Filtered High Gain Interval Observer for LPV Systems with Bounded Uncertainties

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Presentation Overview

Introduction

- Problem of state estimation of uncertain systems
- Existing approaches:
  - Stochastic: *Extended and Unscented Kalman Filters, Particle Filters [György et al. 2014] . . .*
  - Deterministic: *Sliding Modes Observers [Palli et al. 2018], High Gain Observers [Khalil et al. 2014], Interval Observers [Efimov et al. 2016] . . .*

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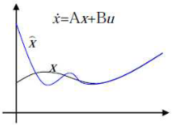
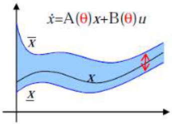


- HGOs had become popular
  - Pros: Easy to design, fast convergence tuned by a unique parameter, robust to disturbances
  - Cons: Sensitive to measurement noise, peaking phenomenon



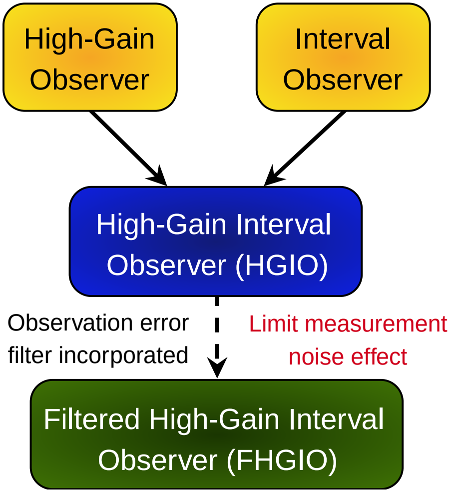
- Interval Observers assume bounded uncertainties [Gouzé et al. 2000]
  - Pros: Guaranteed inclusion $x \in [\underline{x}, \overline{x}]$
  - Cons: Complex design to ensure cooperativity

Objective

Design a simple and arbitrarily fast interval observer.

Improve existing designs:

- Consider the measurement noise and mitigate its effect compared to [Menini et al. 2021]
- Solve an issue of real-time implementation in [Thabet et al. 2022]



High Gain Observer

Commonly designed for systems diffeomorphic to [Khalil et al. 2014]:

$$\begin{cases} \dot{x}(t) = Ax(t) + \varphi(u(t), x(t)) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where $A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and φ locally Lipschitz and lower triangular

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HGO dynamics:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \varphi(u(t), x(t)) + L(y(t) - C\hat{x}(t)) \quad (2)$$

where $L = \theta\Delta_\theta^{-1}K$, $\theta > 1$, $\Delta_\theta = \text{diag}(1, ..., \theta^{-n+1})$ and $K = \begin{bmatrix} k_1 & \dots & k_n \end{bmatrix}^T$.

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Unstructured systems can also be considered [Rajamani 1998]

Interval Observer

Definition: Monotone system

The system $\dot{x} = f(x)$ is monotone if its solutions $x(t, x_0)$ and $x(t, \tilde{x}_0)$, initialized respectively to x_0 and $\tilde{x}_0$ at t_0, are such that:

$$x_0 \leq \tilde{x}_0 \Rightarrow x(t, x_0) \leq x(t, \tilde{x}_0), \quad \forall t \geq t_0$$

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The dynamics $\dot{x} = Ax + \varphi$ is cooperative if A is a Metzler matrix ($\forall i \neq j, A_{ij} \geq 0$) and φ is non-negative.

