Interval estimation for continuous-time LPV switched systems

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Outline

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Context

Switching systems

The topic of diagnosis of complex systems is an important issue in many engineering fields

- Automotive
- Metallurgy
- Aerospace industries

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The class of switching systems is one of important classes of hybrid systems. They involve

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- discrete dynamics

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- continuous
- discrete dynamics

Definition 1

Switched system consists of a finite number of continuous dynamical subsystems combined with a discrete rule that operates switching between these subsystems.

Interval estimation

Switched systems have been studied in the frame of

- Stability
- Stabilization
- Observation

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Remark 1

The problem of unmeasurable state vector estimation is very challenging

Remark 2

A conventional estimator is not possible when the system is subject to uncertainties

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A conventional estimator is not possible when the system is subject to uncertainties

Solution

Interval estimation
[Mazenc et al., 2014, Raïssi et al., 2012, Chebotarev et al., 2015, Efimov et al., 2013a, Wang et al., 2015, Ifqir et al., 2017]

LPV systems

Non-linear systems

Some state estimation methods are based on the approximate linearization which can lead to an unprecedented level of obstruction in practice
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A broad class of nonlinear systems can be presented in a LPV form
[Lee, 1997, Shamma and Xiong, 1999, Marcos and Balas, 2004, Hecker and Varga, 2004, Heemels et al., 2010]

Interval observer / Stability

State estimation based on interval methods had been proposed for :

- Time-invariant and parameter-varying systems
- LPV systems with parametric uncertainty
- Linear time-invariant switched systems with disturbances

References

[Mazenc et al., 2014, Raïssi et al., 2012, Efimov et al., 2013b, Wang et al., 2015, Lamouchi et al., 2018, Ethabet et al., 2017, Ifqir et al., 2017]

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Stability had been treated by :

- Common Lyapunov Functions
- Multiple Lyapunov Functions

References

[Liberzon and Morse, 1999, Narendra and Balakrishnan, 1994, Hespanha and Morse, 1999, Niu and Zhao, 2011]

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Input-to-State Stability and cooperativity of the upper and lower observation errors are ensured

Outline

LPV switched system

Given a system described by:

$$\begin{cases} \dot{x}(t) = A_q(\eta_q)x(t) + B_q(\eta_q)u(t) + w_q(t) \\ y(t) = Cx(t) + v(t) \end{cases}, q \in \mathcal{I} \qquad (1)$$

- $A_q \in \mathbb{R}^{n\times n}$, $B_q \in \mathbb{R}^{n\times l}$ and $C \in \mathbb{R}^{m\times n}$
- q is the index of the active subsystem and assumed to be known
- N is the number of subsystems
- $w_q \in \mathbb{R}^n$ is the state disturbance
- $v \in \mathbb{R}^m$ is the measurement noise.
- $\eta_q = [\eta_{q_1}, ..., \eta_{q_r}]^T$ the collection of measured time varying parameters.

Assumptions

We assume that the matrices $A_q(\eta_q)$, $B_q(\eta_q)$ depend affinely on η_q:

$$\begin{array}{l} A_q(\eta_q) = A_{q0}(\eta_q) + \eta_{q1}A_{q1} + ... + \eta_{qr}A_{qr} \\ B_q(\eta_q) = B_{q0}(\eta_q) + \eta_{q1}B_{q1} + ... + \eta_{qr}B_{qr} \end{array}, q \in \mathcal{I}$$

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Assumption 1

The measurement noise and the state disturbance are assumed to be unknown but bounded with a priori known bounds such that:

$$\underline{w}_q \leq w_q \leq \overline{w}_q, |v(t)| \leq \overline{v}J_m$$

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Assumption 2

$\eta_q = [\eta_{q_1}, ..., \eta_{q_r}]^T$ the collection of measured time varying parameters are constrained in polytopes E_q; E_q depend on the active mode. We denote by $\eta_q^{(i)}$, $i = 1, ..., g$ the vertices of each E_q.

