Union of adjacent contractors

Quentin Brateau

November 03, 2023

ENSTA Bretagne

Context

PhD thesis

Research laboratory

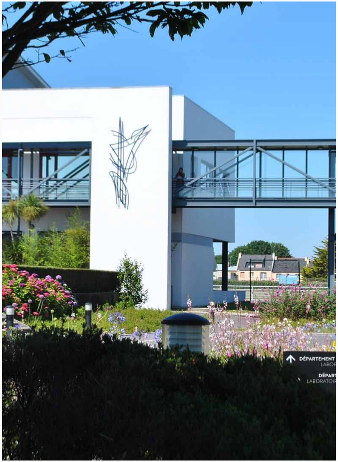
- ENSTA Bretagne, UMR 6285, Lab-STICC, IAO, ROBEX

Supervisiors

- Luc Jaulin
- Fabrice Le Bars

Funding

- AID funding: Jean-Daniel Masson



Goal

AUV

- Control of torpedo-like AUV
- Riptide's micro-uuv

Environment

- Constrained environment
- Pool, harbor, ...

Goals

- Reactivity
- Manoeuvrability

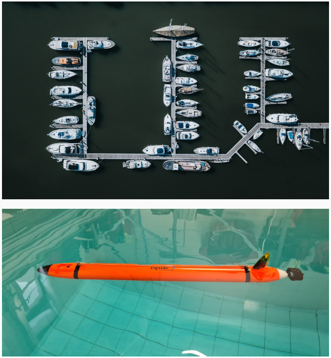


Figure 1: Harbor and Riptide in the ENSTA Bretagne pool

Riptide's sensors

- Proprioceptive

Figure 2: Riptide's echosounder located below the head

Riptide's sensors

- Proprioceptive
  - IMU

Figure 2: Riptide's echosounder located below the head

Robot sensors

Riptide's sensors

- Proprioceptive
  - IMU
- Exteroceptive

Figure 2: Riptide's echosounder located below the head

Robot sensors

Riptide's sensors

- Proprioceptive
  - IMU
- Exteroceptive
  - Pressure sensor

Figure 2: Riptide's echosounder located below the head

Riptide's sensors

- Proprioceptive
  - IMU
- Exteroceptive
  - Pressure sensor
  - Echosounder

Figure 2: Riptide's echosounder located below the head

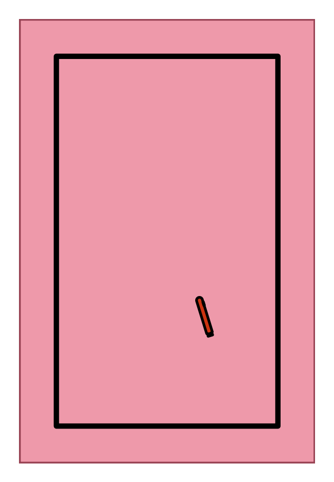


Figure 3: Unknown localization

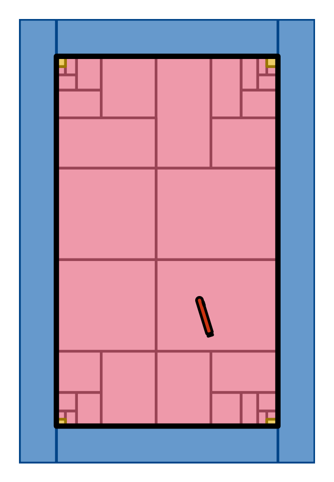


Figure 3: Robot is in the pool

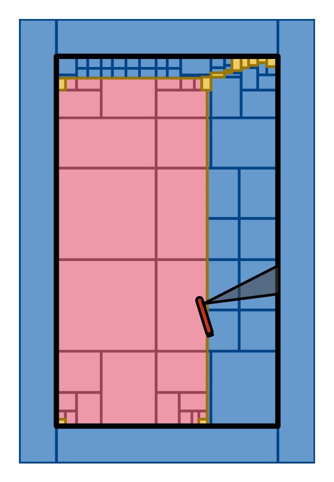


Figure 3: Distance measurement

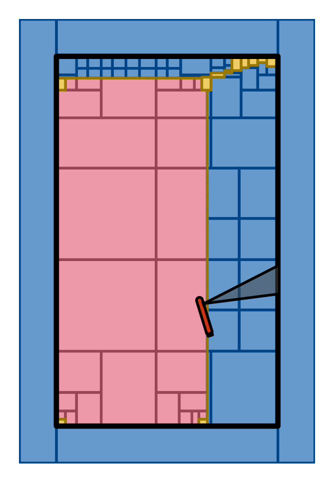


Figure 3: Distance measurement

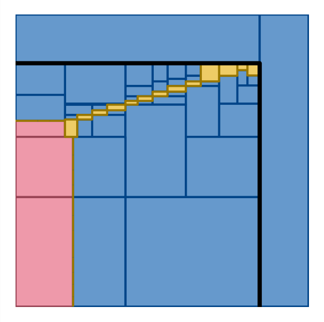


Figure 4: Uncertain area

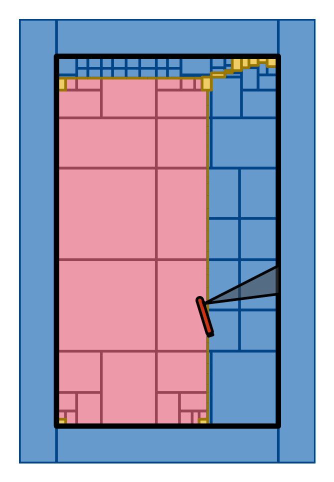


Figure 3: Distance measurement

