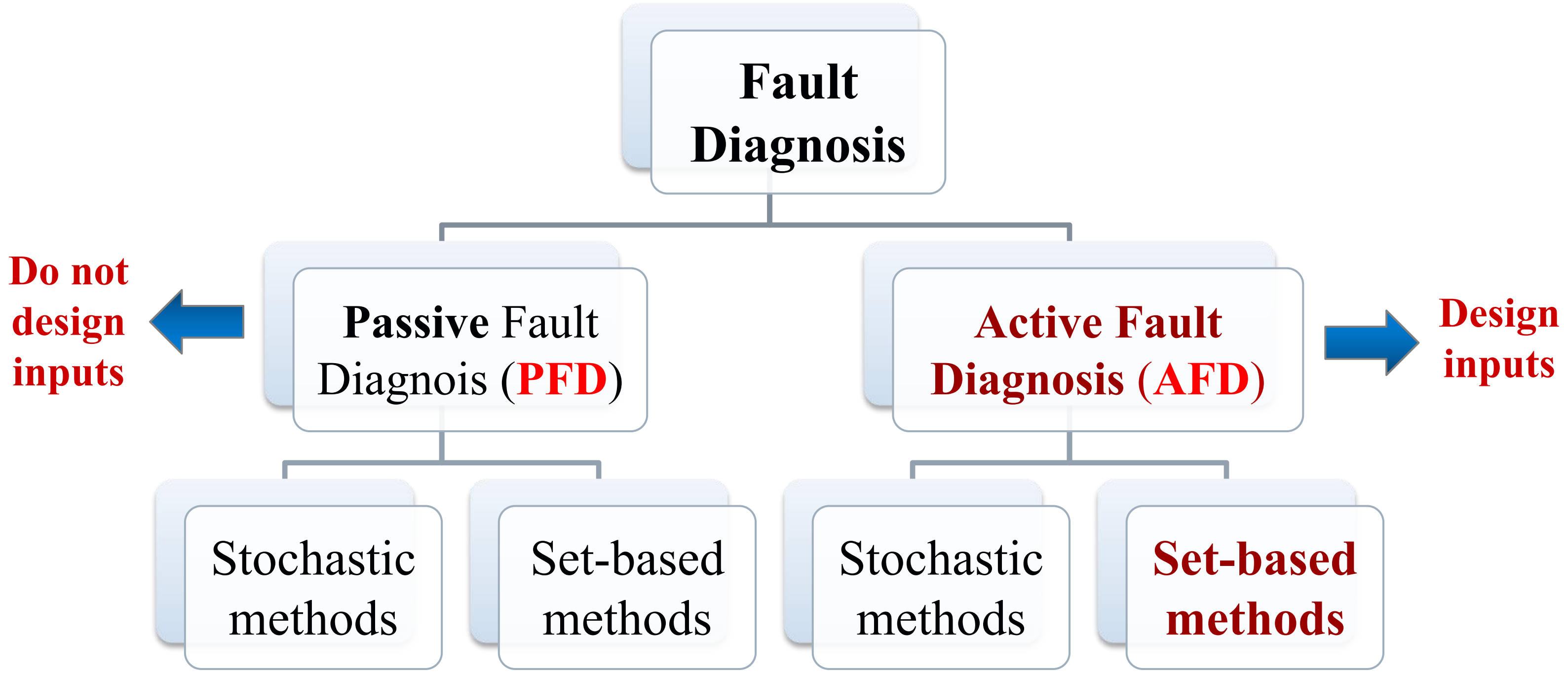
- set separation tendency-based framework

Part 3: Conclusions and Future Work

Part 1: Introduction to Active Fault Diagnosis

- set separation-based active fault diagnosis

Fault Diagnosis



AFD VS PFD

Low conservatism but high complexity.

+

Set-based VS Stochastic

Strong robustness but high conservatism.

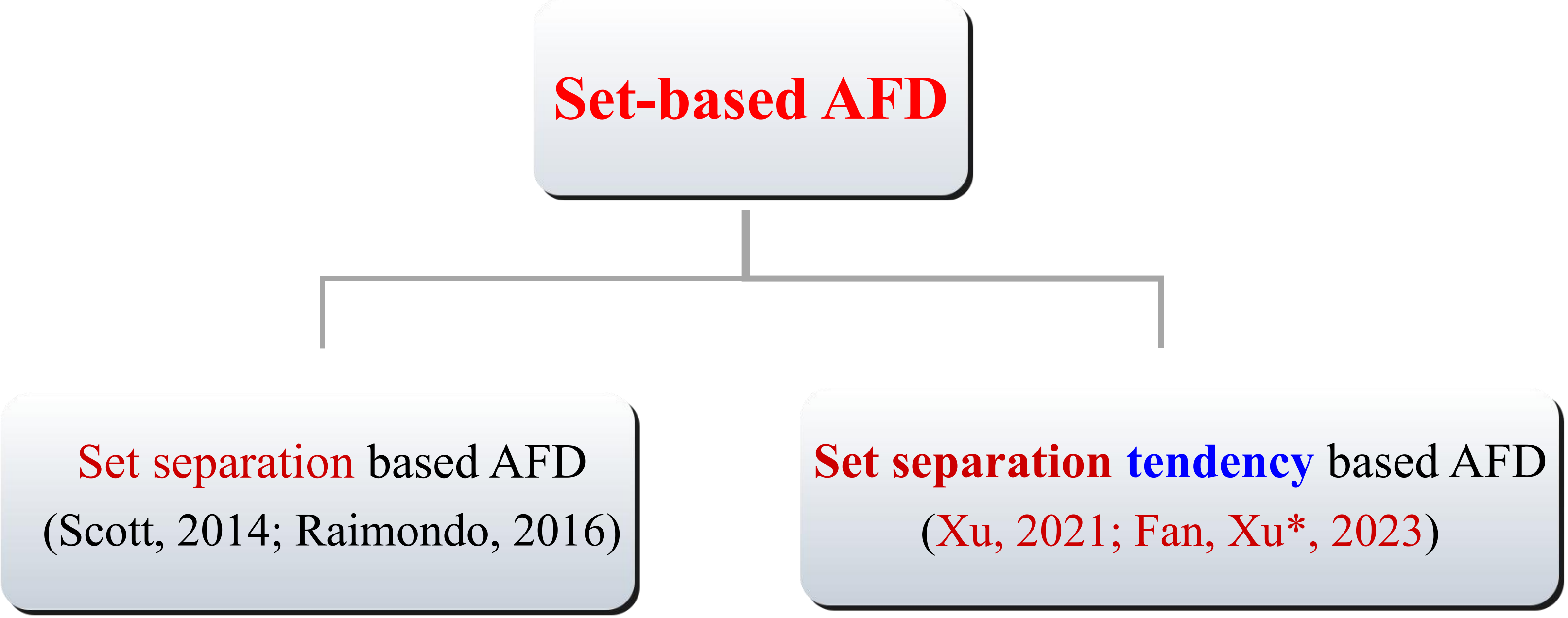
→

Set-based + AFD

Achieve strong robustness with low conservatism.

Y. M. Zhang and J. Jiang. Bibliographical review on reconfigurable fault-tolerant control systems, *Annual Reviews in Control,*32,229-252, 2008.
T.A.N. Heirung and A. Mesbah, Input design for active fault diagnosis, *Annual Reviews in Control*, 47:35 – 50, 2019.

Fault Diagnosis



Set-based AFD

Set separation based AFD
(Scott, 2014; Raimondo, 2016)

Set separation tendency based AFD
(Xu, 2021; Fan, Xu*, 2023)

Classical methods

Our methods

J.K. Scott and R. Findeisen and R.D. Braatz and D.M. Raimondo. Input design for guaranteed fault diagnosis using zonotopes, *Automatica*, 50(6), 1580 - 1589, 2014.
D.M. Raimondo, G.R. Marseglia, R.D. Braatz and J.K Scott. Closed-loop input design for guaranteed fault diagnosis using set-valued observers, *Automatica*, 74:107-117, 2016.
F. Xu. Observer-Based Asymptotic Active Fault Diagnosis: A Two-Layer Optimization Framework, *Automatica*, 125,109558, 2021.
Y.D. Fan, **F. Xu***, X.Q. Wang and B. Liang. Exclusion Tendency-Based Observer Design Framework for Active Fault Diagnosis, *Automatica*, In press, 2023.