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System dynamics:

$$\begin{cases} \dot{x}(t) = Ax(t) + d(t) \\ y(t) = Cx(t) \end{cases} \qquad (3)$$

Interval observer dynamics:

$$\begin{cases} \dot{\overline{x}}(t) = A\overline{x}(t) + L(y(t) - C\overline{x}(t)) + \overline{d}(t) \\ \dot{\underline{x}}(t) = A\underline{x}(t) + L(y(t) - C\underline{x}(t)) + \underline{d}(t) \end{cases} \qquad (4)$$

Context

Continuous Linear Parameter Varying (LPV) system:

$$\begin{cases} \dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) + Ed(t) \\ y(t) = Cx(t) + w(t) \end{cases} \quad (5)$$

$x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$, $y(t) \in \mathbb{R}^{n_y}$,
$d(t) \in \mathbb{R}^{n_d}$, $w(t) \in \mathbb{R}^{n_y}$,
$\rho(t) \in \mathbb{R}^{n_r}$ is unmeasurable.

$$A(\rho(t)) = A_0 + \sum_{i=1}^{n_r} A_i \rho_i(t)$$

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It can be presented as a partially linear system:

$$\begin{cases} \dot{x}(t) = A_0 x(t) + \varphi(u(t), x(t)) + Ed(t) \\ y(t) = Cx(t) + w(t) \end{cases} \quad (6)$$

where $\varphi(u(t), x(t)) = \sum_{i=1}^{n_r} A_i \rho_i(t) x(t) + B(\rho(t))u(t)$.

Assumptions

Assumption 1: Observability

The pair (A_0, C) is observable.

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The input $u(t)$ is bounded $\forall t \in \mathbb{R}^+$.

Assumption 3: Bounded uncertainties

$$x(0) \in x^c(0) \pm x^r(0), \quad \text{with } (x^c(0), x^r(0)) \in \mathbb{R}^{n_x} \times (\mathbb{R}^+)^{n_x}.$$

$$\forall t \in \mathbb{R}^+, \begin{cases} \rho(t) \in \mathbf{0} \pm \mathbf{1}, \\ d(t) \in d^c \pm d^r, \quad \text{with } (d^c, d^r) \in \mathbb{R}^{n_d} \times (\mathbb{R}^+)^{n_d}, \\ w(t) \in w^c \pm w^r, \quad \text{with } (w^c, w^r) \in \mathbb{R}^{n_y} \times (\mathbb{R}^+)^{n_y} \end{cases}.$$

Center-radius interval definition [Combastel et al. 2011]

$$\begin{aligned} \pm : \quad \mathbb{C} \times \mathbb{C}^+ &\rightarrow \mathbb{IC} \\ (c, r) &\mapsto c \pm r = [c - r, c + r] \end{aligned}$$

Design

High-Gain Observer [Khalil et al. 2014] [Rajamani 1998]:

$$\dot{\hat{x}}(t) = (A_0 - LC)\hat{x}(t) + \varphi(u(t), \hat{x}(t)) + E\hat{d}(t) + Ly(t) \tag{7}$$

where L is the observer gain depending on the high-gain parameter θ

- L is computed to ensure stability

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- L is computed to ensure stability
- To make an interval observer, $(A_0 - LC)$ must be Metzler
- Difficult to satisfy both conditions in practice
  $\rightarrow$ Usually overcame by a change of coordinate [Combastel et al. 2011] [Efimov et al. 2016]

Design

Three steps to follow:

1. Gain design of L
   - $(A_0 - LC)$ is Hurwitz with distinct poles $\theta(\xi + i\zeta)$ where $\theta \geq 1$
   - $\min_{\omega \geq 0} \sigma_{min}(A_0 - LC - i\omega I_{n_x}) > \gamma_\varphi$

     $\rightarrow$ Stability and $(A_0 - LC)$ is $\mathbb{C}$-diagonalizable as $v\text{diag}(\theta\xi + i\theta\zeta)v^{-1}$

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2. Change of coordinate
   - $z(t) = \Omega(t)\hat{x}(t)$ with $\Omega(t) = \text{diag}(e^{-i\theta\zeta t})v^{-1}$
   - $\dot{z}(t) = \text{diag}(\theta\xi)z(t) + \Omega(t)(\varphi(u(t), \Omega^{-1}(t)z(t)) + E\hat{d}(t) + Ly(t))$

   $\rightarrow$ Ensure the cooperativity of the error dynamics in the new base

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3. Derivation of the interval observer structure
   - Use the bounds of the uncertainties
   - Center and radius dynamics

Design

Theorem 1: High-Gain Interval Observer (HGIO)