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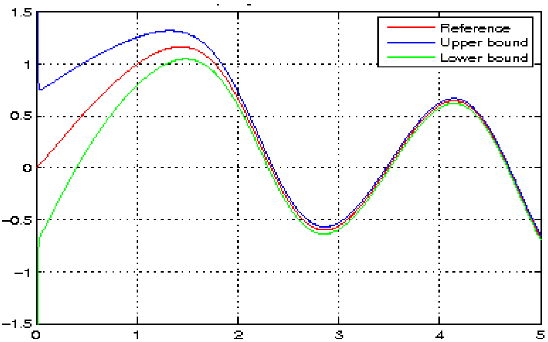
Assumption 3

For all vertices of E_q and for all $q \in \mathcal{I}$, the pairs $(A_q(\eta_q), C)$ are detectable.

- The aim is to estimate two states, an upper state $\overline{x}$ and a lower one $\underline{x}$ such that the solution of the system is between two trajectories without crossing each other and under the assumption that the initial condition x_0 verifies $\underline{x}_0 \leq x_0 \leq \overline{x}_0$ with known $\underline{x}_0, \overline{x}_0 \in \mathbb{R}^n$.



Outline

Interval design for LPV switched system

Interval observer structure

$$\begin{cases} \dot{\overline{x}} = (A_q(\eta_q) - L_q(\eta_q)C)\,\overline{x} + B_q(\eta_q)u + \overline{w}_q + L_q(\eta_q)y + |L_q|\overline{v}J_m \\ \dot{\underline{x}} = (A_q(\eta_q) - L_q(\eta_q)C)\,\underline{x} + B_q(\eta_q)u + \underline{w}_q + L_q(\eta_q)y - |L_q|\overline{v}J_m \end{cases}, q \in \mathcal{I} \quad (2)$$

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The observer gain $L_q(\eta_q)$ has an affine form shown below:

- $L_q(\eta_q) = L_{q0} + \eta_{q1}L_{q1} + \ldots + \eta_{qr}L_{qr}$

where $L_{qj} \in \mathbb{R}^{n\times m}$, $j = 0, 1, \ldots, r$, are constant matrices.

Goal

The observer gains L_q are sought to guarantee that $A_q(\eta_q) - L_q(\eta_q)C$ are Metzler matrices.

Cooperative systems

Lemma 1

Consider the system described by

$$\dot{x}(t) = Ax(t) + u(t) \tag{3}$$

If A is Metzler, the input u verifies $u(t) \geq 0$ and the initial condition x_0 is chosen as $x_0 \geq 0$, then the state x stays nonnegative for all $t \geq 0$. The system (3) is said to be cooperative or nonnegative

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Definition 2

A matrix $A \in \mathbb{R}^{n \times n}$ is called Metzler if there exists $\epsilon \in \mathbb{R}^{+}$ such that

$$A + \epsilon I_n \geq 0 \quad , \quad \forall q \in \mathcal{I}$$

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The interval observer should verify two conditions:

1. Cooperativity: $\underline{x}(t) \leq x(t) \leq \overline{x}(t), \forall t \geq t_0$
2. Stability of $\underline{e} = x - \underline{x}$ and $\overline{e} = \overline{x} - x$

Interval design for LPV switched system

Theorem 1/2

Consider the continuous-time LPV switched system (1), where $A_q(\eta_q)$ and $B_q(\eta_q)$ are affine matrices of η_q, and η_q is supposed to be measured. If there exist matrices P and $Q_q(\eta_q^{(i)})$ that satisfy the conditions:

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(1) $P \in \mathbb{R}^{n\times n}$ is a diagonal positive definite matrix;

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(1) $P \in \mathbb{R}^{n \times n}$ is a diagonal positive definite matrix;

(2) The Metzler property of $A_q(\eta_q) - L_q(\eta_q)C$ is satisfied $\forall\, \eta_q^{(i)}$

$$PA_q(\eta_q^{(i)}) + Q_q(\eta_q^{(i)})C + \epsilon P \geq 0 \quad , \quad \forall q \in \mathcal{I} \tag{4}$$

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(3) The LMI conditions shown in (5) are feasible for all the vertices $\eta_q^{(i)}$ of E_q , $i = 1, ..., g$

$$A_q{}^T(\eta_q^{(i)})P + PA_q(\eta_q^{(i)}) - [C^T Q_q^T(\eta_q^{(i)}) + Q_q(\eta_q^{(i)})C] < 0, \ \forall q \in \mathcal{I} \ , \tag{5}$$