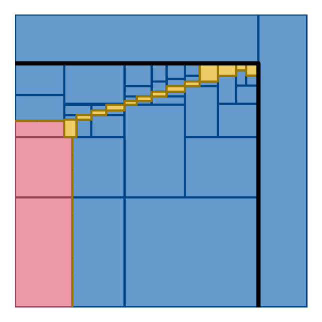


Figure 4: Uncertain area

Problem statement

Why is there this uncertain area?

Geometric contractors

Geometric contractors

Geometric contractors

- Contractors based on geometrical constraints

Geometric contractors

- Contractors based on geometrical constraints
- Usefull in localization

Geometric contractors

- Contractors based on geometrical constraints
- Usefull in localization

Example

Geometric contractors

- Contractors based on geometrical constraints
- Usefull in localization

Example

- CtcCross

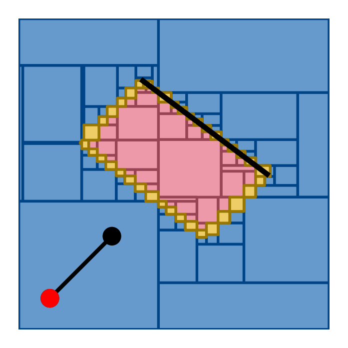


Figure 5: SepCross

Geometric contractors

- Contractors based on geometrical constraints
- Usefull in localization

Example

- CtcCross
- CtcVisible

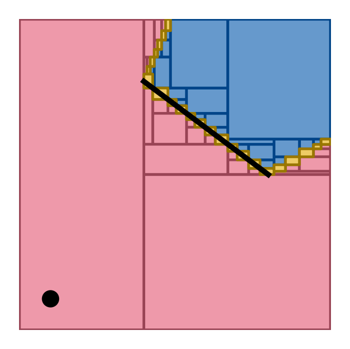


Figure 6: SepVisible

Geometric contractors

- Contractors based on geometrical constraints
- Usefull in localization

Example

- CtcCross
- CtcVisible

Polygon extension

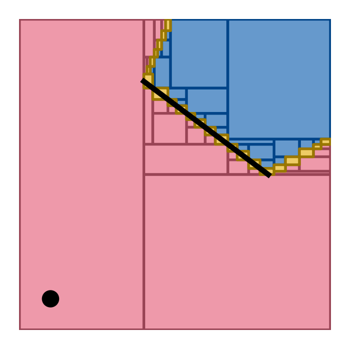


Figure 6: SepVisible

Geometric contractors

- Contractors based on geometrical constraints
- Usefull in localization

Example

- CtcCross
- CtcVisible

Polygon extension

- Contractors on segments

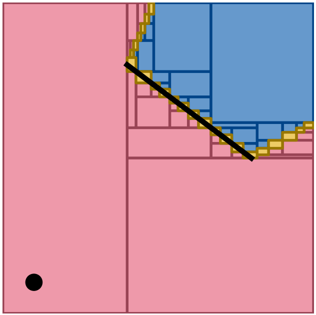


Figure 6: SepVisible

Geometric contractors

- Contractors based on geometrical constraints
- Usefull in localization

Example

- CtcCross
- CtcVisible

Polygon extension

- Contractors on segments
- Extension on polygons

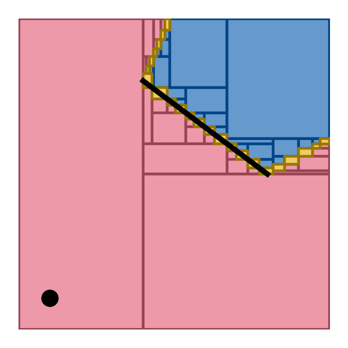


Figure 6: SepVisible

Union problem

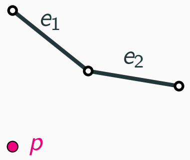
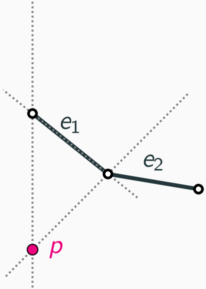