Consider the HGO (7) and Assumptions 1-3. The system:

$$\begin{cases} \dot{z}^c(t) = \mathrm{diag}(\theta\xi) z^c(t) + \Psi^c(t) \\ \dot{z}^r(t) = \mathrm{diag}(\theta\xi) z^r(t) + \Psi^r(t) \end{cases} \quad (8)$$

is an interval observer for (7), named HGIO, with:

$$\begin{cases} \Psi^c(t) = \Omega(t)(B_0 u(t) + Ed^c + Ly(t)) \\ \Psi^r(t) = |\Omega(t)[\ldots A_i \Omega^{-1}(t) z^c(t) + B_i u(t) \ldots, \\ \qquad \ldots A_i \Omega^{-1}(t) \Delta(z^r(t)) \ldots, Ed^r]|\mathbf{1} \end{cases}$$

where $\forall v \in \mathbb{C}^n, \Delta(v) = \begin{bmatrix} \mathrm{diag}(v^R) & i.\mathrm{diag}(v^I) \end{bmatrix} \in \mathbb{C}^{n \times 2n}$.

...

Design

Theorem 1: High-Gain Interval Observer (HGIO)

...

Moreover, the following inclusion properties hold:

$$\begin{aligned} \forall t \in \mathbb{R}^+, z^r(t) \geq 0 \wedge z(t) \in z^c(t) \pm z^r(t) \subset \mathbb{C}^{n_x} \\ \forall t \in \mathbb{R}^+, \hat{x}^r(t) \geq 0 \wedge \hat{x}(t) \in \hat{x}^c(t) \pm \hat{x}^r(t) \subset \mathbb{R}^{n_x} \end{aligned} \quad (9)$$

with:

$$\forall t \in \mathbb{R}^+, \hat{x}^c(t) = \Omega^{-1}(t) z^c(t), \hat{x}^r(t) = \Omega^{-1}(t) \diamond z^r(t) \quad (10)$$

where $A \diamond B = |A||B| + 2|A^I||B^I|$.

Stability Analysis

Upper error dynamics:

$$\dot{\overline{e}} = \dot{z}^c + \dot{z}^r - \dot{z}$$
$$= \mathrm{diag}(\theta\xi)e + \Omega E d^c + |\Omega E d^r| - \Omega E \hat{d}$$
$$+ |\Omega[\ldots A_i \Omega^{-1} z^c \ldots]| \mathbf{1} + |\Omega[\ldots A_i \Omega^{-1} \Delta(z^r) \ldots]| \mathbf{1} - \Omega \Sigma_{i=1}^{n_r} A_i \rho_i \Omega^{-1} z$$
$$+ |\Omega[\ldots B_i u \ldots]| \mathbf{1} - \Omega \Sigma_{i=1}^{n_r} B_i \rho_i u$$

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&+ |\Omega[\ldots A_i \Omega^{-1} z^c \ldots]|\mathbf{1} + |\Omega[\ldots A_i \Omega^{-1} \Delta(z^r) \ldots]|\mathbf{1} - \Omega\Sigma_{i=1}^{n_r} A_i \rho_i \Omega^{-1} z \\
&+ |\Omega[\ldots B_i u \ldots]|\mathbf{1} - \Omega\Sigma_{i=1}^{n_r} B_i \rho_i u
\end{aligned}
$$

$\overline{e}(0) > 0 \Rightarrow \overline{e}(t) > 0$ (same for $\underline{e}(t) > 0$)

Cooperativity of the errors dynamics $\Rightarrow z(t) \in z^c(t) \pm z^r(t)$

Stability Analysis

HGIO radius dynamics:

$$\dot{z}^r(t) = \text{diag}(\theta\xi)z^r(t) + \Psi^r(t)$$

with $\Psi^r(t) = |\Omega(t)[\dots A_i\Omega^{-1}(t)z^c(t) + B_i u(t)\dots, \dots A_i\Omega^{-1}(t)\Delta(z^r(t))\dots, Ed^r]|\mathbf{1}.$