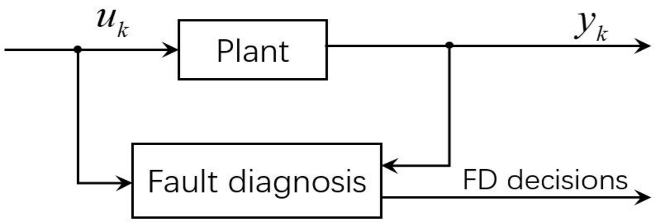
Set Separation-Based Active Fault Diagnosis

System Models and Set-Wise Dynamics

System models (healthy and faulty modes):

used to model actuator faults

$$x_{k+1} = Ax_k + B\boxed{G_i} u_k + E\omega_k,$$
$$y_k = Cx_k + F\eta_k.$$



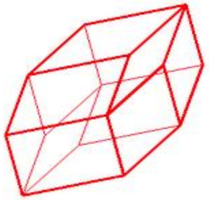
General scheme of fault diagnosis

Set-wise models (healthy and faulty modes):

$$\hat{X}_{k+1}^{i} = A\hat{X}_k^i \oplus B\mathbf{G}_i u_k \oplus EW,$$
$$\hat{Y}_k^i = C\hat{X}_k^i \oplus FV, i \in \mathbb{I} = \{0, 1, 2, \ldots, n_u\}.$$

Zonotopic sets of disturbances, noises and inputs:

$$\omega_k \in W = \langle \omega^c, H_\omega \rangle, \eta_k \in V = \langle \eta^c, H_\eta \rangle, u_k \in U.$$



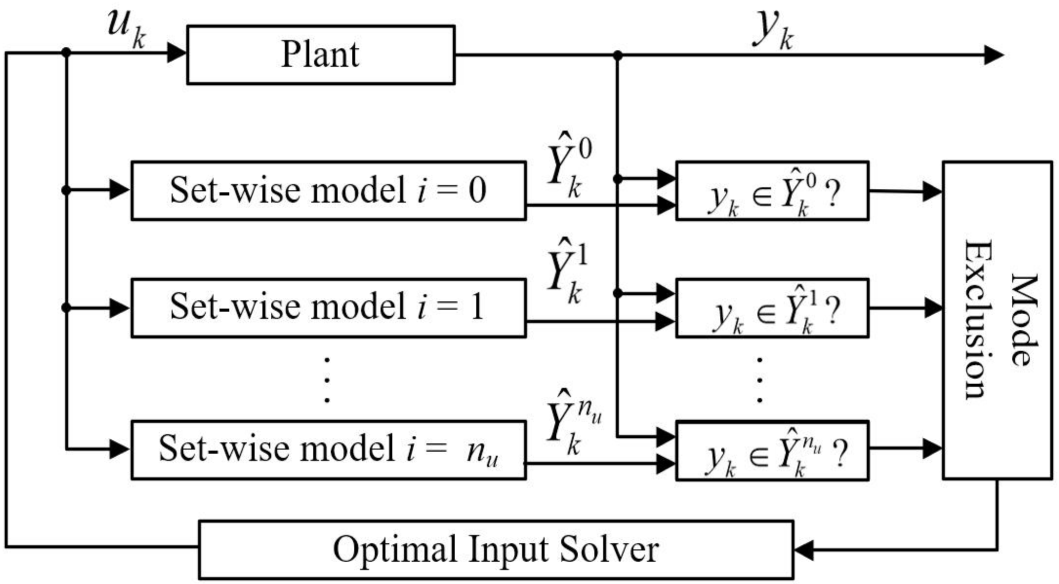
Zonotope: $Z = g \oplus H\mathbb{B}^m = \langle g, H \rangle$

Fault interval in the i-*th* actuator: $G_i \in \mathbf{G}_i$

J.K. Scott and R. Findeisen and R.D. Braatz and D.M. Raimondo. Input design for guaranteed fault diagnosis using zonotopes, *Automatica*, 50(6), 1580 - 1589, 2014.
D.M. Raimondo, G.R. Marseglia, R.D. Braatz and J.K Scott. Closed-loop input design for guaranteed fault diagnosis using set-valued observers, *Automatica*, 74:107 -117, 2016.

Set Separation-Based Active Fault Diagnosis

Scheme of Set Separation-Based Active Fault Diagnosis (AFD)



Final mode left indicates fault

Active fault diagnosis criterion:

$$y_k \in \hat{Y}_{k+N}^j, \, y_k \notin \hat{Y}_{k+N}^i \forall i \in \mathbb{I}, i \neq j$$

J.K. Scott and R. Findeisen and R.D. Braatz and D.M. Raimondo. Input design for guaranteed fault diagnosis using zonotopes, *Automatica*, 50(6), 1580 - 1589, 2014.
D.M. Raimondo, G.R. Marseglia, R.D. Braatz and J.K Scott. Closed-loop input design for guaranteed fault diagnosis using set-valued observers, *Automatica*, 74:107 -117, 2016.

Set Separation-Based Active Fault Diagnosis

Guaranteed Fault Diagnosis Conditions for AFD Input Design

Set separation diagnosis conditions:

$$\hat{Y}^{i}_{k+N} \cap \hat{Y}^{j}_{k+N} = \emptyset, \forall i, j \in \mathbb{I}, i \neq j$$

Input design

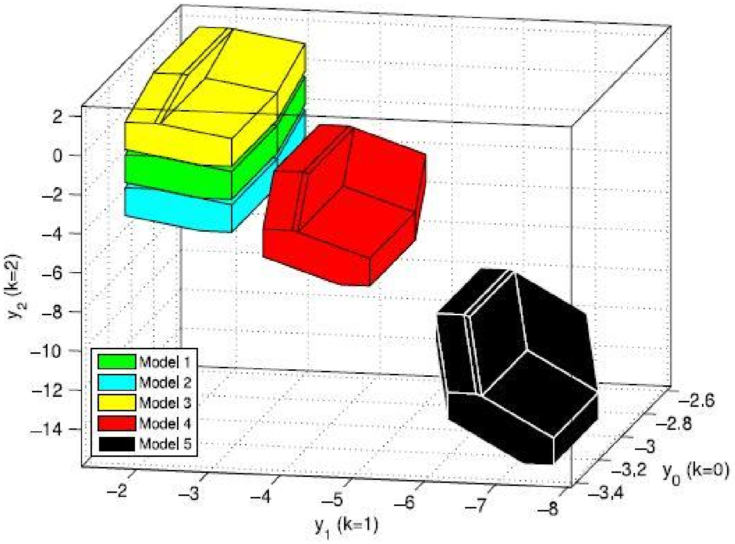
Actively design an *N*-step input sequence:

$$\min_{u_{k:k+N-1}} u^{T}_{k:k+N-1} Q u_{k:k+N-1}$$

$$\text{s.t.} \boxed{\hat{Y}^{i}_{k+N} \cap \hat{Y}^{j}_{k+N} = \emptyset, \forall i, j \in \mathbb{I}, i \neq j,}$$

$$u_{k:k+N-1} \in \mathbf{U} = U \times U \times \ldots \times U.$$

Key problem: How to handle this set separation constraint?



Five (healthy and faulty) modes and a two-step separating input sequence [Scott, 2014]

J.K. Scott and R. Findeisen and R.D. Braatz and D.M. Raimondo. Input design for guaranteed fault diagnosis using zonotopes, *Automatica*, 50(6), 1580 - 1589, 2014.
D.M. Raimondo, G.R. Marseglia, R.D. Braatz and J.K Scott. Closed-loop input design for guaranteed fault diagnosis using set-valued observers, *Automatica*, 74:107 -117, 2016.