Interval design for LPV switched system

Theorem 2/2

$Q_q(\eta_q^{(i)})$ are affine matrices of $\eta_q^{(i)}$ given by

$$Q_q(\eta_q^{(i)}) = Q_{q0} + \eta_1^{(i)} Q_{q1} + \dots + \eta_r^{(i)} Q_{qr} \tag{6}$$

where $Q_{qj} \in \mathbb{R}^{n \times m}, j = 0, 1, \dots, r$ are constant matrices,

then the observer gains $L_{qj}, j = 0, 1, \dots, r$ can be obtained as:

$$L_{qj} = P^{-1} Q_{qj} \tag{7}$$

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Using the proposed theorem:

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Interval design for LPV switched system

Using the proposed theorem:

- The observer gain $L_q(\eta_q)$ is calculated in real time.
- The stability is ensured by a common Lyapunov function.
- By assuming that the scheduling vector is described by a convex combination and its parametric uncertainties belongs to polytopes, we will prove that resolution of LMIs becomes less conservative.

Interval design for LPV switched system

Proof

- $A_q(\eta_q)$ depends affinely of $\eta_q \Rightarrow A_q(\eta_q)$ can be written as a convex combination form [Hetel et al., 2006].

$$\Rightarrow \quad A_q(\eta_q) = \lambda_1 A_q({\eta_q}^{(1)}) + \ldots + \lambda_g A_q({\eta_q}^{(g)}) = \sum_{i=1}^{g} \lambda_i A_q(\eta_q^{(i)})$$

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where $\lambda_i \geq 0$ and $\lambda_1 + \ldots + \lambda_g = 1$

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- $A_q({\eta_q}^{(i)})$ represent the vertices of the state matrices of each polytope E_q

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- $A_q({\eta_q}^{(i)})$ represent the vertices of the state matrices of each polytope E_q
- $L_q({\eta_q}^{(i)})$ represent the vertices of the observer gain.

Interval design for LPV switched system

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$$\begin{aligned} \dot{\overline{e}}(t) = \dot{\overline{x}} - \dot{x} &= (\sum_{i=1}^{g} \lambda_i [(A_q({\eta_q}^{(i)}) - L_q({\eta_q}^{(i)})C)])\overline{e} + \overline{\chi}_q \\ \dot{\underline{e}}(t) = \dot{x} - \dot{\underline{x}} &= (\sum_{i=1}^{g} \lambda_i [(A_q({\eta_q}^{(i)}) - L_q({\eta_q}^{(i)})C)])\underline{e} + \underline{\chi}_q \end{aligned} \quad (8)$$

where:

$$\begin{aligned} \overline{\chi}_q &= \overline{w}_q - w_q + L_q(\eta_q)v + |L_q|\overline{v}J_m \\ \underline{\chi}_q &= w_q - \underline{w}_q - L_q(\eta_q)v + |L_q|\overline{v}J_m \end{aligned} \quad (9)$$

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- $\overline{w}_q \geq |w_q|$ and $\overline{v} \geq |v| \Rightarrow$ $\overline{\chi}_q(t) \geq 0$, $\underline{\chi}_q(t) \geq 0$.

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1 Cooperativity

Define the estimation errors $\overline{e}(t) = \overline{x} - x$ and $\underline{e}(t) = x - \underline{x}$

$$\begin{aligned} \dot{\overline{e}}(t) = \dot{\overline{x}} - \dot{x} &= (\sum_{i=1}^{g} \lambda_i[(A_q({\eta_q}^{(i)}) - L_q({\eta_q}^{(i)})C)])\overline{e} + \overline{\chi}_q \\ \dot{\underline{e}}(t) = \dot{x} - \dot{\underline{x}} &= (\sum_{i=1}^{g} \lambda_i[(A_q({\eta_q}^{(i)}) - L_q({\eta_q}^{(i)})C)])\underline{e} + \underline{\chi}_q \end{aligned} \quad (8)$$

where:

$$\begin{aligned} \overline{\chi}_q &= \overline{w}_q - w_q + L_q(\eta_q)v + |L_q|\overline{v}J_m \\ \underline{\chi}_q &= w_q - \underline{w}_q - L_q(\eta_q)v + |L_q|\overline{v}J_m \end{aligned} \quad (9)$$

- If there exist $\epsilon \in \mathbb{R}^+$ such that $A_q({\eta_q}^{(i)}) - L_q({\eta_q}^{(i)})C + \epsilon I_n \geq 0 \Rightarrow$ $A_q(\eta_q^{(i)}) - L_q(\eta_q^{(i)})C$ are Metzler matrices
- $\overline{w}_q \geq |w_q|$ and $\overline{v} \geq |v| \Rightarrow \overline{\chi}_q(t) \geq 0$, $\underline{\chi}_q(t) \geq 0$.