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Sufficient condition

Proposition 1: HGIO Radius Non-Divergence

Let $H = \sum_{i=1}^{q} \|vA_iv^{-1}\| \in (\mathbb{R}^+)^{n_x}$. If $\text{diag}(\theta\xi)$ is Hurwitz and if the Metzler matrix $[\text{diag}(\theta\xi) + H, H; H, \text{diag}(\theta\xi) + H]$ is Hurwitz, then $\forall t \in \mathbb{R}^+, 0 \leq z^r(t) \leq \bar{z}^r(t)$ where $\bar{z}^r(t)$ follows a stable dynamics and so is $z^r(t)$.

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- *Measurement noise amplification*: one drawback of HGOs
- Several methods to overcome this issue [Astolfi et al. 2021] [Zemouche et al. 2019] ...

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- Several methods to overcome this issue [Astolfi et al. 2021] [Zemouche et al. 2019] ...
- An output observation error filter is incorporated in the HGO as in [Tréangle et al. 2019]:

$$\dot{\tilde{x}}(t) = M\tilde{x}(t) + \Phi(u(t), \tilde{x}(t)) + N\tilde{u}(t) \tag{11}$$

where $\tilde{x}(t) = \begin{bmatrix} \hat{x}^T(t) & \eta^T(t) \end{bmatrix}^T$, $\tilde{u}(t) = \begin{bmatrix} \hat{d}^T(t) & y^T(t) \end{bmatrix}^T$, $M = \begin{bmatrix} A_0 & \bar{L} \\ -\theta C & D \end{bmatrix}, \ldots$

$\eta \in \mathbb{R}^{n_y}$ is the filter state,

$\bar{L}$ and D are the observation and filter gains.

Design

FHGIO design procedure stays quite similar to the HGIO.

Three steps to follow:

1. Gains design of $\bar{L}$ and D
   - M Hurwitz with distinct poles $\theta(\bar{\xi} + i\bar{\zeta})$ where $\theta \geq 1$
   - $\min_{\omega \geq 0} \sigma_{min}(M - i\omega I_{n_x+n_y}) > \gamma_\Phi$

     $\rightarrow$ Stability and M is $\mathbb{C}$-diagonalizable as $\bar{v}\text{diag}(\theta\bar{\xi} + i\theta\bar{\zeta})v^{-1}$

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2. Change of coordinate
   - $z(t) = \bar{\Omega}(t)\tilde{x}(t)$ with $\bar{\Omega}(t) = \text{diag}(e^{-i\theta\bar{\zeta}t})\bar{v}^{-1}$
   - $\dot{\tilde{z}}(t) = \text{diag}(\theta\bar{\xi})\tilde{z}(t) + \bar{\Omega}(t)(\phi(u(t), \bar{\Omega}^{-1}(t)\tilde{z}(t)) + N\tilde{u}(t))$

     $\rightarrow$ Ensure the cooperativity of the errors dynamics in the new base

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     - $\rightarrow$ Ensure the cooperativity of the errors dynamics in the new base
3. Derivation of the interval observer structure
   - Use the bounds of the uncertainties
   - Center and radius dynamics

Design

Theorem 2: Filtered High-Gain Interval Observer (FHGIO)

Consider the FHGO (11) and Assumptions 1-3. The system:

$$\begin{cases} \dot{\tilde{z}}^c(t) = \mathrm{diag}(\theta\bar{\xi})\tilde{z}^c(t) + \bar{\Psi}^c(t) \\ \dot{\tilde{z}}^r(t) = \mathrm{diag}(\theta\bar{\xi})\tilde{z}^r(t) + \bar{\Psi}^r(t) \end{cases} \tag{12}$$

is an interval observer for (11), named FHGIO, with:

$$\begin{cases} \bar{\Psi}^c(t) = \bar{\Omega}(t)(\Phi^c(u(t)) + N\tilde{u}^c(t)) \\ \bar{\Psi}^r(t) = |\bar{\Omega}(t)[\dots \Phi_i^r(u(t), \bar{\Omega}^{-1}\tilde{z}^c(t), \bar{\Omega}^{-1}\Delta(\tilde{z}^r(t)) \dots, N\tilde{u}^r]|\mathbf{1} \end{cases}$$