Figure 7: Sepvisible on a polygon



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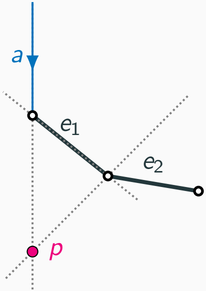

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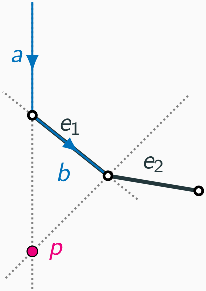

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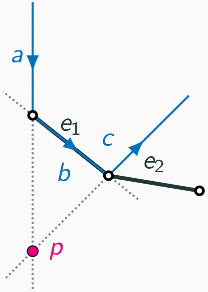

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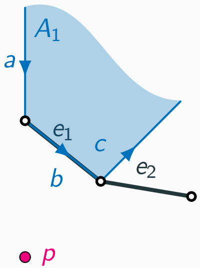

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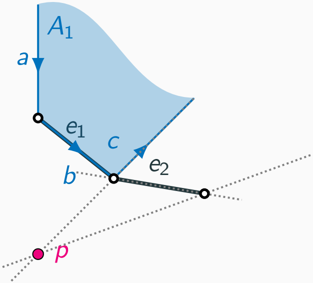

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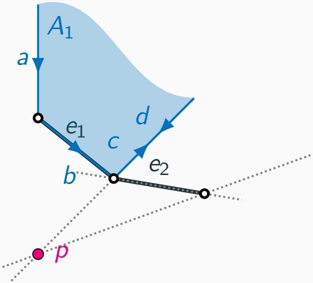

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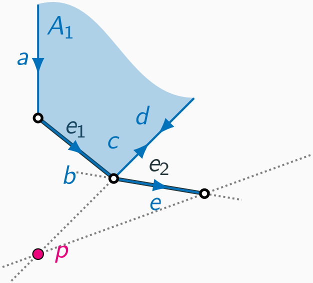

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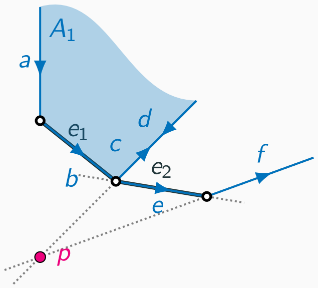

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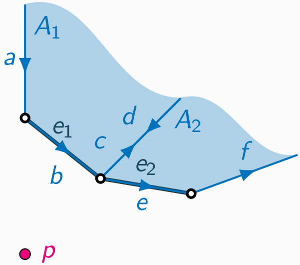

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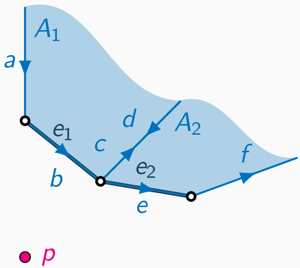
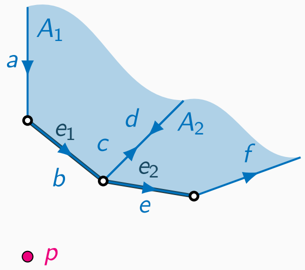


Figure 7: Sepvisible on a polygon

Masked area

$$A = A_1 \cup A_2$$



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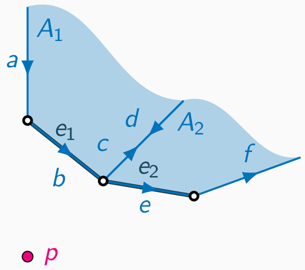
Masked area

$$A = A_1 \cup A_2$$

Boundary aproach



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Masked area

$$A = A_1 \cup A_2$$

Boundary aproach

- $\partial A_1 = a + b + c$

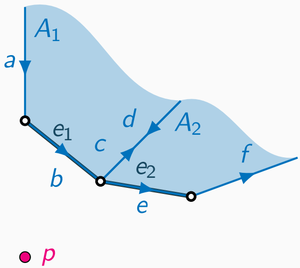


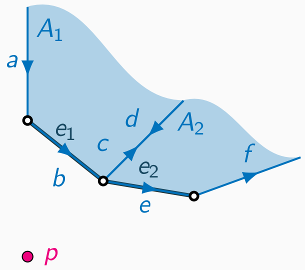
Figure 7: Sepvisible on a polygon

Masked area

$$A = A_1 \cup A_2$$

Boundary aproach

- $\partial A_1 = a + b + c$
- $\partial A_2 = d + e + f$



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Masked area

$$A = A_1 \cup A_2$$

Boundary aproach

- $\partial A_1 = a + b + c$
- $\partial A_2 = d + e + f$
- $\partial A \neq \partial A_1 + \partial A_2$