$\Rightarrow$ It follows that $\overline{e}(t) \geq 0$ and $\underline{e}(t) \geq 0 \Rightarrow \underline{x}(t) \leq x(t) \leq \overline{x}(t)$.

Interval design for LPV switched system

2 Stability

Interval design for LPV switched system

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Consider the Lyapunov function $V(\bar{e}) = \bar{e}(t)^T P \bar{e}(t)$ with $P = P^T > 0$.
The derivative of V is given by:

$$\begin{aligned}\dot{V}(\bar{e}) &= \bar{e}^T[(A_q(\eta_q) - L_q(\eta_q)C)^T P + P(A_q(\eta_q) - L_q(\eta_q)C)]\bar{e} - \\ &\quad 2\bar{e}^T P w_q + 2\bar{e}^T P L_q(\eta_q) v + 2\bar{e}^T P \overline{w}_q + 2\bar{e}^T P |L_q| \overline{v} J_m\end{aligned}$$

Lemma

Consider two vectors $u,\ v \in \mathbb{R}^n$, then:

$$2u^T M v \leq \frac{1}{\varrho} u^T M u + \varrho v^T M v$$

holds for any constant $\varrho > 0$ and any positive definite matrix M.

- The derivative of V satisfies:

$$\dot{V}(\bar{e}) \leq \bar{e}^T B_1 \bar{e} + C_1$$

Interval design for LPV switched system

where:

$$B_1 = (A_q(\eta_q) - L_q(\eta_q)C)^T P + P(A_q(\eta_q) - L_q(\eta_q)C)$$

Interval design for LPV switched system

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$$B_1 = (A_q(\eta_q) - L_q(\eta_q)C)^T P + P(A_q(\eta_q) - L_q(\eta_q)C)$$

$$C_1 = -\varrho_q w_q^T P w_q + \varrho_q \overline{w}_q^T P \overline{w}_q + \varrho_q v^T {L_q}^T P L_q v + \varrho_q {J_m}^T \overline{v} \mid L_q \mid^T P \mid L_q \mid \overline{v} J_m$$

Interval design for LPV switched system

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- $B_1 < 0$

Interval design for LPV switched system

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- $B_1 < 0$
- Under the assumption that the uncertainties w_q and v are bounded, C_1 is also bounded, the system (8) is ISS and the upper and lower error estimation are bounded.

Interval design for LPV switched system

Results of the contribution

- Stability and cooperativity properties have been relaxed thanks to the polytopic shape of the system parameters, the observer has been modeled taking into account the uncertain state matrix and not its upper and lower bounds.

Interval design for LPV switched system

Results of the contribution

- Stability and cooperativity properties have been relaxed thanks to the polytopic shape of the system parameters, the observer has been modeled taking into account the uncertain state matrix and not its upper and lower bounds.
- LMIs and cooperativity conditions are expressed on the vertices of each polytope in order to avoid any infinite dimensional problem due to the time varying measured parameters.

Outline

Consider the LPV switched system (1) with:

- $N = 3$

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- $\eta_q = [\eta_{q1} \ \eta_{q2}]^T$

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Based on the representation Eq (10), matrices of the system are chosen as bellow :

$$A_{10}(\eta_1) = \begin{bmatrix} -10\eta_{12} & 0.1 \\ -3 & -\eta_{12} \end{bmatrix},\ A_{11} = \begin{bmatrix} -0.5 & 1.5 \\ -0.5 & -1 \end{bmatrix},\ A_{12} = \begin{bmatrix} 2 & -0.5 \\ -0.5 & -1.5 \end{bmatrix}$$

$$B_{10}(\eta_1) = \begin{bmatrix} -\eta_{12} & 1 \\ 1 & 0 \end{bmatrix},\ B_{11} = \begin{bmatrix} -2 & 2 \\ 1 & 0.5 \end{bmatrix},\ B_{12} = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