$\dots$

Design

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Moreover, the following inclusion properties hold:

$$\forall t \in \mathbb{R}^+, \tilde{z}^r(t) \geq 0 \wedge \tilde{z}(t) \in \tilde{z}^c(t) \pm \tilde{z}^r(t) \subset \mathbb{C}^{n_x+n_y}$$
$$\forall t \in \mathbb{R}^+, \tilde{x}^r(t) \geq 0 \wedge \tilde{x}(t) \in \tilde{x}^c(t) \pm \tilde{x}^r(t) \subset \mathbb{R}^{n_x+n_y} \quad (13)$$

with:

$$\forall t \in \mathbb{R}^+, \tilde{x}^c(t) = \bar{\Omega}^{-1}(t)\tilde{z}^c(t), \tilde{x}^r(t) = \bar{\Omega}^{-1}(t) \diamond \tilde{z}^r(t) \quad (14)$$

Stability Analysis

Cooperative errors dynamics $\Rightarrow$ inclusion property

FHGIO radius dynamics:

$$\dot{\tilde{z}}^r(t) = \text{diag}(\theta\bar{\xi})\tilde{z}^r(t) + \bar{\Psi}^r(t)$$

with $\bar{\Psi}^r(t) = |\bar{\Omega}(t)[\ldots \Phi_i^r(u(t), \bar{\Omega}^{-1}\tilde{z}^c(t), \bar{\Omega}^{-1}\Delta(\tilde{z}^r(t)) \ldots, N\tilde{u}^r]|\mathbf{1}$.

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Sufficient condition

Proposition 2: FHGIO Radius Non-Divergence

Let $\bar{H} = \sum_{i=1}^{q} \|\bar{v}\bar{A}_i\bar{v}^{-1}\| \in (\mathbb{R}^+)^{n_x+n_y}$. If $\text{diag}(\theta\bar{\xi})$ is Hurwitz and if the Metzler matrix $[\text{diag}(\theta\bar{\xi}) + \bar{H}, \bar{H}; \bar{H}, \text{diag}(\theta\bar{\xi}) + \bar{H}]$ is Hurwitz, then $\forall t \in \mathbb{R}^+, 0 \leq \tilde{z}^r(t) \leq \bar{\tilde{z}}^r(t)$ where $\bar{\tilde{z}}^r(t)$ follows a stable dynamics and so is $\tilde{z}^r(t)$.

Numerical example

Consider the LPV system:

$$\begin{cases} \dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) + Ed(t) \\ y(t) = Cx(t) + w(t) \end{cases} \tag{15}$$

where

$A_0 = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}$, $A_1 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $B_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 0.5 \\ -0.2 \end{bmatrix}$,

$E = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ and $C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Numerical example

Consider the LPV system:

$$\begin{cases} \dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) + Ed(t) \\ y(t) = Cx(t) + w(t) \end{cases} \quad (15)$$

where

$$A_0 = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}, A_1 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0.5 \\ -0.2 \end{bmatrix},$$

$E = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ and $C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

$u(t) = \sin(2\pi t)$ and $\rho(t) = \cos(2\pi 0.5t)$.
$d(t) = d^c + d^r \sin(2\pi 0.2t)$ with $d^c = 0$ and $d^r = 0.3$.
$w(t) = w^c + w^r \sin(2\pi 200t)$ with $w^c = 0$ and $w^r = 0.5$.
$x(0) = [0, 0, 0]^T$, $x^c(0) = [0, 0, 0]^T$ and $x^r(0) = [0.1, 0.1, 0.1]^T$.