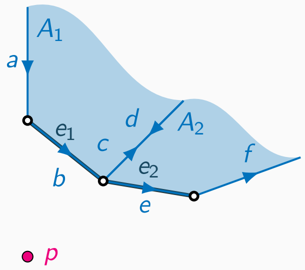


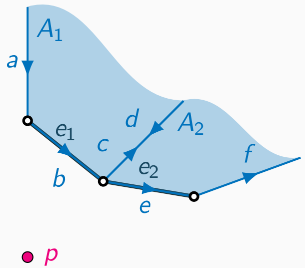
Figure 7: Sepvisible on a polygon

Masked area

$$A = A_1 \cup A_2$$

Boundary aproach

- $\partial A_1 = a + b + c$
- $\partial A_2 = d + e + f$
- $\partial A = a + b + c + d + e + f$



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Masked area

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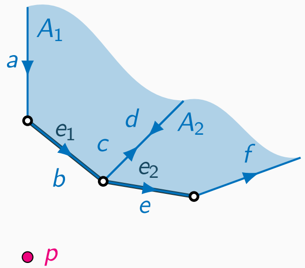

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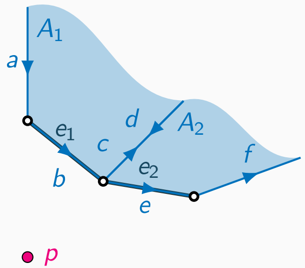
Masked area

$$A = A_1 \cup A_2$$

Boundary aproach

- $\partial A_1 = a + b + c$
- $\partial A_2 = d + e + f$
- $\partial A = a + b + c + (-c) + e + f$



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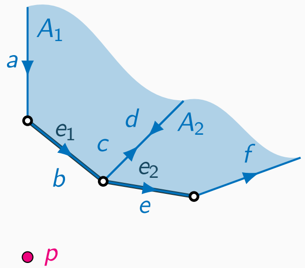

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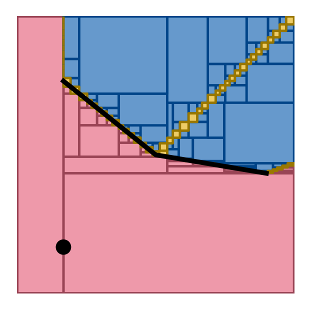
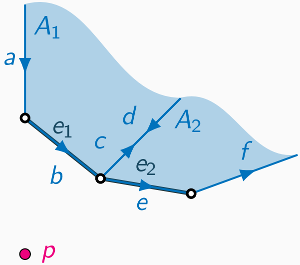


Figure 8: SepVisible on a polygon



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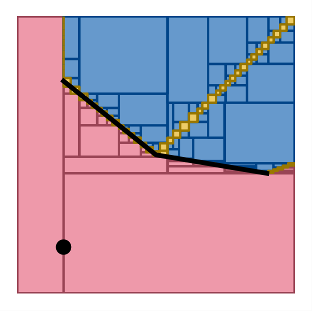
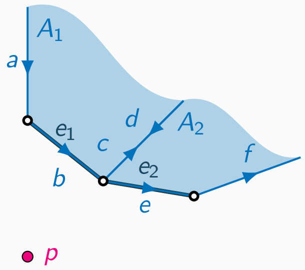


Figure 8: SepVisible on a polygon

Solution

Union problem



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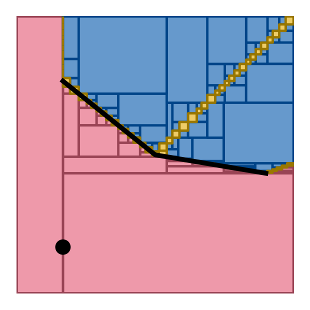
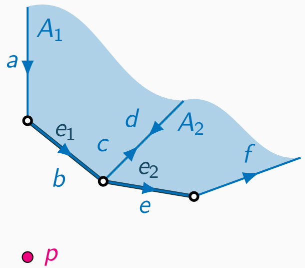


Figure 8: SepVisible on a polygon

Solution

- $C_{A \cup B} \neq C_A \cup C_B$



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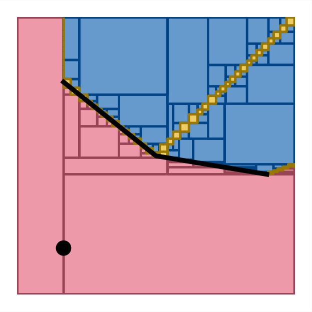

Figure 8: SepVisible on a polygon

Solution

- $C_{A \cup B} \neq C_A \cup C_B$
- $c + d = \mathbb{R}^2$

Union of contractors

Set definition

- $A : \{x_1 + 3 \cdot x_2 \in [-\infty; 0]\}$

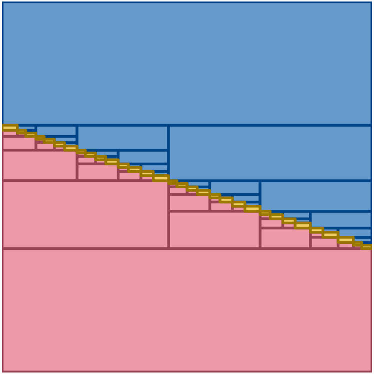

Figure 9: A

Set definition

- $A : \{x_1 + 3 \cdot x_2 \in [-\infty; 0]\}$
- $B : \left\{(x_1 + 0.5)^2 + x_2^2 - 2^2 \in [-\infty; 0]\right\}$