$$A_{20}(\eta_2) = \begin{bmatrix} -15\eta_{22} & 10 \\ -2 & -3\eta_{21} \end{bmatrix},\ A_{21} = \begin{bmatrix} -0.5 & -2 \\ 1.5 & -2 \end{bmatrix},\ A_{22} = \begin{bmatrix} 0.5 & 2 \\ -1 & -1.4 \end{bmatrix}$$

$$B_{20}(\eta_2) = \begin{bmatrix} -\eta_{22} & 2 \\ 1.5 & 0 \end{bmatrix},\ B_{21} = \begin{bmatrix} -1.5 & 1 \\ 1 & 0.5 \end{bmatrix},\ B_{22} = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

$$A_{30}(\eta_3) = \begin{bmatrix} -3.5\eta_{32} & 5 \\ -10 & -2\eta_{31} \end{bmatrix},\ A_{31} = \begin{bmatrix} -0.5 & 1.5 \\ -0.5 & -1 \end{bmatrix},\ A_{32} = \begin{bmatrix} 2 & -0.5 \\ -0.5 & -1.5 \end{bmatrix}$$

$$B_{30}(\eta_3) = \begin{bmatrix} -2\eta_{32} & 1.5 \\ 1 & 0 \end{bmatrix},\ B_{31} = \begin{bmatrix} -1 & 2.5 \\ 1 & 0.75 \end{bmatrix},\ B_{32} = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

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- $u = [1 \quad 1]^T$ is the known input

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- $v(t) = 0.09cos(2t)$

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- The state initial conditions are set as $x(0) = [0, 0]^T$ such that :

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$$\underline{x}(0) \leq x(0) \leq \overline{x}(0)$$

- The measured time varying parameters η_q for $q = 1, 2, 3$ are defined by:

$$\eta_1(t) = \begin{pmatrix} |sin(t)| + 2 \\ |cos(2t)| + 2 \end{pmatrix} \quad \eta_2(t) = \begin{pmatrix} |2cos(0.8t)| + 2 \\ |2cos(2t)| + 2 \end{pmatrix} \quad \eta_3(t) = \begin{pmatrix} |3sin(3t)| \\ |3cos(t)| \end{pmatrix}$$

The Lyapunov matrix is given by:

$$P = \begin{bmatrix} 2.50 & 0 \\ 0 & 3.88 \end{bmatrix}$$

The observer gains $L_q(\eta_q)$ are computed using the expression (7)

$$L_{10} = \begin{pmatrix} -8.1 & 6.33 \end{pmatrix}^T, \quad L_{11} = \begin{pmatrix} 21.46 & -12.87 \end{pmatrix}^T, \quad L_{12} = \begin{pmatrix} 2.47 & -5.28 \end{pmatrix}^T$$

$$L_{20} = \begin{pmatrix} 21.06 & -11.45 \end{pmatrix}^T, \quad L_{21} = \begin{pmatrix} -5.18 & 6.31 \end{pmatrix}^T, \quad L_{22} = \begin{pmatrix} -3.93 & -3.83 \end{pmatrix}^T$$

$$L_{30} = \begin{pmatrix} 19.97 & -13.43 \end{pmatrix}^T, \quad L_{31} = \begin{pmatrix} -4.57 & 3.87 \end{pmatrix}^T, \quad L_{32} = \begin{pmatrix} -0.57 & 0.85 \end{pmatrix}^T$$