Results

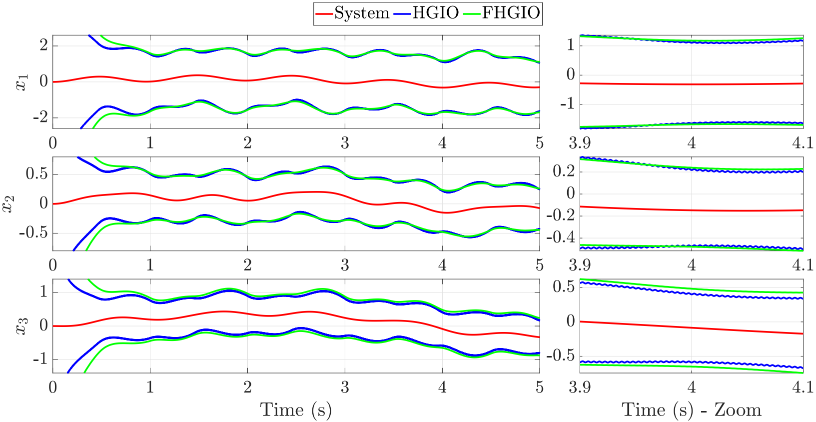

Figure: HGIO and FHGIO state estimations for $\theta = 10$

Results

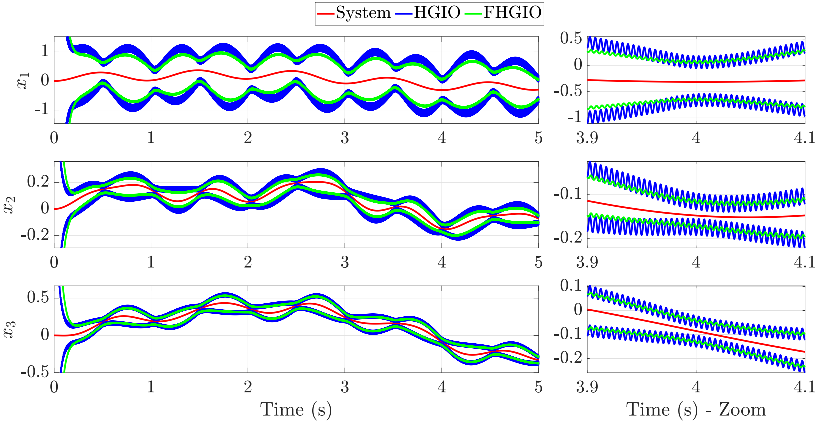

Figure: HGIO and FHGIO state estimations for $\theta = 30$

Comparison

For an LPV system:

$$\begin{cases} \dot{x} = (A_0 + \Delta A(\rho(t)))x + b(t) \\ y = Cx + v(t) \end{cases} \tag{16}$$

L_2 Interval Observer from [Chebotarev et al. 2015]:

$$\begin{cases} \dot{\underline{x}} = (A_0 - \underline{L}C)\underline{x} + (\underline{\Delta A}^+\underline{x}^+ - \overline{\Delta A}^+\underline{x}^- - \underline{\Delta A}^-\overline{x}^+ + \overline{\Delta A}^-\underline{x}^-) + \underline{L}y - |\underline{L}|VE_p + \underline{b}(t) \\ \dot{\overline{x}} = (A_0 - \overline{L}C)\overline{x} + (\overline{\Delta A}^+\overline{x}^+ - \underline{\Delta A}^+\overline{x}^- - \overline{\Delta A}^-\underline{x}^+ + \underline{\Delta A}^-\underline{x}^-) + \overline{L}y + |\overline{L}|VE_p + \overline{b}(t) \end{cases} \tag{17}$$

$\underline{L}$ and $\overline{L}$ such that $\begin{bmatrix} \underline{b} \\ \overline{b} \end{bmatrix} \to \begin{bmatrix} \underline{x} \\ \overline{x} \end{bmatrix}$ has a L_2 norm less then γ.