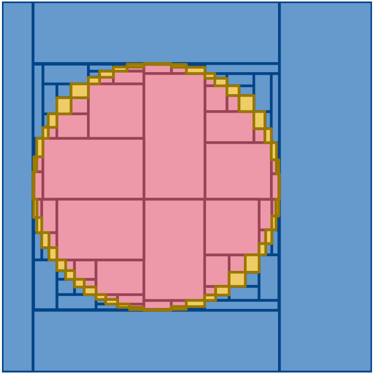

Figure 9: B

Set definition

- $A : \{x_1 + 3 \cdot x_2 \in [-\infty; 0]\}$
- $B : \left\{(x_1 + 0.5)^2 + x_2^2 - 2^2 \in [-\infty; 0]\right\}$
- $C : \left\{(x_1 - 0.5)^2 + x_2^2 - 2^2 \in [-\infty; 0]\right\}$

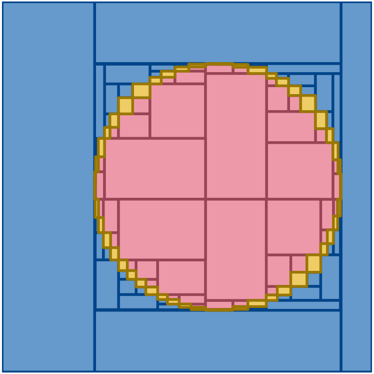

Figure 9: C

Set definition

- $A : \{x_1 + 3 \cdot x_2 \in [-\infty; 0]\}$
- $B : \left\{(x_1 + 0.5)^2 + x_2^2 - 2^2 \in [-\infty; 0]\right\}$
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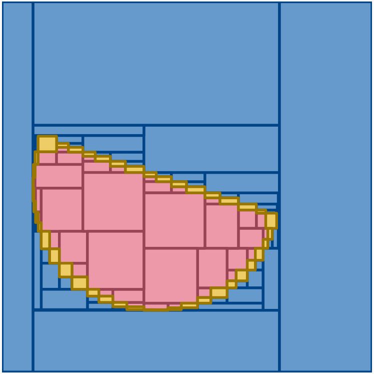

Figure 9: $A \cap B$

Set definition

- $A : \{x_1 + 3 \cdot x_2 \in [-\infty; 0]\}$
- $B : \left\{(x_1 + 0.5)^2 + x_2^2 - 2^2 \in [-\infty; 0]\right\}$
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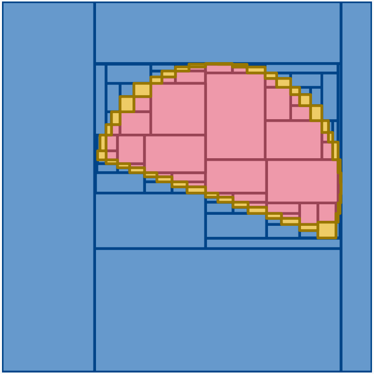


Figure 9: $\bar{A} \cap C$

Set definition

- $A : \{x_1 + 3 \cdot x_2 \in [-\infty; 0]\}$
- $B : \left\{(x_1 + 0.5)^2 + x_2^2 - 2^2 \in [-\infty; 0]\right\}$
- $C : \left\{(x_1 - 0.5)^2 + x_2^2 - 2^2 \in [-\infty; 0]\right\}$

Set expression

$$Z = (A \cap B) \cup (\bar{A} \cap C)$$

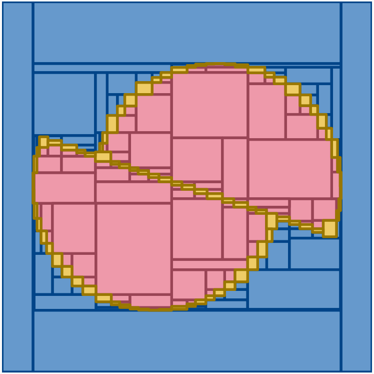


Figure 9: $Z = (A \cap B) \cup (\bar{A} \cap C)$

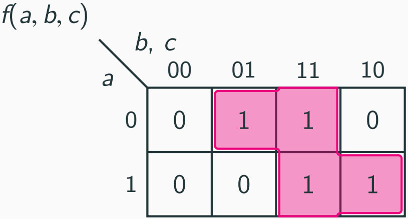


Figure 10: Karnaugh map - Z

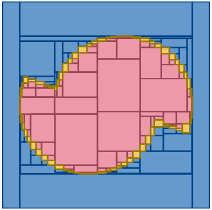
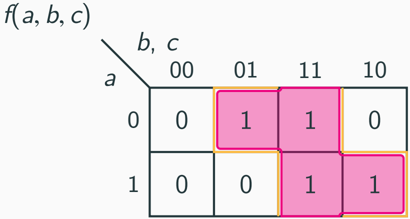