Set Separation-Based Active Fault Diagnosis

The Origin of Computational Complexity

Original AFD input design problem:

$$\min_{u_{k:k+N-1}} u_{k:k+N-1}^T Q u_{k:k+N-1}$$

$$\text{s.t.} \boxed{\hat{Y}_{k+N}^i \cap \hat{Y}_{k+N}^j = \varnothing, \forall i, j \in \mathbb{I}, i \neq j,}$$

$$u_{k:k+N-1} \in \mathbf{U} = U \times U \times \ldots \times U.$$

Transformated into → Solution in [Scott, 2014]

Solvable mixed-integer problem

with mixed-integer constraints

Computational complexity

Exponentially increase → Integer variables

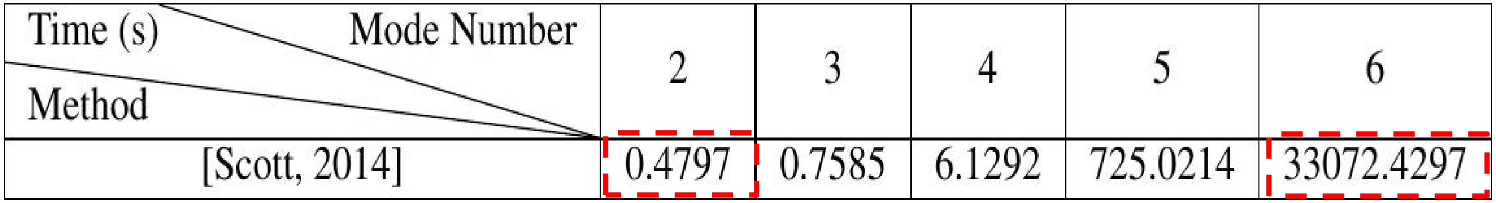
1) Number of faulty modes
2) Length of AFD input sequence
3) Dimension of system

J.K. Scott and R. Findeisen and R.D. Braatz and D.M. Raimondo. Input design for guaranteed fault diagnosis using zonotopes, *Automatica*, 50(6), 1580 - 1589, 2014.
D.M. Raimondo, G.R. Marseglia, R.D. Braatz and J.K Scott. Closed-loop input design for guaranteed fault diagnosis using set-valued observers, *Automatica*, 74:107 -117, 2016.

Set Separation-Based Active Fault Diagnosis

Illustration of Computational Complexity

A rough illustration on **computational complexity** of set separation-based methods (a two-input, two-state and two-output example):



Time (s) Mode Number Method	2	3	4	5	6
[Scott, 2014]	0.4797	0.7585	6.1292	725.0214	33072.4297

Key problem: How to **reduce computational complexity** of AFD?

J.K. Scott and R. Findeisen and R.D. Braatz and D.M. Raimondo. Input design for guaranteed fault diagnosis using zonotopes, *Automatica*, 50(6), 1580 - 1589, 2014.
D.M. Raimondo, G.R. Marseglia, R.D. Braatz and J.K Scott. Closed-loop input design for guaranteed fault diagnosis using set-valued observers, *Automatica*, 74:107 -117, 2016.

Part 2: Observer-Based Active Fault Diagnosis:

- set separation tendency-based framework

Reduce Computational Complexity

1. Observer-Based Active Fault Diagnosis Framework

The proposed idea of overcoming the computational complexity problem:

Discard N-step **set separation** condition:

$$\hat{Y}^{i}_{k+N} \cancel{\cap} \hat{Y}^{j}_{k+N} = \varnothing$$

Propose a novel notion **“set separation tendency”**

Establish a **new** AFD input design framework

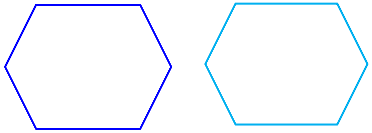
F. Xu. Observer-Based Asymptotic Active Fault Diagnosis: A Two- Layer Optimization Framework, *Automatica*, 125,109558, 2021.

Reduce Computational Complexity

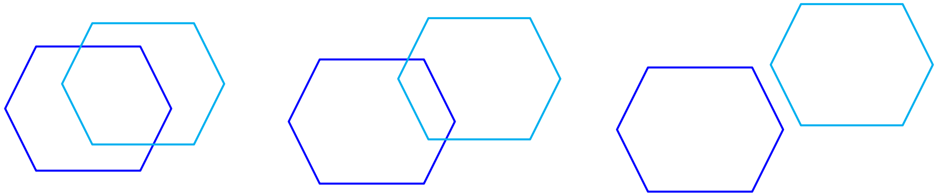
1. Observer-Based Active Fault Diagnosis Framework

Design an N-step input sequence to implement output sets separation at a time:

$$\hat{Y}^{i}_{k+N} \cap \hat{Y}^{j}_{k+N} = \varnothing$$



Design a one-step input at a time but repeat N times till AFD is finally achieved:



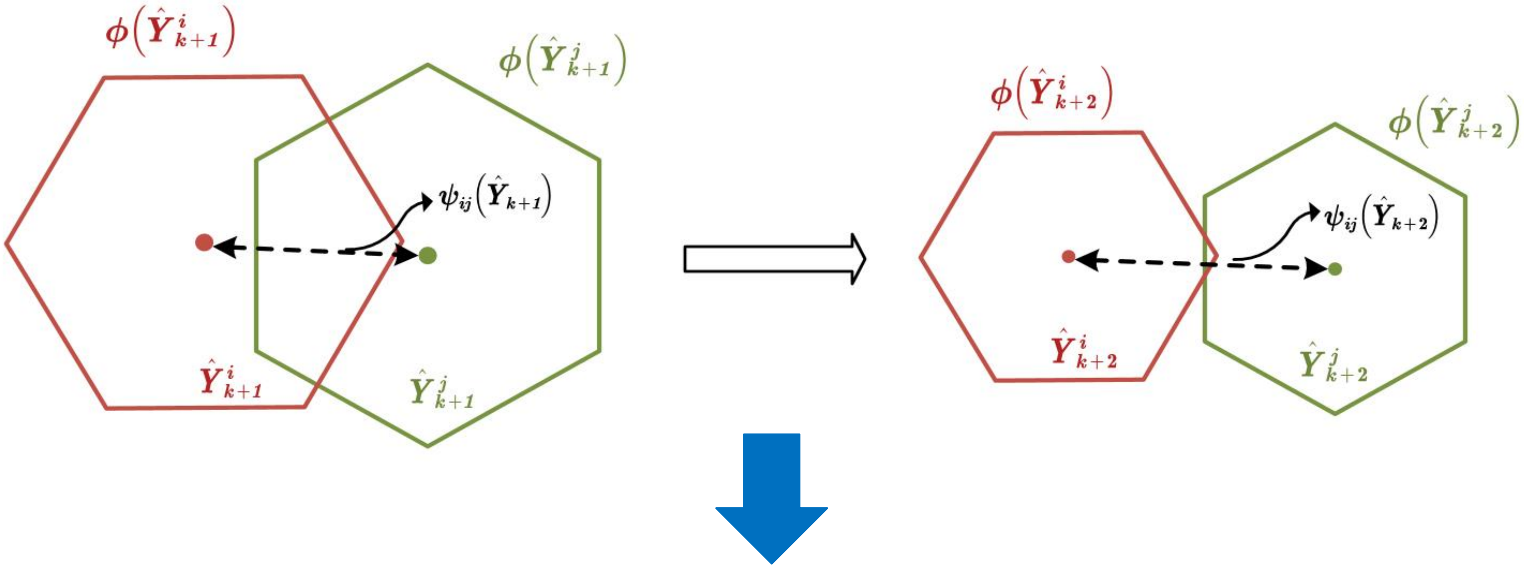
Key problem: How to **achieve fast increase** of set separation tendency?

F. Xu. Observer-Based Asymptotic Active Fault Diagnosis: A Two- Layer Optimization Framework, *Automatica*, 125,109558, 2021.