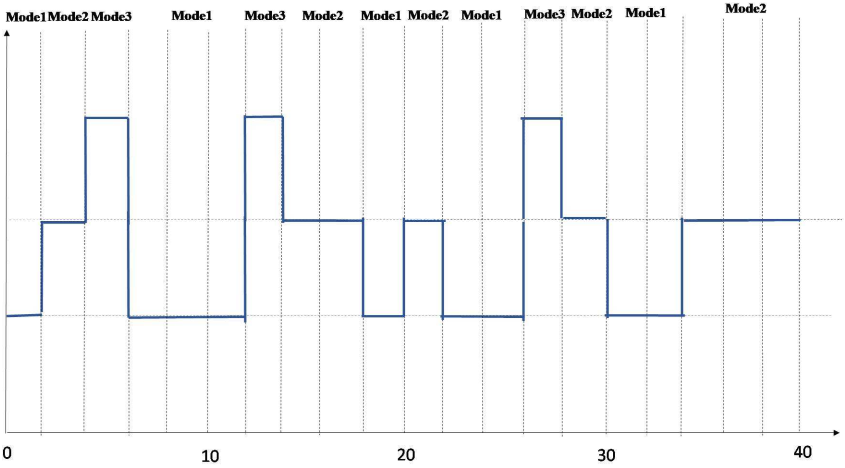

Figure: Evolution of the switching signal

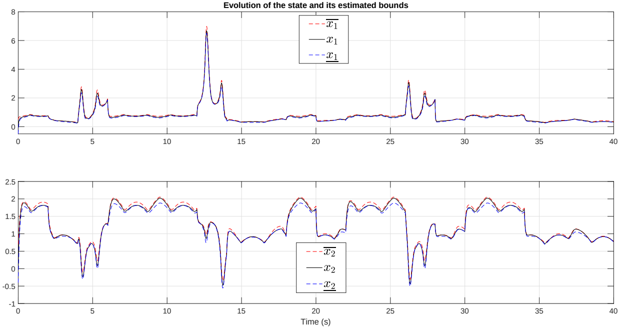

Figure: Evolutions of the state x and the estimated upper and lower bounds $\overline{x}$ and $\underline{x}$.

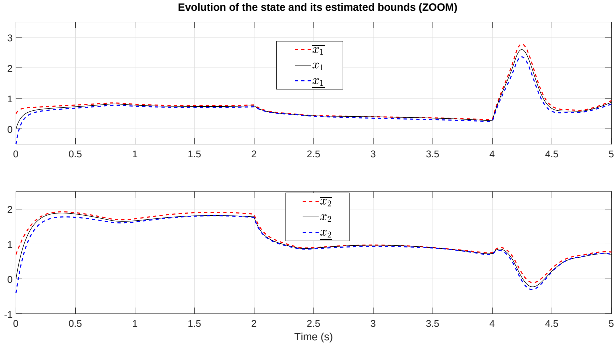

Figure: Evolutions of the state x and the estimated upper and lower bounds $\overline{x}$ and $\underline{x}$ (ZOOM).

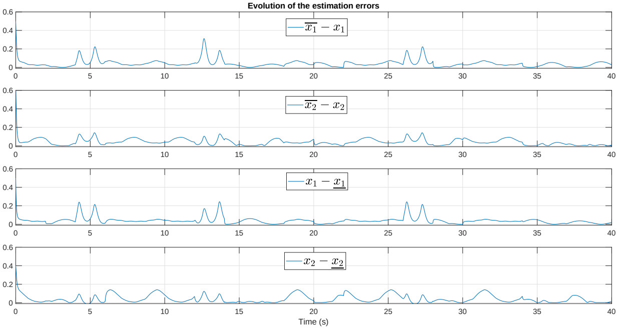

Figure: Evolutions of the etimatation errors

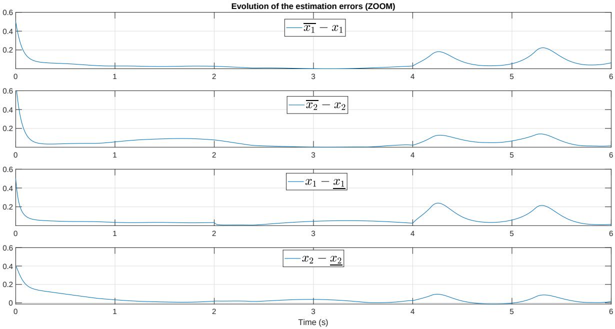

Figure: Evolutions of the estimation errors(ZOOM)

Outline

Conclusions

- An interval state estimation for continuous-time LPV switched system with polytopic parametric uncertainties has been developed.

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- Upper and lower bounds of the state has been determined in order to guarantee both cooperativity and ISS.

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- An interval state estimation for continuous-time LPV switched system with polytopic parametric uncertainties has been developed.
- Upper and lower bounds of the state has been determined in order to guarantee both cooperativity and ISS.
- The conservatism has been relaxed thanks to the polytopic form of parametric uncertainties.

Thank You For Your Attention

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