Figure 11: Z



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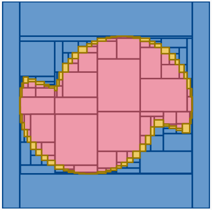

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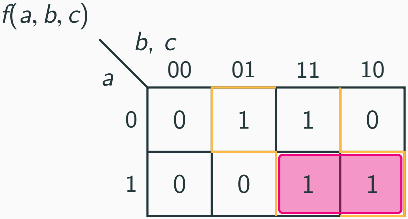

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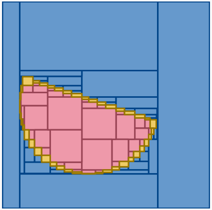


Figure 11: $A \cap B$

Disjunctive Normal Form

$$f(a, b, c) = (a \wedge b)$$

Karnaugh map

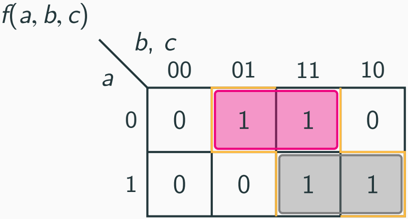


Figure 10: Karnaugh map - Z

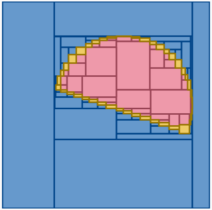


Figure 11: $\bar{A} \cap C$

Disjunctive Normal Form

$$f(a,b,c) = (a \wedge b) \vee (\neg a \wedge c)$$

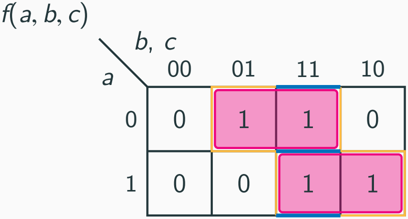


Figure 10: Karnaugh map - Z

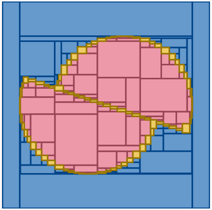
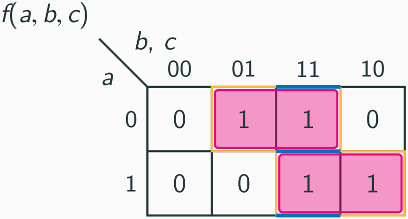


Figure 11: $(A \cap B) \cup (\bar{A} \cap C)$

Disjunctive Normal Form

$$f(a, b, c) = (a \wedge b) \vee (\neg a \wedge c)$$

Karnaugh map



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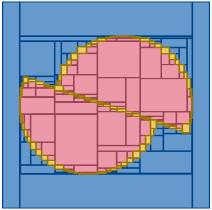