Reduce Computational Complexity

1. Observer-Based Active Fault Diagnosis Framework

The proposed idea to achieve fast increase of set separation tendency:



Minimize size & Maximize centers distance

Key problem: only have one design variable u_k but two objectives.

F. Xu. Observer-Based Asymptotic Active Fault Diagnosis: A Two- Layer Optimization Framework, *Automatica*, 125,109558, 2021.

Reduce Computational Complexity

1. Observer-Based Active Fault Diagnosis Framework

[Scott, 2014] uses set-wise system models:

$$\hat{X}^{i}_{k+1} = A\hat{X}^{i}_{k} \oplus B\mathbf{G}_{i}\boxed{u_{k}} \oplus EW$$

$$\hat{Y}^{i}_{k} = C\hat{X}^{i}_{k} \oplus FV, \; i \in \mathbb{I}.$$

[Xu, 2021] proposes to use set-valued observers:

$$\hat{X}^{i}_{k+1} = (A - \boxed{L^{i}}C)\hat{X}^{i}_{k} \oplus B\mathbf{G}_{i}\boxed{u_{k}} \oplus L^{i}y_{k} \oplus (-LF)V \oplus EW$$

$$\hat{Y}^{i}_{k} = C\hat{X}^{i}_{k} \oplus FV, \; i \in \mathbb{I}.$$

Observer gain *L*

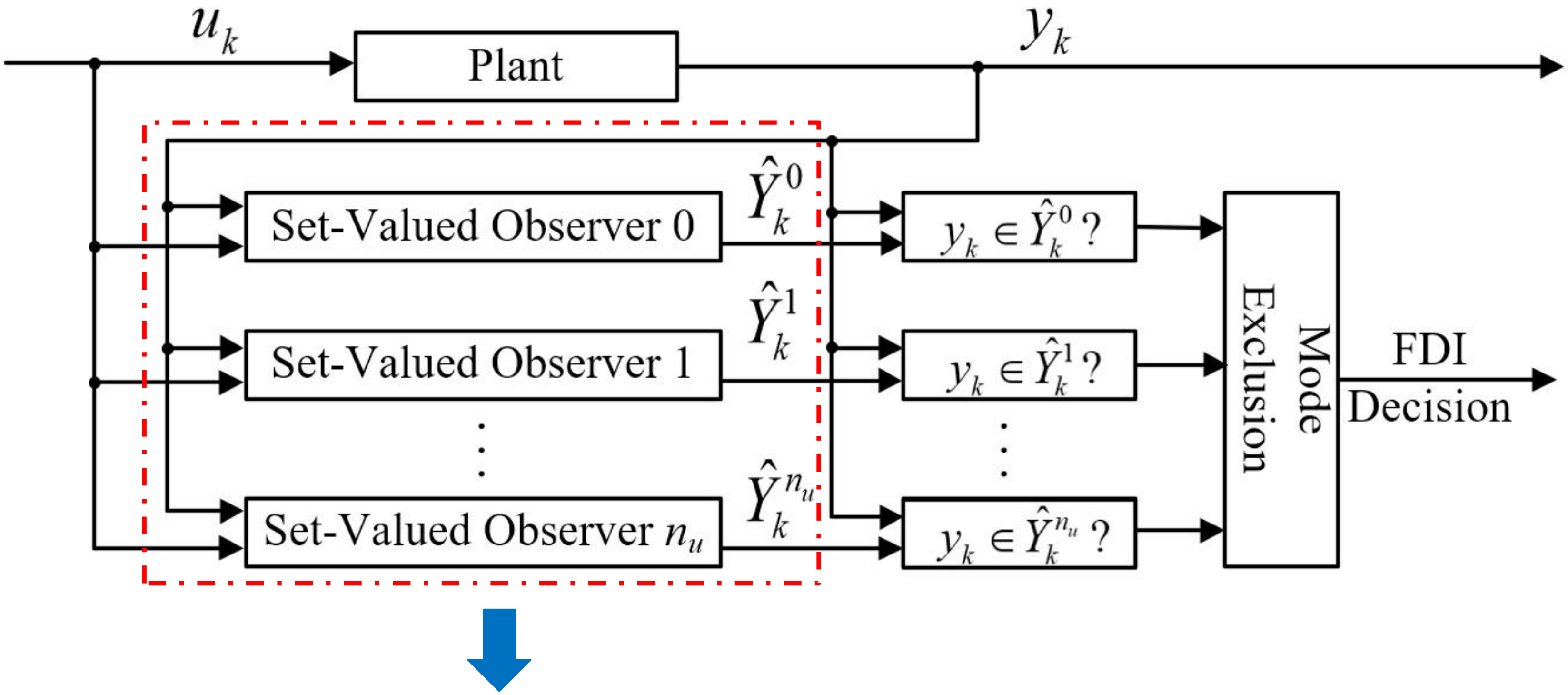
Two design parameters *L* and *u* are obtained.

F. Xu. Observer-Based Asymptotic Active Fault Diagnosis: A Two- Layer Optimization Framework, *Automatica*, 125,109558, 2021.

Reduce Computational Complexity

1. Observer-Based Active Fault Diagnosis Framework

The proposed Observer-based AFD framework



Minimize size & Maximize centers distance

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Reduce Computational Complexity

1. Observer-Based Active Fault Diagnosis Framework

Compute the centers distance of all output sets:

$$\Psi_{k+1}(L,u)=\sum_{i=0}^{n_u-1}\sum_{j=i-1}^{n_u}\psi\left(\hat{Y}_{k+1}^{ij}\right)=\sum_{i=0}^{n_u-1}\sum_{j=i+1}^{n_u}\left\|\hat{y}_{k+1}^{i,c}-\hat{y}_{k+1}^{j,c}\right\|_2^2$$

Define the *F*-norm size of all output sets:

$$\Phi_{k+1}(L,\ u)=\sum_{i=0}^{n_u}\phi\left(\hat{Y}_{k+1}^{i}\right)=\sum_{i=0}^{n_u}\left\|\hat{H}_{k+1}^{i,y}\right\|_F^2=\Phi_{k+1}(L)+\Phi_{k+1}(u)$$

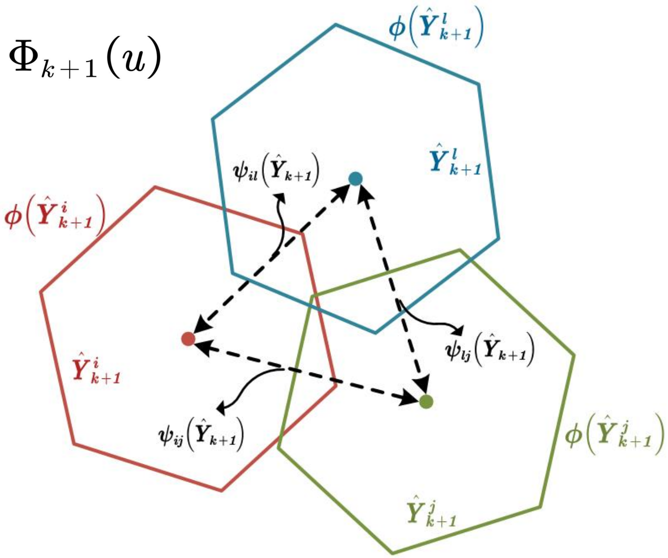
Center-generator matrix form of set-wise dynamics:

$$\hat{x}_{k+1}^{i,c}=(A-L^iC)\hat{x}_k^{i,c}+\mathrm{mid}(B\mathbf{G}_i)u_k+L^iy_k+E\omega^c-L^iF\eta^c,$$

$$\hat{H}_{k+1}^{i,x}=[(A-L^iC)\hat{H}_k^{i,x}\ \ \mathrm{diag}\left(\mathrm{rad}(B\mathbf{G}_i)u_k\right)\ \ EH_{\varpi}\ \ -L^iFH_{\bar{\eta}}],$$

$$\hat{y}_{k+1}^{i,c}=C\hat{x}_{k+1}^{i,c}+F\eta^c,$$

$$\hat{H}_{k+1}^{i,y}=[C\hat{H}_{k+1}^{i,x}\ \ FH_{\bar{\eta}}].$$



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Reduce Computational Complexity