Comparison

For an LPV system:

$$\begin{cases} \dot{x} = (A_0 + \Delta A(\rho(t)))x + b(t) \\ y = Cx + v(t) \end{cases} \quad (16)$$

L_2 Interval Observer from [Chebotarev et al. 2015]:

$$\begin{cases} \dot{\underline{x}} = (A_0 - \underline{L}C)\underline{x} + (\underline{\Delta A}^+ \underline{x}^+ - \overline{\Delta A}^+ \underline{x}^- - \underline{\Delta A}^- \overline{x}^+ + \overline{\Delta A}^- \underline{x}^-) + \underline{L}y - |\underline{L}|VE_p + \underline{b}(t) \\ \dot{\overline{x}} = (A_0 - \overline{L}C)\overline{x} + (\overline{\Delta A}^+ \overline{x}^+ - \underline{\Delta A}^+ \overline{x}^- - \overline{\Delta A}^- \underline{x}^+ + \underline{\Delta A}^- \underline{x}^-) + \overline{L}y + |\overline{L}|VE_p + \overline{b}(t) \end{cases} \quad (17)$$

$\underline{L}$ and $\overline{L}$ such that $\begin{bmatrix} \underline{b} \\ \overline{b} \end{bmatrix} \to \begin{bmatrix} \underline{x} \\ \overline{x} \end{bmatrix}$ has a L_2 norm less then γ.

Example with:

$$A_0 + \Delta A(\rho(t)) = \begin{bmatrix} \epsilon\cos(t) & 1 + \epsilon\sin(x_3) & \varepsilon\sin(x_2) \\ \varepsilon\sin(x_3) & -0.5 + \epsilon\sin(t) & 1 + \epsilon\cos(2t) \\ \varepsilon\sin(x_2) & 0.3 + \varepsilon\cos(2t) & -1 + \epsilon\sin(t) \end{bmatrix}, \quad \begin{matrix} \epsilon = 0.01 \\ \varepsilon = 0.001 \end{matrix}$$

$C = [1 \ \ 0 \ \ 0]$, $b(t) = \begin{bmatrix} 6\cos(x_1) \\ \sin(t) + 0.1\sin(x_3) \\ -\cos(3t) + 0.1\sin(2x_2) \end{bmatrix}$ and $v(t) = 0.1(sin(5t) + cos(3t))/2$.

Results

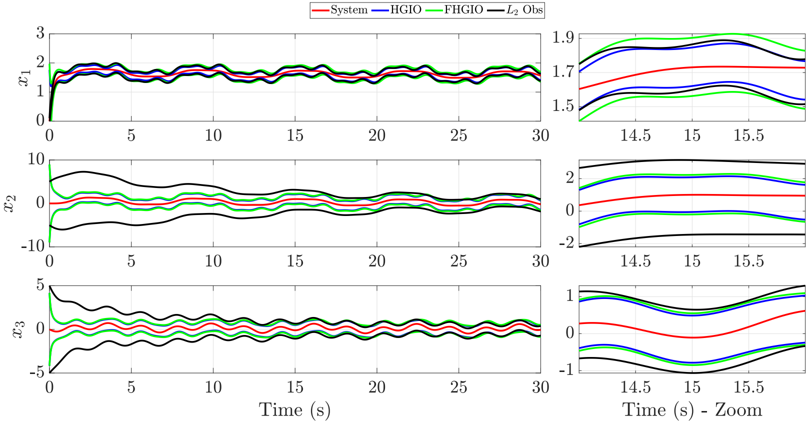

Figure: Comparison bewteen HGIO/FHGIO and the L_2 Interval Observer

Conclusion and perspectives

Accepted conference article [Hugo et al. 2023]

Conclusion

- Design of a High-Gain Interval Observer:
  - Arbitrarily fast convergence and width tuned by the high-gain parameter
  - Design of gain and change of coordinates provided
  - Sufficient condition given for the radius non-divergence
- Extension to a Filtered High-Gain Interval Observer:
  - Observation error filter incorporated in the design
  - Measurement noise amplification mitigated

Conclusion and perspectives

Accepted conference article [Hugo et al. 2023]

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Perspectives

- Extension to systems with sampled and delay measurements
- Application to real systems such as autonomous vehicles

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Thank You

Questions? Comments?