Figure 11: $(A \cap B) \cup (\bar{A} \cap C)$

Disjunctive Normal Form

$$f(a, b, c) = (a \wedge b) \vee (\neg a \wedge c)$$

Boundary Preserving Form

$$f(a, b, c) = (a \wedge b) \vee (\neg a \wedge c)$$

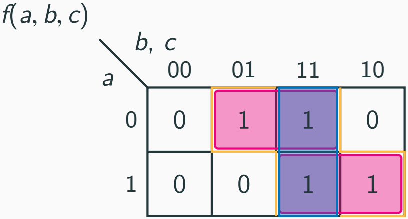

Figure 10: Karnaugh map - Z

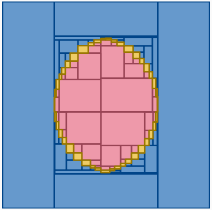

Figure 11: $B \cap C$

Disjunctive Normal Form

$$f(a, b, c) = (a \wedge b) \vee (\neg a \wedge c)$$

Boundary Preserving Form

$$f(a, b, c) = (a \wedge b) \vee (\neg a \wedge c) \vee (b \wedge c)$$

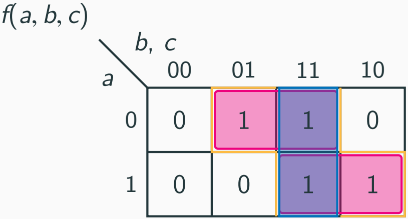

Figure 10: Karnaugh map - Z

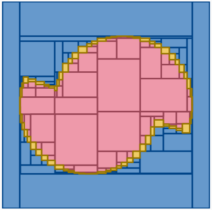

Figure 11: Z

Disjunctive Normal Form

$$f(a, b, c) = (a \wedge b) \vee (\neg a \wedge c)$$

Boundary Preserving Form

$$f(a, b, c) = (a \wedge b) \vee (\neg a \wedge c) \vee (b \wedge c)$$

Adjacent Sets

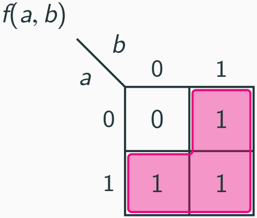

Figure 12: Karnaugh map - $A \cup B$

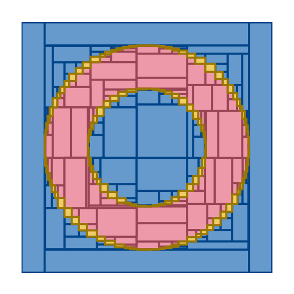

Figure 13: Expected $A \cup B$

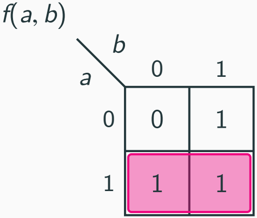

Figure 12: Karnaugh map - $A \cup B$

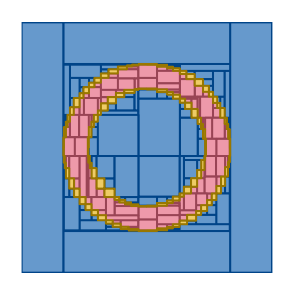

Figure 13: A

Adjacent Sets

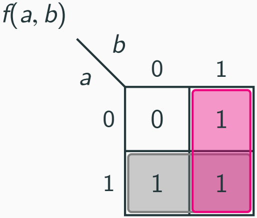

Figure 12: Karnaugh map - $A \cup B$

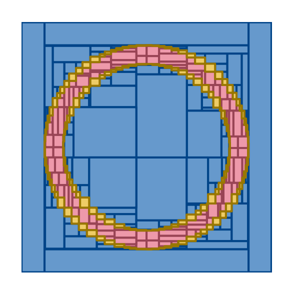

Figure 13: B

Adjacent Sets

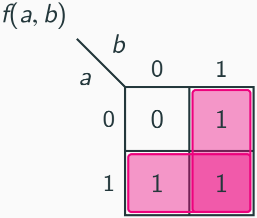

Figure 12: Karnaugh map - $A \cup B$

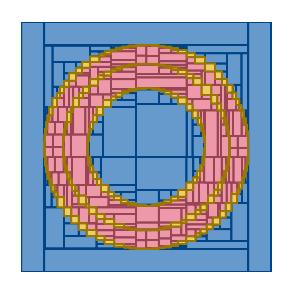

Figure 13: $A \cup B$

$f(a, b, c)$

a \ b, c	00	01	11	10
0	0	1	1	0
1	0	0	1	1

Figure 14: Karnaugh map of the expression

Boundary approach

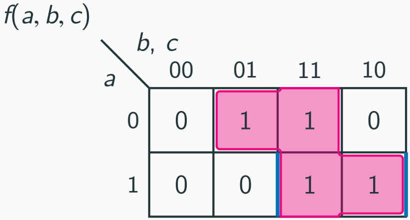

Figure 14: Karnaugh map of the expression

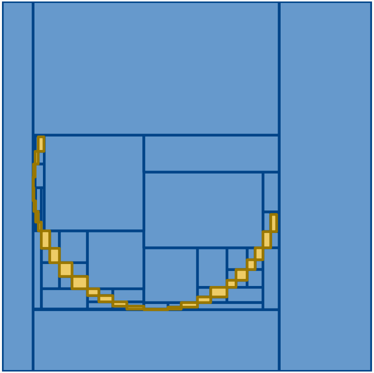

Figure 15: $\partial B \cap A$

$f(a, b, c)$

a \ b, c	00	01	11	10
0	0	1	1	0
1	0	0	1	1

Figure 14: Karnaugh map of the expression

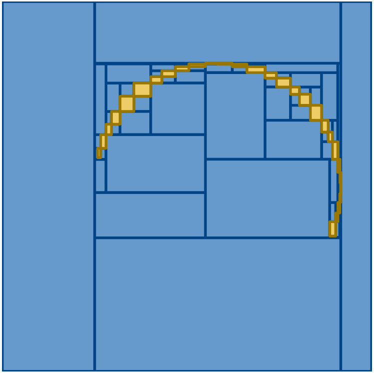

Figure 15: $\partial C \cap \bar{A}$

Boundary approach

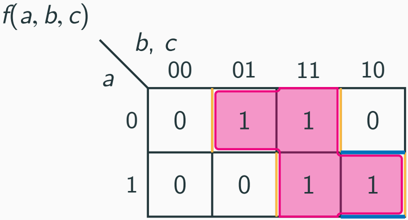

Figure 14: Karnaugh map of the expression

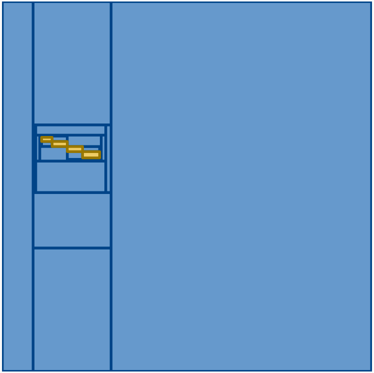