1. Observer-Based Active Fault Diagnosis Framework

Step 1): Minimize the first part of the size of output sets:

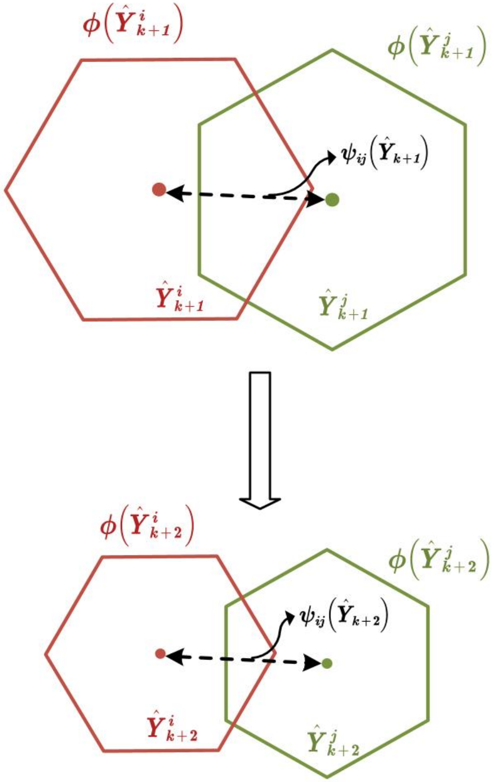
$$\min_{L_k^0, L_k^1, \ldots, L_k^{n_u}} \quad \Phi_{k+1}(L)$$

$$\frac{\partial \Phi_{k+1}}{\partial L_k^i} = 0$$

Analytical solution of optimal gain:

$$L_k^{i,KF} = A\hat{H}_k^{i,x}(\hat{H}_k^{i,x})^T C^T (FH_\eta (H_\eta)^T F^T + C\hat{H}_k^{i,x}(\hat{H}_k^{i,x})^T C^T)^{-1}$$

Online function value evaluation



F. Xu. Observer-Based Asymptotic Active Fault Diagnosis: A Two- Layer Optimization Framework, *Automatica*, 125,109558, 2021.
C. Combastel. Zonotopes and Kalman observers: Gain optimality under distinct uncertainty paradigms and robust convergence, *Automatica*, 55:265-273, 2015.

Reduce Computational Complexity

1. Observer-Based Active Fault Diagnosis Framework

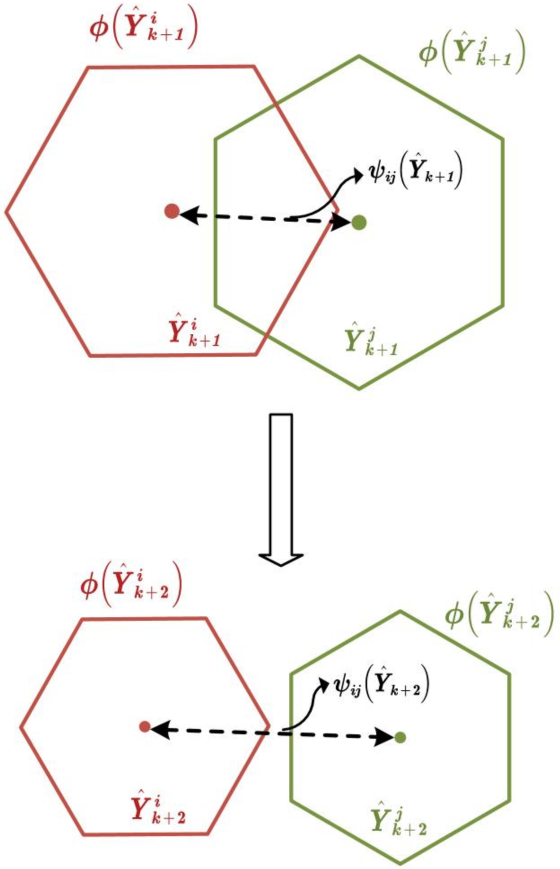
Step 2): Maximize their centers distance:

$$\max_{u_k \in \mathrm{U}} \Psi_{k+1}(L_k^{i,KF},\ u)$$

$\mathbf{U}$ is convex input-constraint polytope

Vertices enumeration

$\mathbf{U}^*$ is the set of vertices that achieve maximum



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Reduce Computational Complexity

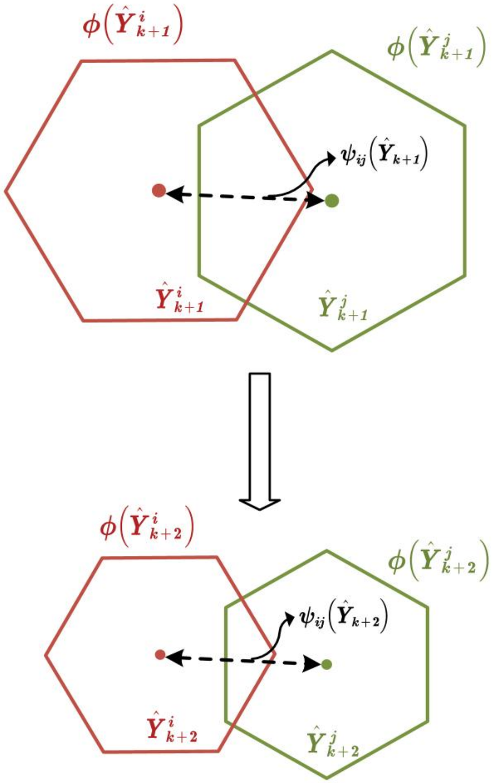
1. Observer-Based Active Fault Diagnosis Framework

Step 3): Further minimize the second part of the size of output sets:

$$\min_{u_k \in \mathbf{U}^*} \Phi_{k+1}(u)$$

Online function value evaluation

Optimal AFD input



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Reduce Computational Complexity

1. Observer-Based Active Fault Diagnosis Framework

Logic of the proposed method in [Xu,2021]:

1) Design L_k to minimize the size of output sets;

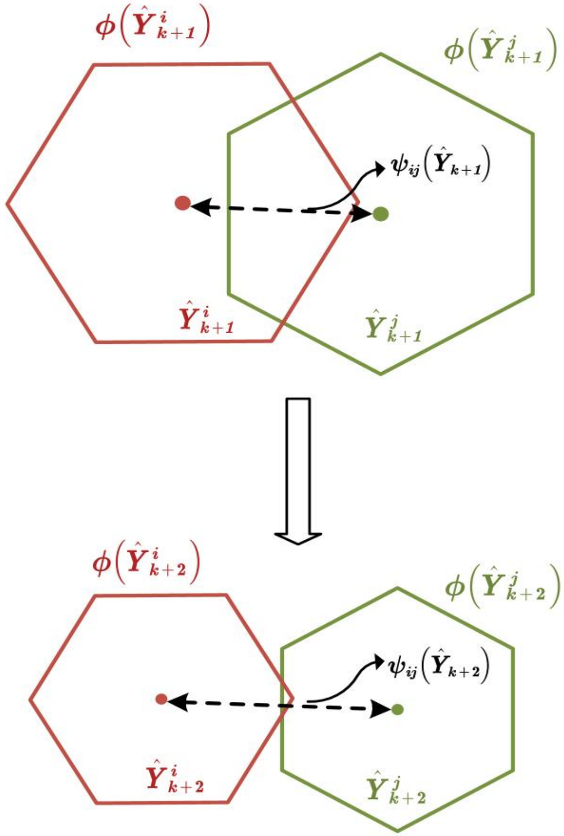
2) Design u_k to maximize centers distance simultaneously.

Online function value evaluation

+

Vertices enumeration

Discard mixed-integer constraints



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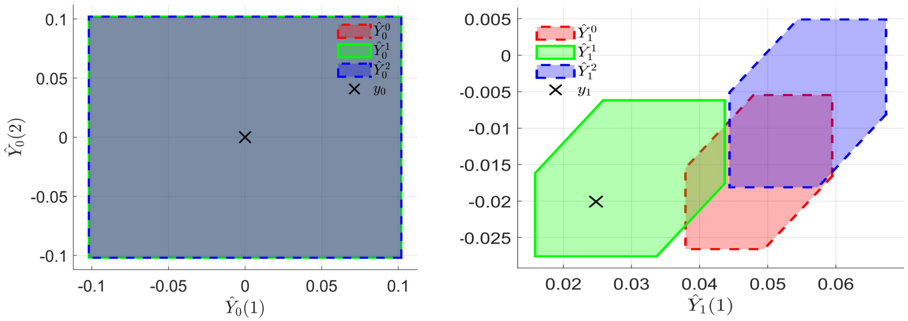
Reduce Computational Complexity

1. Observer-Based Active Fault Diagnosis Framework

Illustration of [Xu, 2021]:

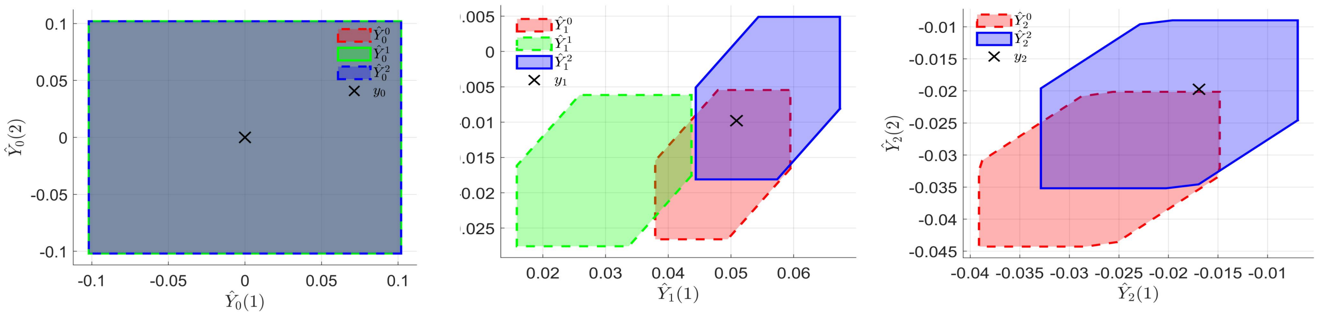
(A two-input, four-state and two-output system with three actuator modes (healthy + two faulty modes)

fault 1:



Computing time: 0.0021s

fault 2:



Computing time: 0.0091s

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Reduce Computational Complexity

1. Observer-Based Active Fault Diagnosis Framework

A rough comparison of computing time between [Xu, 2021] and [Scott, 2014]:

[Xu, 2021]:

1) Successful diagnosis of fault 1: 0.0021s;

2) Successful diagnosis of fault 2: 0.0091s

[Scott, 2014]:

1) Find a feasible separating input sequence: 0.71s;

2) Design optimal input sequece by solving mixed-integer quadratic problem: >2000s (unsolvable)

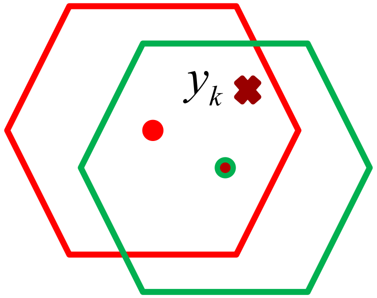
F. Xu. Observer-Based Asymptotic Active Fault Diagnosis: A Two- Layer Optimization Framework, *Automatica*, 125,109558, 2021.

Optimize Fault Diagnosis Performance

2. Exclusion Tendency-Based Design for Optimal Observer Gain

Motivation: Minimal size of output sets does not mean the optimal fault diagnosis performance.
Idea: **Deform output estimation set (OES)** to optimize observer gains.
Objective: Further improve the performance of fault diagnosis.

After establishing observer-based AFD framework → What we need to do next?



↓

Further improve fault diagnosis performance

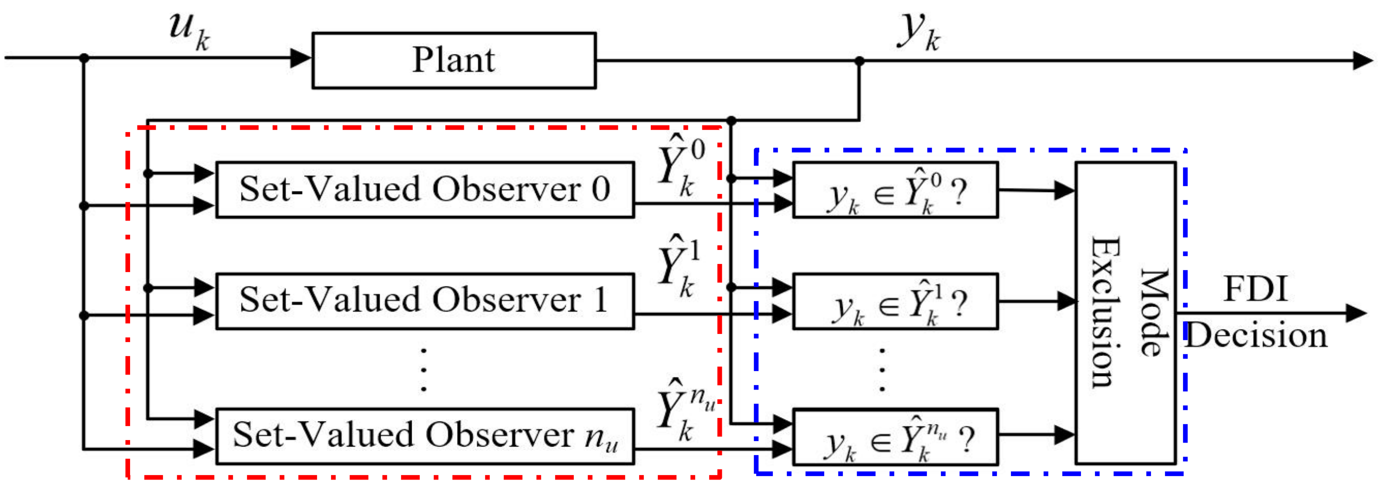
←

Optimize observer gains for fault diagnosis

Y.D. Fan, **F. Xu***, X.Q. Wang and B. Liang. Exclusion Tendency-Based Observer Design Framework for Active Fault Diagnosis, *Automatica*, In press, 2023.

Optimize Fault Diagnosis Performance

2. Exclusion Tendency-Based Design for Optimal Observer Gain



Step 1

Design gains to enlarge separation tendency of output sets

Step 2

Test fault diagnosis criterion to exclude unmatched modes

Question: Can we design gains to directly exclude unmatched modes such that the two steps above are fused into one step?

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Optimize Fault Diagnosis Performance

2. Exclusion Tendency-Based Design for Optimal Observer Gain

How to determine if the output is inconsistent with a certain model? →

Fault diagnosis criterion:

$$y_{k+1} \notin \hat{Y}^{i}_{k+1}, i \in \mathbb{I}$$

How to determine if the output is not in a set? →

Solve an **LP problem** [Scott, 2014]**:**

$$\begin{aligned} \hat{\delta}^i_{k+1} := \min_{\delta^i_{k+1}, \xi} \ & \delta^i_{k+1}, \\ \text{s.t. } & y_{k+1} = \hat{y}^{i,c}_{k+1} + \hat{H}^{i,y}_{k+1}\xi, \\ & \|\xi\|_\infty \leq \delta^i_{k+1}. \end{aligned}$$

Zonotopic representation: $\boxed{\hat{Y}^i_{k+1} = \langle \hat{y}^{i,c}_{k+1}, \hat{H}^{i,y}_{k+1} \rangle}$

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Optimize Fault Diagnosis Performance