Figure 15: $\partial A \cap B \cap \bar{C}$

Boundary approach

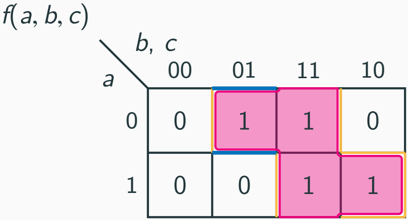

Figure 14: Karnaugh map of the expression

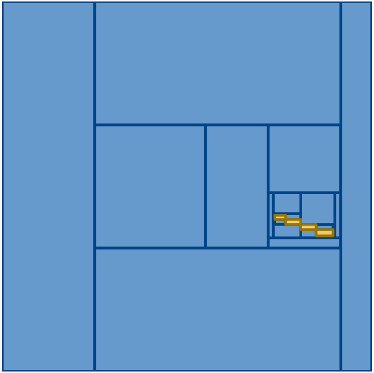

Figure 15: $\partial A \cap \bar{B} \cap C$

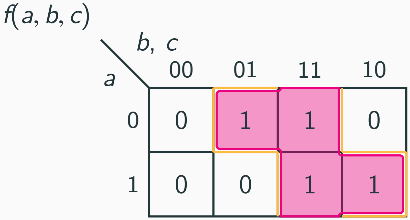

Figure 14: Karnaugh map of the expression

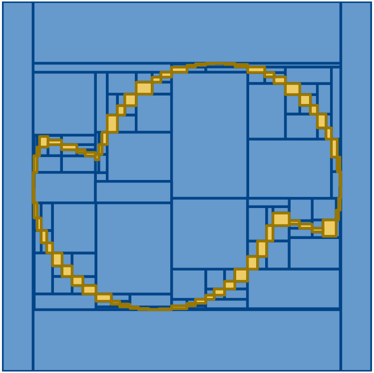

Figure 15: ∂Z

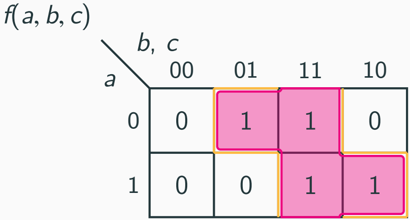

Figure 14: Karnaugh map of the expression

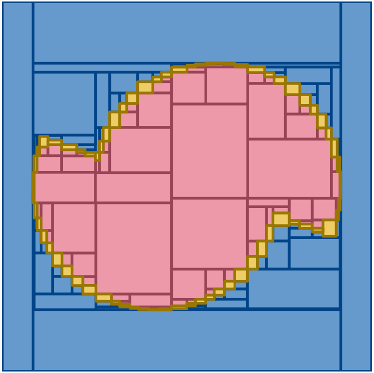

Figure 15: Z

Problem statement

Problem statement

- Union contractors problem

Outlook

Problem statement

- Union contractors problem
- Adjacent contractors

Outlook

Problem statement

- Union contractors problem
- Adjacent contractors
- Fake boundary

Outlook

Problem statement

- Union contractors problem
- Adjacent contractors
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Geometric contractors

Problem statement

- Union contractors problem
- Adjacent contractors
- Fake boundary

Geometric contractors

- Define $C_{A \cup B}$

Outlook

Problem statement

- Union contractors problem
- Adjacent contractors
- Fake boundary

Geometric contractors

- Define $C_{A \cup B}$
- Add constraint $A \cup \bar{A} = \mathbb{R}^2$

Outlook

Problem statement

- Union contractors problem
- Adjacent contractors
- Fake boundary

Geometric contractors

- Define $C_{A \cup B}$
- Add constraint $A \cup \bar{A} = \mathbb{R}^2$

General union

Outlook

Problem statement

- Union contractors problem
- Adjacent contractors
- Fake boundary

Geometric contractors

- Define $C_{A \cup B}$
- Add constraint $A \cup \bar{A} = \mathbb{R}^2$

General union

- Boundary Preserving Form

Outlook

Problem statement

- Union contractors problem
- Adjacent contractors
- Fake boundary

Geometric contractors

- Define $C_{A \cup B}$
- Add constraint $A \cup \bar{A} = \mathbb{R}^2$

General union

- Boundary Preserving Form
- Boundary approach + Predicate

Questions?

Union of adjacent contractors

Quentin Brateau

November 03, 2023

ENSTA Bretagne