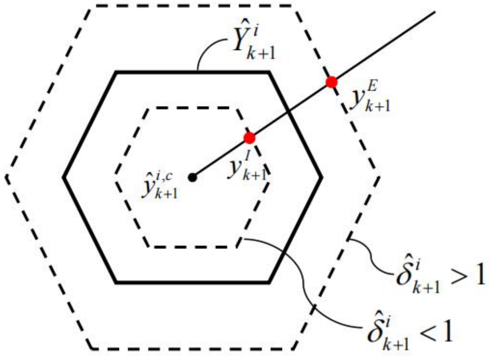
2. Exclusion Tendency-Based Design for Optimal Observer Gain

The value $\hat{\delta}^i_{k+1}$ is the scaling factor of $\hat{Y}^i_{k+1}$ around its center $\hat{y}^{i,c}_{k+1}$:

$$\begin{cases} \text{if } \hat{\delta}^i_{k+1} > 1 \quad \text{then } y_{k+1} \notin \hat{Y}^i_{k+1} \\ \text{if } \hat{\delta}^i_{k+1} \leq 1 \quad \text{then } y_{k+1} \in \hat{Y}^i_{k+1} \end{cases}$$

$\hat{\delta}^i_{k+1} = 1$ serves as a boundary between inclusion and exclusion;

Define $\hat{\delta}^i_{k+1}$ as the **exclusion tendency** of y_{k+1} from $\hat{Y}^i_{k+1}$.



The proposed idea to design optimal observer gains for fault diagnosis:

Increase exclusion tendency of the output from each inconsistant output set such that:

$$\hat{\delta}^i < 1 \quad \Rightarrow \quad \hat{\delta}^i > 1$$

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Optimize Fault Diagnosis Performance

2. Exclusion Tendency-Based Design for Optimal Observer Gain

The proposed idea of designing optimal gains:

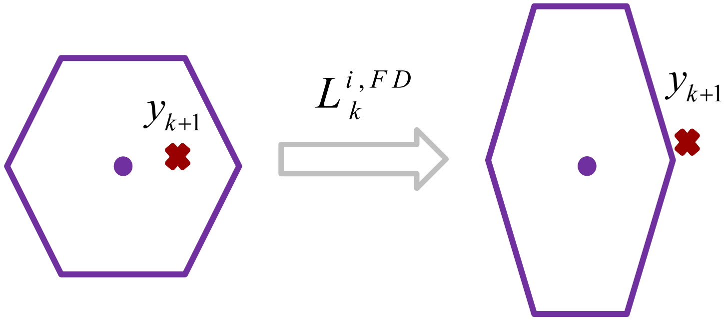
$$\begin{aligned}
\hat{x}^{i,c}_{k+1} =& (A - \boxed{L^i_k}C)\hat{x}^{i,c}_k + L^i_k(y_k - F\eta^c) \\
& + \text{mid}(B\mathbf{G}_i)u_k + E\omega^c, \\
\hat{H}^{i,x}_{k+1} =& [(A - \boxed{L^i_k}C)\hat{H}^{i,x}_k, -L^i_k F H_\eta, \\
& \text{diag}(\text{rad}(B\mathbf{G}_i)u_k), EH_\omega], \\
\hat{y}^{i,c}_{k+1} =& C\hat{x}^{i,c}_{k+1} + F\eta^c, \\
\hat{H}^{i,y}_{k+1} =& [C\hat{H}^{i,x}_{k+1}, FH_\eta],
\end{aligned}$$

$\boxed{L^i_k}$ can change the center $\hat{y}^{i,c}_{k+1}$ and generator matrix $\hat{H}^{i,y}_{k+1}$ of $\hat{Y}^i_{k+1}$,thus affect the exclusion tendency $\hat{\delta}^i_{k+1}$

Optimize L^i_k such that $\hat{\delta}^i_{k+1}$ is maximized

$$\hat{\delta}^{i,*}_{k+1} := \max_{L^i_k} \hat{\delta}^i_{k+1} = \max_{L^i_k} \min_{\delta^i_{k+1},\xi} \delta^i_{k+1}.$$

$$L^{i,FD}_k = \arg\max_{L^i_k}\min_{\delta^i_{k+1},\xi} \delta^i_{k+1}$$



$$\hat{\delta}^i_{k+1} := \min_{\delta^i_{k+1},\xi} \delta^i_{k+1},$$

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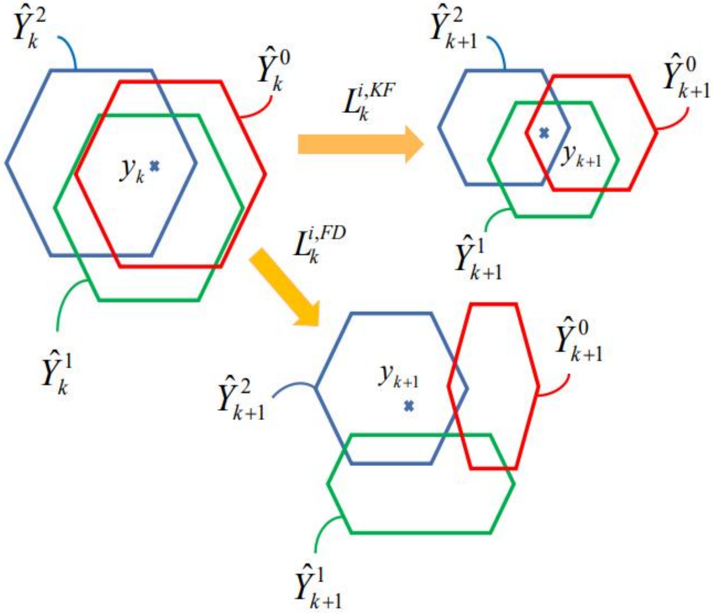
Optimize Fault Diagnosis Performance

2. Exclusion Tendency-Based Design for Optimal Observer Gain

An illustration of the significance of $L_k^{i,FD}$ compared with $L_k^{i,KF}$:

- ◆ $L_k^{i,FD}$ that maximizes $\hat{\delta}_{k+1}^{i}$ boosts the exclusion y_{k+1} of from $\hat{Y}_{k+1}^{i}$
- ◆ Compared with $L_k^{i,FD}$, the size of each obtained from $L_k^{i,KF}$ is minimized, $\hat{Y}_{k+1}^{i}$ is still included in the intersection re y_{k+1}



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2. Exclusion Tendency-Based Design for Optimal Observer Gain

Dual transformation of the optimization problem:

$$\hat{\delta}^{i,*}_{k+1} := \max_{L^i_k} \hat{\delta}^i_{k+1} = \max_{L^i_k} \boxed{\min_{\delta^i_{k+1},\xi} \delta^i_{k+1}}.$$

Bi-level max-min problem, which is difficult to solve

Due to the strong duality of the inner layer, it can be transformed into a **single-level max problem**

$$\begin{aligned}
\hat{\delta}^{i,*}_{k+1} = \max_{L^i_k,\lambda^i_{n,k},v^i_k} \; & \boxed{{v^i_k}^T C L^i_k}\left(y_k - \hat{y}^{i,c}_k\right) + {v^i_k}^T c^{i,\Delta}_k \\
s.t.\; & {\lambda^i_{1,k}}^T - {\lambda^i_{2,k}}^T = -({v^i_k}^T CA - \boxed{{v^i_k}^T C L^i_k} C)\hat{H}^{i,x}_k, \\
& {\lambda^i_{3,k}}^T - {\lambda^i_{4,k}}^T = \boxed{{v^i_k}^T C L^i_k} F H_\eta, \\
& {\lambda^i_{5,k}}^T - {\lambda^i_{6,k}}^T = -{v^i_k}^T H^{i,\Delta}_k, \\
& \lambda^i_{n,k} \geq 0,\; n = 1, ..., 6, \\
& \sum_{n=1}^{6} {\lambda^i_{n,k}}^T \mathbf{1} = 1,\;\; \forall i \in \mathbb{I}.
\end{aligned}$$

Bi-linear terms

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2. Exclusion Tendency-Based Design for Optimal Observer Gain

Based on a transformation $(\gamma_k^i)^T = (v_k^i)^T C L_k^i$, we obtain the following **linear problem:**

$$\bar{\delta}_{k+1}^{i,*} := \max_{\gamma_k^i,\lambda_{n,k}^i,v_k^i} \boxed{\gamma_k^{i\,T}}\left(y_k - \hat{y}_k^{i,c}\right) + v_k^{i\,T} c_k^{i,\Delta}$$

$$\text{s.t. } \lambda_{1,k}^{i\ T} - \lambda_{2,k}^{i\ T} = -v_k^{i\,T} C A \hat{H}_k^{i,x} + \boxed{\gamma_k^{i\,T}} C \hat{H}_k^{i,x},$$

$$\lambda_{3,k}^{i\ T} - \lambda_{4,k}^{i\ T} = \boxed{\gamma_k^{i\,T}} F H_\eta,$$

$$\lambda_{5,k}^{i\ T} - \lambda_{6,k}^{i\ T} = -v_k^{i\,T} H_k^{i,\Delta},$$

$$\lambda_{n,k}^i \geq 0, n = 1, \ldots, 6,$$

$$\sum_{n=1}^{6} \lambda_{n,k}^{i\ T} \mathbf{1} = 1, \ \forall i \in \mathbb{I}.$$

$$\xrightarrow{(v_k^{i,*})^T C L_k^i = (\gamma_k^{i,*})^T}$$

The observer gain $L_k^{i,FD}$ has the form:

Where $L_k^{i,FD} = L_k^{i,0} + L_k^{i,1} R_k^i$

$$L_k^{i,0} = \frac{C^T v_k^{i,*} \gamma_k^{i,*T}}{v_k^{i,*T} C C^T v_k^{i,*}}, \quad L_k^{i,1} = I_{n_x} - \frac{C^T v_k^{i,*} v_k^{i,*T} C}{v_k^{i,*T} C C^T v_k^{i,*}}$$

R_k^i is a free variable which can be further optimized to improve performance.

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2. Exclusion Tendency-Based Design for Optimal Observer Gain

Further improvements of observer performance:

(I) Minimization of F-radius:

Minimize the F-radius of the state estimation set for more accurate results:

$$\min_{R_k^i} \| \hat{X}_{k+1}^i \|_F^2 = \mathrm{tr}\left(\hat{H}_{k+1}^{i,x\,T} \hat{H}_{k+1}^{i,x} \right), \forall i \in \mathbb{I}.$$

The derivative of $\|\hat{X}_{k+1}^i\|_F^2$ with respect to R_k^i is

$$\frac{\partial \|\hat{X}_{k+1}^i\|_F^2}{\partial R_k^i} = 0 \longrightarrow \text{the optimal } R_k^{i,*}$$

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2. Exclusion Tendency-Based Design for Optimal Observer Gain

(II) Suppress divergence of observer dynamics:

The observer gain is designed to suppress the divergence of estimation error. Consider the Lyapunov stability to ensure the stable dynamics:

$$\min_{R_k^i,\alpha} \ \alpha$$

$$s.t. \begin{bmatrix} -W & \left(A - L_k^{i,FD}C\right)^T W \\ \star & -W \end{bmatrix} \preceq \alpha I$$

$W \succ 0$

$\star$ denotes the symmetric term

The dynamics of ***SVO*** is stable at the *k-th* step if the optimal solution

$$\alpha^* < 0$$

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2. Exclusion Tendency-Based Design for Optimal Observer Gain

(III) Stability guarantee:

The stability-guaranteeing approach is to add the Lyapunov stability condition into the optimization problem:

$$\hat{\delta}_{k+1}^{i,L} = \max_{L_k^i, \lambda_{n,k}^i, v_k^i} \; v_k^{i\,T} C L_k^i \left(y_k - \hat{y}_k^{i,c} \right) + v_k^{i\,T} c_k^{i,\Delta}$$

$$\text{s.t. } \lambda_{1,k}^{i\ \ T} - \lambda_{2,k}^{i\ \ T} = -(v_k^{i\,T} C A - v_k^{i\,T} C L_k^i C) \hat{H}_k^{i,x},$$

$$\lambda_{3,k}^{i\ \ T} - \lambda_{4,k}^{i\ \ T} = v_k^{i\,T} C L_k^i F H_{\eta},$$

$$\lambda_{5,k}^{i\ \ T} - \lambda_{6,k}^{i\ \ T} = -v_k^{i\,T} H_k^{i,\Delta},$$

$$\lambda_{n,k}^{i} \geq 0, n = 1, \ldots, 6,$$

$$\sum_{n=1}^{6} \lambda_{n,k}^{i\ \ T} \mathbf{1} = 1,$$

$$\begin{bmatrix} -W & (A - L_k^i C)^T W \\ \star & -W \end{bmatrix} \preceq 0, \;\; \forall i \in \mathbb{I}.$$

$L_k^{i,FD}$ given by the left problem guarantees **stable observer dynamics**

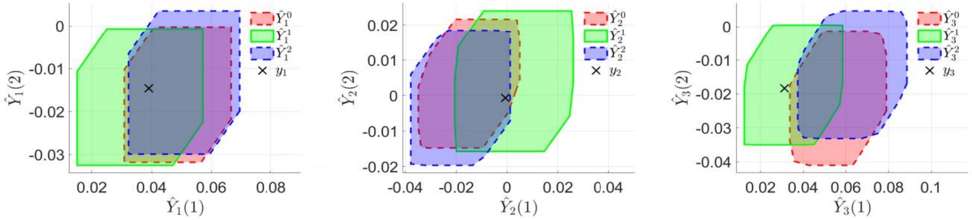
Optimize Fault Diagnosis Performance

2. Exclusion Tendency-Based Design for Optimal Observer Gain

Illustrative example :

A two-input, four-state and two-output system with three actuator modes (healthy and two faulty modes)

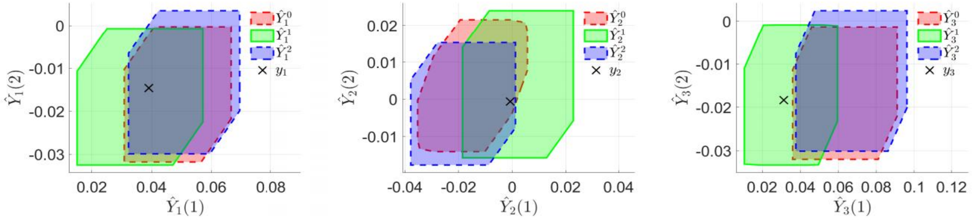
Proposed method:



(a) $k=1$ (b) $k=2$ (c) $k=3$

Design principle (I)

Computing time: 1.579s

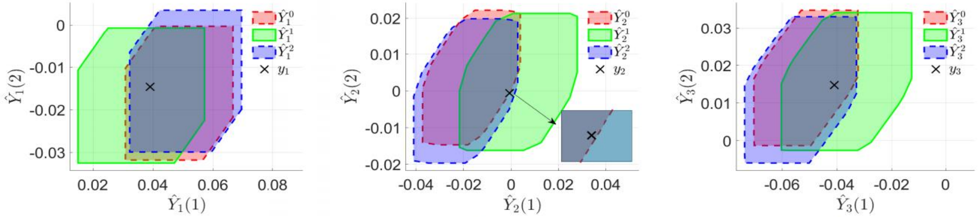


(d) $k=1$ (e) $k=2$ (f) $k=3$

Design principle (II)

Computing time: 2.64s

Previous method:



(a) $k=1$ (b) $k=2$ (c) $k=3$

Computing time: 0.0013s

See the original paper for detailed comparison

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Part 3: Conclusions and Future Work

Conclusions and Future Work

Conclusions:

1) Propose a separation tendency-based observer AFD framework that has lower complexity than classical methods using set separation condition.
2) Propose an exclusion tendency-based framework to design optimal observer gains to improve fault diagnosis performance.
3) Separation tendency-based methods may loss some AFD performance compared with set separation-based methods.

Future Work:

Make a detailed comparison of AFD performance between set separation tendency-based methods and set separation-based methods.

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Thanks for your attention！